Optimal control of an epidemic model of leptospirosis with time delay

Syed farasat saddiq[1] , Muhammad Altaf Khan[2], Saeed Islam[2], Naeam Khalid[1], Syed Inayat Ali Shah[1]

[1]Department of Mathematics, Islamia College University Peshawar, Khyber Pakhtunkhwa, Pakistan.
[2]Department of Mathematics, Abdul Wali Khan University Mardan, Khyber Pakhtunkhwa , Pakistan
altafdir@gmail.com

Abstract: In this paper, we consider a leptospirosis epidemic model with non linear incidences by applying the optimal control techniques and time delay. First we formulate the control epidemic model with time delay and using the control for the infectious host. We want to control the infection in the population and maximize the recovered population. For the eradication of the infection, we use two control variables, to minimize the infection and maximize the population of susceptible and recovered individuals. Find the existence of the control problem and then we characterize the optimal control problem by using the well known method of Pontryagin's Maximum Principle. The numerical simulation of both the system were solved by using backward runge-kutta order four scheme for the solution of the problem numerically.

[Syed farasat saddiq, Muhammad Altaf Khan, Saeed Islam, Naeam Khalid. Syed Inayat Ali Shah *Life Sci J* 2025;22(12):29-36]. ISSN 1097-8135 (print); ISSN 2372-613X (online). http://www.lifesciencesite.com. 03. doi:10.7537/marslsj221225.03

Keywords: Leptospirosis; Pontryagin's Maximum Principle; Time Delay; Optimal control; Numerical simulations

Subject classification: 92D25, 49J15, 93D20

1. Introduction

Mathematical modeling of population dynamics an important research area. Mathematical modeling of infectious disease is one of the important research area now-a-days. The basic and important concern for mathematical models in epidemiology is qualitative analysis, the persistence, performance, asymptotic stability and the existence and uniqueness for the models. Many influential results related to this area have been established and can be found in many articles and books. The first epidemic model for the spread of infectious disease was introduced by [12]. They divided the population in three classes the Susceptible, infected and recovered. They assumed that the susceptible population in a fixed population become infected by contacted with infected individuals, infected individuals either die or recover at a constant rate. Their models consists of three differential equations of ODE's which represent the rate of change in their respective population.

In recent years, some mathematical models incorporating delayed effects have been studied. smith in [24] and Thieme [25] in (1990) derived a scalar delay differential equations for the population of immature and mature age classes. The maturation period is regarded as a time delay. Using the same idea, a system of delayed differential equations for mature population in a patchy environment has been proposed in So et al [26]. More recent studies consider an epidemic model with density dependence to describe disease transmission in variable population size, which can be found in cooke et al [6,8,28]. Zaman et al [21] study an SIR epidemic model with control strategies and using delay. Zaman et al. [10] studied the stability and optimal vaccination of a controlled SIR epidemic model without time delays. Many mathematical models have been proposed to study the optimal control and delay such as [3,5,10,29,30,8,9,11]

In this paper we consider a leptospirosis epidemic model [31] with time delay to prevent the spread of disease by using optimal treatment strategies. In order to do this, we first introduce a control variable representing the optimal treatment for infectious host and set an optimal control for our model. Moreover, we show the existence of an optimal control for this control problem and the infection in a community dies out by using the possible control treatment. We also analyzed the optimal control and optimality system using optimal control techniques. For the numerical simulation we use the real data of Thailand. Our optimal control strategies decrease infection in the population and increase the susceptible and recovered individuals in the population.

The paper is organized as follows. In Section 2 we study the basic model and applying the optimal control and time delay. Find the jacobian and Hamiltonian to show the existence of the proposed model. Numerical Simulation of the model with the complete description of the parameters is discussed in

Section 3. In the last Section 4 we wind up our work with the conclusion and references.

Optimal Control Techniques in Delay Model

To begin the optimal control procedure, it is necessary to have a model which describe the population dynamics. Youshida and Hara [27] considered an SIR model with time delay. We use an epidemic model of leptospirosis disease model to set our optimal control model. We have a population which consists of five differential equations. The system have two categories, Human and Vector. The human population consists of three sub-classes Susceptible S_h Infected I_h and Recovered R_h . The human population is denoted by N_1 with $N_1 = S_h + I_h + R_h$. The vector population is denoted by N_2 consists of two classes that is susceptible S_v and infected I_v , and $N_2 = S_v + I_v$. The model consists of a system of non-linear differential equation is given in the following.

$$\frac{dS_h}{dt} = b_1 - \mu_h S_h - \frac{\beta_1 S_h I(t)}{N_1(t)} - \frac{\beta_2 S_h I_v(t)}{N_1(t)} + \lambda_h R_h,$$

$$\frac{dI_h}{dt} = \frac{\beta_1 S_h I(t)}{N_1(t)} + \frac{\beta_2 S_h I_v(t)}{N_1(t)} - (\mu_h + \delta_h + \gamma_h) I_h(t),$$

$$\frac{dR_h}{dt} = \gamma_h I_h(t) - (\mu_h + \lambda_h) R_h,$$

$$\frac{dS_v}{dt} = b_2 - \gamma_v S_v - \frac{\beta_3 S_v I_h(t)}{N_2(t)},$$

$$\frac{dI_v}{dt} = \frac{\beta_3 S_v I_h(t)}{N_2(t)} - (\gamma_v + \delta_v) I_v,$$

Here b_1 is the birth rate of human population, β_1, β_2, β_3 respectively represent the transmission coefficient between human, susceptible human and infected vector and susceptible vector and infected human.Natural mortality rate of human population is represented by μ_h. λ_h is constant of proportionality where the infected human become susceptible again. Disease death rate for human population is denoted by δ_h. Natural mortality rate of vector population is shown by γ_v. δ_v is the disease death rate of vector. α_1 the parameter measure inhibitory effect of human vector population and α_2 the parameter measure inhibitory effect of human population. b_2 is the birth rate for vector population.

The total dynamics of human population is represented by N_h given by,

$$N_h = b_1 - \mu_h N_1 - \delta_h I_h,$$

and the total dynamics of vector population is denoted by N_h and given by,

$$N_v = b_2 - \gamma_v N_v - \delta_v I_v.$$

Next we apply the optimal control and delay to our proposed model (1), we will derive an optimal control model to fit our control strategy. The theoretical foundation of optimal control models with underlying dynamics given by ordinary differential equations was developed by Pontragin and his co-worker in Moscow in 1950 [9]. So by Pontryagin's Maximum principle, its extensions and appropriate numerical methods we well set an optimal control problem in the time delayed model of leptospirosis disease. Our main goal is to investigate an effective treatment strategy to control infection diseases. We can make an epidemic model which satisfy that the number of infected individuals is not larger than the susceptible population and want to increase the recovered individuals from the infection. The definition of the control variables u_1 and u_2 is given by,

$u_1(t)$ Represents (cover all cuts, water dry, full-cover boots, shoes and long sleeve shirts when handling animals).

$u_2(t)$ Represents (wash hands thoroughly on a regular basis and shower after work),

To do this, we set an optimal control problem, with the control set defined by

$$U = \{(u_1(t), u_2(t)) \in L^2(0,T) : 0 \le u_1(t), u_2(t) \le 1, 0 \le t \le T\}.$$

where $u_1(t), u_2(t)$ is Lebesgue measurable and called a control variable.

$$J_\xi(u) = \int_0^T [(A_0 I_h + A_1 I_v + A_2 S_v) + \frac{1}{2}(\xi_1 u_1^2 + \xi_2 u_2^2)] dt.$$

subject to the control system

$$\frac{dS_h}{dt} = b_1 - \mu_h S_h - \frac{\beta_1 S_h I(t-h)}{N_1(t-h)} - \frac{\beta_2 S_h I_v(t-h)}{N_1(t-h)} + \frac{w u_1(t) I_h(t)}{N_1(t)} + \lambda_h R_h,$$
$$\frac{dI_h}{dt} = \frac{\beta_1 S_h I(t-h)}{N_1(t-h)} + \frac{\beta_2 S_h I_v(t-h)}{N_1(t-h)} - (\mu_h + \delta_h + \gamma_h) I_h(t) - \frac{u_1(t) I_h(t)}{N_1(t)},$$
$$\frac{dR_h}{dt} = \gamma_h I_h(t) - (\mu_h + \lambda_h) R_h + \frac{(1-w) u_1(t) I_h(t)}{N_1(t)},$$
$$\frac{dS_v}{dt} = b_2 - \gamma_v S_v - \frac{\beta_3 S_v I_h(t-h)}{N_2(t-h)} - \grave{o}_1 u_2(t) S_v(t),$$
$$\frac{dI_v}{dt} = \frac{\beta_3 S_v I_h(t-h)}{N_2(t-h)} - (\gamma_v - \delta_v) I_v - \grave{o}_2 u_2(t) I_v(t),$$

$$S_h(0) \ge 0, \quad I_h(0) \ge 0, \quad R_h(0) \ge 0, \quad S_v(0) \ge 0 \quad I_v(0) \ge 0.$$

Here ξ_1, ξ_2 are the positive constants to keep balanced of the sized of infected human individuals, infected vector individuals, susceptible vector individuals, I_h, I_v, S_v at time t and $w \in [0,1]$ and $\grave{o}_1$ and $\grave{o}_2$ are positive constants.

In epidemic dynamics, stability, existence and optimal control theory are important research area. At first we will show the existence of solutions for the control system (6). In this control problem , we assume the restriction on the control variables such that $0 \le u_1, u_2 \le 1$, where $(u_1, u_2) \ge 0$ for all $t \in [0,T]$. The non-negative constants h represents a time delay on the infected individuals I and the total individual human population is N_1 and N_1 during the spread of diseases.

Susceptible individuals acquire infection at a per capit $\beta_1 I_h(t-h) N_1(t-h), \beta_2 I_v(t-h) N_1(t-h).$ In our model the incidence rate is $\beta_1 S_h I_h(t-h) N_1(t-h)$ and $\beta_2 I_v(t-h) N_1(t-h)$ and $\frac{\beta_3 S_v I_h(t-h)}{N_2(t-h)}$. This incidences rate seems more reasonable than $\beta_1 I_h(t) N_1(t), \beta_2 I_v(t) N_1(t)$ because the force of infection is proportional to $\frac{I_h(t-h)}{N_1(t-h)}$ with time delay. Note that in some epidemic models, bilinear incidence rate $\beta_1 S_h(t) I_h(t)$ and standard incidence rate $\beta_1 S_h(t) I_h(t)/N$ are frequently used. Actually the infection probability per contact is likely influenced by the number of infected individual because more infected individuals can increase infection risk. For instance, during SARS outbreak in 2003. The Chinese government did a lot of protection measures and control polices: closing schools, closing restaurants, postponing conferences, isolating infection etc. These actions greatly reduced the contact number per unit time. Then we write the control system (6) in the following form:

$$\frac{dW(t)}{dt} = AW + F(W(t))$$

Where

$$W(t) = \begin{bmatrix} S_h(t) \\ I_h(t) \\ R_h(t) \\ S_v(t) \\ I_v(t) \end{bmatrix},$$

$$\begin{bmatrix} -\frac{\beta_1 S_h I_h(t-h)}{N_1(t-h)} - \frac{\beta_2 S_v I_v(t-h)}{N_1(t-h)} + b_1 + \frac{w u_1 I_h}{N_1} \\ \frac{\beta_1 S_h I_h(t-h)}{N_1(t-h)} + \frac{\beta_2 S_v I_v(t-h)}{N_1(t-h)} - \frac{u_1 I_h}{N_1} \\ \frac{(1-w) u_1 I_h}{N_1} \\ -\frac{\beta_3 S_v I_h(t-h)}{N_2(t-h)} + b_2 \\ \frac{\beta_3 S_v I_h(t-h)}{N_2(t-h)} \end{bmatrix}$$

$$A = \begin{bmatrix} -\mu_h & 0 & \lambda_h & 0 & 0 \\ 0 & -P_2 & 0 & 0 & 0 \\ 0 & \gamma_h & -P_3 & 0 & 0 \\ 0 & 0 & 0 & -\gamma_v - \grave{o}_1 u_2 & 0 \\ 0 & 0 & 0 & 0 & -P_1 - \grave{o}_2 u_2 \end{bmatrix}$$

The second term on the right hand side of equation (8) satisfies,

$$|F(W_1)-F(W_2)| \le C_1(|(S_{1h}(t)-S_{2h}(t))|+C_2|(I_{1h}(t)-I_{2h}(t))|$$
$$+C_3|(R_{1h}(t)-R_{2h}(t))|+C_4|(S_{1v}(t)-S_{2v}(t))|$$
$$+C_5|(I_{1v}(t)-I_{2v}(t))|),$$
$$\le C(|(S_{1h}(t)-S_{2h}(t))|+|(I_{1h}(t)-I_{2h}(t))|+|(R_{1h}(t)-R_{2h}(t))|$$
$$+|(S_{1v}(t)-S_{2v}(t))|+|(I_{1v}(t)-I_{2v}(t))|),$$

where the positive constant $C=\max(C_1,C_2,C_3,C_4,C_5)$ is independent of the state variables . Also we have

$$|G(W_1)-G(W_2)| \le C|W_1-W_2|,$$

where $C=C_1+C_2+C_3+C_4+C_5+\|M\|<\infty.$ So, it follows

that the function G is uniformly Lipschitz continuous.

From the definition of control variables and non-negative initial conditions we

can see that a solution of the system (5) exists see [19].

Now, we consider the control system (6) with the initial conditions (7) to show the existence of

the control problem. Note that for bounded Lebesgue measurable controls and non-negative initial

conditions, non-negative bounded solutions to the state system exists [19].

$$L(I_h,I_v,S_v,u_1,u_2)=(A_0 I_h+A_1 I_v+A_2 S_v)+\frac{1}{2}(\xi_1 u_1^2+\xi_2 u_2^2).$$

We seek for the minimum value of the Lagrangian and the Hamiltonian for the control system is given by

$$M=L(I_h,S_v,I_v,u_1,u_2)+\lambda_1\frac{dS_h}{dt}+\lambda_2\frac{dI_h}{dt}$$
$$+\lambda_3\frac{dR_h}{dt}+\lambda_4\frac{dS_v}{dt}+\lambda_5\frac{dI_v}{dt}.$$

In order to find an optimal control pair, we consider the optimal control problem (6-7). First we have to find the Lagrangian and Hamiltonian for the optimal control problem (6-7). Actually, the Lagrangian of the optimal control problem is given by

Theorem: Let $S_h^*(t), I_h^*(t), R_h^*(t), S_v^*(t)$ and $I_v^*(t)$ be the state variables with associated optimal solutions with the corresponding optimal control variables $u_1^*(t), u_2^*(t)$ for the optimal control problem (4-6). Then there exists adjoint variables $\lambda_i, i=1,2...5.$ satisfying

$$\frac{d\lambda_1}{dt}=\lambda_1(t)(\mu_h+\frac{\beta_1 I_h^*(t-h)}{N_1^*(t-h)}+\frac{\beta_2 I_v^*(t-h)}{N_1^*(t-h)}-\frac{wu_1^*(t)I_h^*(t)}{(N_1^*(t))^2})$$
$$+\lambda_2(t)(\frac{-\beta_1 I_h^*(t-h)}{N_1^*(t-h)}-\frac{\beta_2 I_v^*(t-h)}{N_1^*(t-h)}+\frac{u_1^*(t)I_h^*(t)}{(N_1^*(t))^2}))$$
$$+\lambda_3(t)((1-w)u_1^*(t)\frac{I_h^*(t)}{(N_1^*(t)^2)})-A_o,$$
$$\frac{d\lambda_2}{dt}=\lambda_1(t)(\frac{wu_1^*(T)(s_h^*(t)+R_h^*(t))}{(N_1^*(t))^2}+\lambda_h)$$
$$+\lambda_2(t)(\frac{u_1^*(t)I_h^*(t)^2}{(N_1^*)})+\lambda_3(t)(-\gamma_h+\frac{(1-w)u_1^*(S_h^*+R_h^*)}{(N_1^*(t))^2}),$$
$$\frac{d\lambda_3}{dt}=\lambda_1(t)(\frac{wu_1^*(t)I_h^*(t)}{(N_1^*(t))}-\lambda_h)+\lambda_2(t)(\frac{u_1^*(t)I_h^*(t)}{(N_1^*(t))})$$
$$+\lambda_3(t)(\frac{(1-w)u_1^*(t)I_h^*(t)}{(N_1^*(t))^2})$$
$$\frac{d\lambda_4}{dt}=\lambda_4(t)(\gamma_v+\frac{\beta_3 I_h^*(t-h)}{N_2^*(t-h)}+\grave{o}_1 u_2^*(t))+\lambda_5(t)(-\frac{\beta_3 I_h^*(t-h)}{N_2^*(t-h)})-A_1,$$
$$\frac{d\lambda_5}{dt}=\lambda_5(t)\Big((\gamma_v+\delta_v)+\grave{o}_2 u_2(t)\Big)-A_2$$

with the transversality or boundary conditions

$$\lambda_i(T)=0, i=1,2...5. \qquad (11)$$

And the optimal control variables is given as

$$u_1^*(t)=\max(\min\Big(\frac{\frac{-wI_h^*(t)\lambda_1}{N_1^*(t)}+\frac{I_h^*(t)\lambda_2}{N_1^*(t)}-\frac{\lambda_3(1-w)I_h^*(t)}{N_1^*(t)}}{\xi_1},1,0\Big)$$

$$u_2^*(t)=\max(\min\Big((\frac{\lambda_5\grave{o}_1 S_v^*(t)+\grave{o}_2 I_v^*(t)\lambda_4}{\xi_2},1),0\Big).$$

Proof: To prove the above result, i.e the adjoint equation and the transversallity conditions, we use the Hamiltonian (9). The adjoint system was obtained by Pontryagin's Maxmimum Principle [19].

$$\frac{d\lambda_1}{dt}=-\frac{\partial M}{\partial S_h},\ \ .....,\frac{d\lambda_5}{dt}=-\frac{\partial M}{\partial I_v}$$

with $\lambda_i(T)=0.$ To obtained the required characterization of the optimal control given by (12-13), solving the equations,

$$\frac{\partial M}{\partial u_1}=0, and\ \frac{\partial M}{\partial u_2}=0$$

in the interior of the control set and by the control space U, we derive the equation (10-13). □

Substituting the corresponding derivatives in the above equations and after the arrangement we get the adjoint equations (10-13).

In addition, the second derivative of the Lagrangian with respect to u_1^*, u_2^* is positive, which shows that the optimal problem is minimum at control u_1^*, u_2^*. By substituting the value of u_1^*, u_2^* in the control system (6) we get the following system

$$\frac{dS_h}{dt} = b_1 - \mu_h S_h - \frac{\beta_1 S_h I(t-h)}{N_1(t-h)} - \frac{\beta_2 S_h I_v(t-h)}{N_1(t-h)} + \frac{w\max(\min\Big(\frac{\frac{-wI_h^*(t)\lambda_1}{N_1^*(t)} + \frac{I_h^*(t)\lambda_2}{N_1^*(t)} - \frac{\lambda_3(1-w)I_h^*(t)}{N_1^*(t)}}{\xi_1},1,0\Big)I_h(t)}{N_1(t)} + \lambda_h R_h,$$

$$\frac{dI_h}{dt} = \frac{\beta_1 S_h I(t-h)}{N_1(t-h)} + \frac{\beta_2 S_h I_v(t-h)}{N_1(t-h)} - (\mu_h + \delta_h + \gamma_h) I_h(t) - \frac{\max(\min\Big(\frac{\frac{-wI_h^*(t)\lambda_1}{N_1^*(t)} + \frac{I_h^*(t)\lambda_2}{N_1^*(t)} - \frac{\lambda_3(1-w)I_h^*(t)}{N_1^*(t)}}{\xi_1},1,0\Big)I_h(t)}{N_1(t)},$$

$$\frac{dR_h}{dt} = \gamma_h I_h(t) - (\mu_h + \lambda_h) R_h + \frac{(1-w)\max(\min\Big(\frac{\frac{-wI_h^*(t)\lambda_1}{N_1^*(t)} + \frac{I_h^*(t)\lambda_2}{N_1^*(t)} - \frac{\lambda_3(1-w)I_h^*(t)}{N_1^*(t)}}{\xi_1},1,0\Big)I_h(t)}{N_1(t)},$$

$$\frac{dS_v}{dt} = b_2 - \gamma_v S_v - \frac{\beta_3 S_v I_h(t-h)}{N_2(t-h)} - \grave{o}_1 \max(\min\Big((\frac{\lambda_5 \grave{o}_1 S_v^*(t) + \grave{o}_2 I_v^*(t)\lambda_4}{\xi_2},1),0\Big)$$

$$S_v(t), \frac{dI_v}{dt} = \frac{\beta_3 S_v I_h(t-h)}{N_2(t-h)} - (\gamma_v - \delta_v) I_v - \grave{o}_2 \max(\min\Big((\frac{\lambda_5 \grave{o}_1 S_v^*(t) + \grave{o}_2 I_v^*(t)\lambda_4}{\xi_2},1),0\Big) I_v(t),$$

With the Hamiltonian M^* at $(I_h^*, S_v^*, I_v^*, u_1^*, u_2^*, \lambda_{1'}, \lambda_{2'}, \lambda_{3'}, \lambda_{4'}, \lambda_{5'})$ is

$$M^* = A_o I_h^* + A_1 I_v^* + A_2 S_v^* + \frac{1}{2}(\xi_1 \{\frac{\frac{-wI_h^*(t)\lambda_1}{N_1^*(t)} + \frac{I_h^*(t)\lambda_2}{N_1^*(t)} - \frac{\lambda_3(1-w)I_h^*(t)}{N_1^*(t)}}{\xi_1}\})^2$$
$$+\xi_2(\max(\min\Big((\frac{\lambda_5 \grave{o}_1 S_v^*(t) + \grave{o}_2 I_v^*(t)\lambda_4}{\xi_2},1),0\Big))^2$$
$$+\lambda_1\lambda_{1'} + \lambda_2\lambda_{2'} + \lambda_3\lambda_{3'} + \lambda_4\lambda_{4'} + \lambda_5\lambda_{5'}.$$

Numerical Simulation and Summary

In this we present the numerical simulations of the proposed model (1) and the delay control model (6) by using Runge-Kutta method. Solving first the model (1) and then solving the proposed model (6). Using the adjoint equation (10) with the boundary conditions (11) solving numerically by Runge-kutta order four backward scheme. The constants used in the objective functional with their numerical values we assumed in the numerical simulation is $A_2 = 0.001, A_3 = 0.002, \xi_1 = 0.7, \xi_2 = 0.3$ and $\grave{o}_2 = 0.0031$. The values of parameters used in the numerical simulations are presented in **Table 1.**

In this simulation the bold line shows the system with no control and the dashed line shows the system with control throughout Figure 1 to Figure 5. Figure 6 and Figure 7 represents the control variable u_1 and u_2 respectively. The aim of this paper was to control the infection in the host population by using the control variables in the form of treatment or suggestion. The control shows in the Figure 1 that the population of susceptible human increases and Figure 2 the infection in the host human decreases. Figure 3 shows the recovered individuals of human population which increases. Also the population of susceptible vector and infected vector and susceptible vector also decreases in Figure 4 and Figure 5.

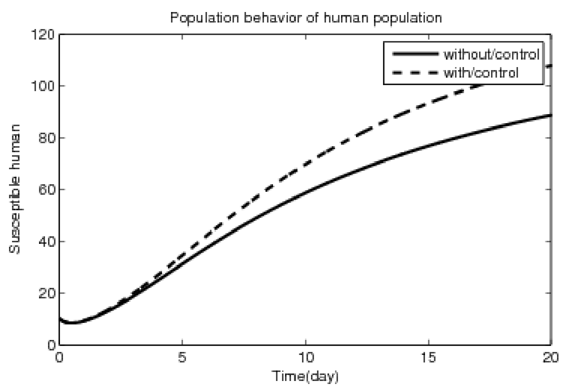

Figure .1. Represents the comparision of susceptible human in both the system without control and control.

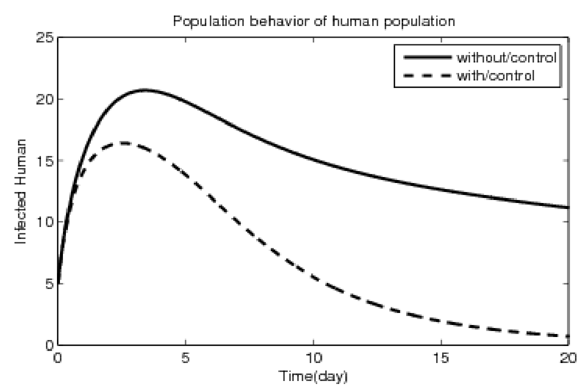

Figure 2. Represents the comparision of infected human in both the system without control and control.

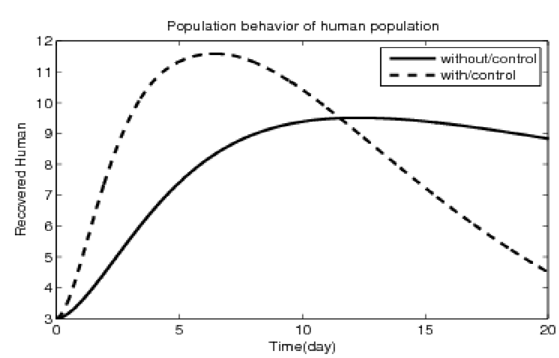

Figure 3. Represents the comparision of recovered human in both the system without control and control.

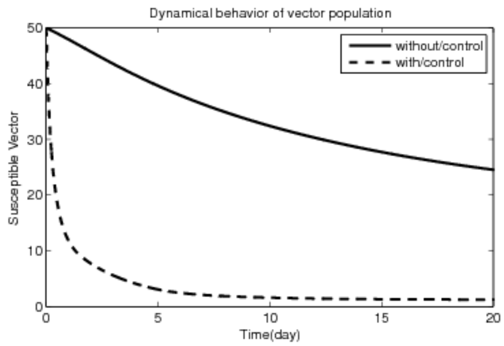

Figure 4. Represents the comparision of susceptible vector in both the system without control and control.

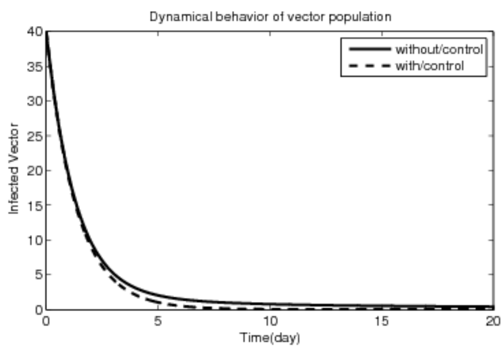

Figure 5. Represents the comparision of infected vector in both the system without control and control

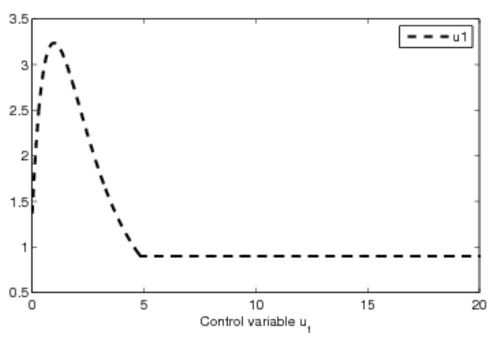

Figure 6. Represents the the contro variable **u1**

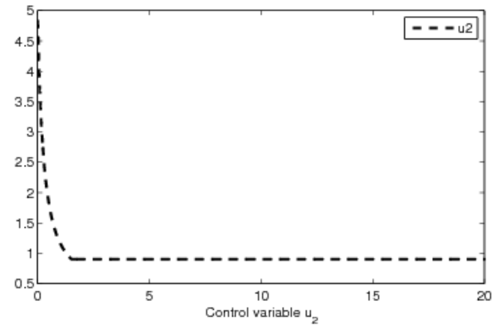

Figure 6. Represents the the contro variable **u2**

In this paper we consider an epidemic model by applying the optical control and time delay. First we formulate the model and the applying the time delay and optimal control with control variables u_1, u_2. Then we showed the existence of the control system and find the numerical solution of the both the system without control and control. Finally we conclude our work by references.

Corresponding Author:
Muhammad Altaf Khan Phd Fellow
Department of Mathematics
Abdul Wali Khan University Mardan
Pakistan.
E-mail: altafdir@gmail.com

References

1. Anderson, R.M., May, R.M., 1979. Population biology of infectious diseases: Part I. Nature 280 (5721), 361–367.
2. Bachar,M., Dorfmayr, A., 2004. HIV treatment models with time delay C. R. Biol. 327, 983–994.
3. Baker, C.T.H., Paul, C.A.H., Will, D.R., 1995. ssues in the numerical solution of evolutionary delay differential equations, Adv. Comput. Math. 3, 171–196.
4. Birkhoff, G., Rota, G.C., 1989. Ordinary Differential Equations, fourth ed. John Wiley and Sons, New York.
5. Brauer, F., Castillo-Chavez, C., 2001.Mathematical Models in Population Biology and Epidemiology Springer, New York, USA.
6. Chwell, G., Hengartner, N.W., Castillo-Chavez, C., Fenimore, P.W., Hyman, J.M.,

2004. The basic reproductive number of Ebola and the effects of public health measures: the case of Congo and Uganda J. Theor. Biol. 229, 119–126.
7. Cooke, K., van den Driessche, P., Zou, X., 1999. Interaction of maturation delay and nonlinear birth in population and epidemic models, J. Math. Biol. 39, 332–352.
8. Halanay, A., 1968. Optimal control for systems with time lag, SIAM J. Control 6, 215–234.
9. Hethcote, H.W., van den Driessche, P., 2000. Two SIS epidemiologic models with delays, J. Math. Biol. 40, 3–26.
10. Kamien, M.I., Schwartz, N.L., 2000. Dynamics Optimization: The calculus of Variations and Optimal Control in Economics and Management, Elsevier Science, The Netherland.
11. Zaman, G., Kang, Y.H., Jung, I.H., 2008. Stability analysis and optimal vaccination of an SIR epidemic model, Biosystems 93, 240–249.
12. Katri, P., Ruan, S., 2004. Dynamics of human T-cell lymphotropic virus I (HTLV-I) infection of CD4+ T-cells, C. R. Biol. 327, 1009–1016.
13. Kermack,W.O., Mckendrick, A.G., 1927. Contribution to the mathematical theory of epidemics, Proc. R. Soc. Lond. A 115, 700–721.
14. Lekone, P.E., Finkenstädt, B.F., 2006. Statistical inference in a stochastic epidemic SEIR model with control intervention: Ebola as a case study,Biometrics 62, 1170–1177.
15. Lenhart, S., John, T.W., 2007. Optimal Control Applied to Biological Models. Chapman and Hall/CRC, Mathematical and Computational Biology Series, London, UK.
16. Linda, J.S.A., Amy, M.B., 2000. Comparison of deterministic and stochastic SIS and SIR models in discrete time, Math. Biosci. 163, 1–33.
17. Linda, J.S.A., 2007. An Introduction to Mathematical Biology, Pearson Education Ltd., USA, pp. 123–127.
18. Lukes, D.L., 1982. Differential Equations: Classical to Controlled. Mathematics in Science and Engineering}, vol. 162. Academic Press, New York, p. 182.
19. Ma, Z., Liu, J., Li, J., 2003. Stability analysis for differential infectivity epidemic models, Nonlinear Anal. Real World Appl. 4, 841–856.
20. Ma,W., Takeuchi, Y., Hara, T., Beretta, E., 2002. Permanence of an SIR epidemic model with distributed time delays}, Tohoku Math. J. 54, 581–591.
21. May, R.M., 1983. Parasitic infections as regulators of animal populations, Am. Sci. 71, 36–45.
22. G. Zaman, Y. H. Kang, Il Hyo Jung, Optimal treatment of an SIR epidemic model with time delay}, BioSystems 98 (2009) 43–50.
23. Mills, C.E., Robins, J.M., Lipsitch, M., 2004. Transmissibility of 1918 pandemic influenza, Nature 432 (7019), 904–906.
24. Milner, F.A., Pugliese, A., 1999. Periodic solutions: a robust numerical method for an S–I–R model of epidemics, J. Math. Biol. 39, 471–492.
25. Smith, H.L., 1994. A structured population model and related function–differential equation: global attractors and uniform persistence, J. Dyn. Diff. Eqns. 6, 71–99.
26. Smith, H.L., Thieme, H., 1990. Monotone semiflows in scalar non-quasi-monotone functional deferential equations}, J. Math. Anal. Appl. 21, 673–692.
27. So, J.W.-H., Wu, J., Zou, X., 2001. Structured population on two patches: modeling dispersion and delay}, J. Math. Biol. 43, 37–51.
28. G. Zaman, M. A. Khan, S. Islam,et al, mdeling dynamical interaction between leptospirosis infected vector and human population ,Vol 6 2012 no.26, 1287-1302.
29. M. A. Khan, et al, Optimal Campaign in Leptospirosis Epidemic by Multiple Control Variables, *Applied Mathematics*, 2012, 3, 1655-1663. (http://www.SciRP.org/journal/am)
30. Hamid Sharif Nia, et al, Relationship of some risk factors and symptoms in patients with acute coronary syndrome, Life Science Journal 2012;9(4)
31. Global Analysis of leptospirosis epidemic model with nonlinear incidences ,submitted, 2012.

Table 1. Parameters value used in the numerical simulation of the optimal control problem.

.

Notation	Parmeters Description	Value
b_1 β_1 β_2 β_3 μ_h λ_h δ_h γ_v δ_v b_2	Recruitment rate for human population Transmission rate for human population Transmission rate for vector population Transmission rate for S_v and I_h Natural mortality rate of human population Proportionality constant Disease death rate for human population Natural mortality rate of vector population Disease death rate for vector population Recruitment rate for vector population	13 0.01 0.95 0.09 0.00001 0.02 0.051 0.051 0.051 3

10/25/2025