# Some Portfolio Selection Models Applied to Russian Stock Exchange Market

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**Abstract:** In this paper we consider different portfolio selection models: the basic Markowitz model, modified models with alternative risk measures and Black-Litterman model, which is able to take into account expert views specified by confidence level. We use asymmetric risk measures like Value-at-Risk, Mean Absolute Deviation and Semivariance in modified models. In our experiments we took the data of Russian Stock Exchange market on the example of Moscow Interbank Currency Exchange (MICEX). The genetic algorithms were also chosen as the instrument for solving optimization problems. Finally we compare the efficiency of considered models.

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#### 1. Introduction

Optimal portfolio selection is a key challenge in the activities of commercial banks, pension funds, insurance companies, etc. Today there exist many approaches to form investment portfolio's structure. In this paper we make review of some quantitative approaches and present the results of experiments with portfolio selection models where that approaches were implemented. All experiments were made on the Russian stock exchange market.

First, let us describe some portfolio investment models. For that we need a basic set of notations, which will be used in considered models. Let  $i = \overline{1, n}$  denotes the different risky assets where *n* is the amount of the assets in portfolio. Let *x* denotes the  $n \times 1$  vector of portfolio weights and  $x_i$ is the share of the portfolio invested in asset *i*. Thus:

$$0 \le x_i \le 1 \sum_{i=1}^n x_i = 1$$
 (1)

Let  $\mu_i$  denotes the expected return of asset i,  $\sigma_{ij}$  are the coefficients of the variance-covariance  $n \times n$  matrix V and  $R_{const}$  is the determined by investor level of return for portfolio. Let  $t = \overline{1,T}$ denotes the time periods of historical data and  $\mu_{it}$  is the return of asset i at time t. Thus, expected portfolio's return and portfolio's return at time tlook like:

$$R = \sum_{i=1}^{n} x_{i} \mu_{i} \qquad \text{and} \qquad R_{t} = \sum_{i=1}^{n} x_{i} \mu_{it} \qquad ((2))$$

H. Markowitz described own model in his famous paper [1]. The key idea of this model is to balance the expected return and risk. He offered to use the mean return as expected return and Variance as risk measure. There exist also several assumptions, which could be considered at large in original paper. The mathematical model is set out below.

$$\min_{x} \sum_{i=1}^{n} \sum_{j=1}^{n} x_{i} x_{j} \sigma_{ij}$$

$$\sum_{i=1}^{n} x_{i} \mu_{i} \ge R_{const}$$

$$\sum_{i=1}^{n} x_{i} = 1$$
(3)

Here the objective function represents the Variance of portfolio. In practice, solving this problem one can get solution, which consists of negative weights  $x_i$  in portfolio. Negative  $x_i$  could be interpreted like call to complete «short sale» contract. However, we can observe in practice that sometimes we can't or not allowed to complete «short sale» contract. For example, Russian Financial markets administration disallowed ability to complete «short sale» contracts at the period of World Financial Crisis 2008 [2]. That could be reclaimed very easy by adding positive constraint like  $x_i > 0$ ,

but in this case the complexity of the optimization problem increases. Using the Variance as a risk measure is the other disadvantage of Markowitz model. Minimizing the Variance, we can lose portfolio, which periodically has high positive returns.

#### 2. Material and Methods.

Consider the alternative risk measures, which could be used as objective function. One of them is Semivariance. It's possible to eliminate the disadvantage of the Variance using this measure. Semivariance is the expected value of the squared negative deviations of possible outcomes from the expected return [3]. The definition is derived as follows:

$$SV(R) = E[\min(R - E[R], 0)], \qquad (4)$$

where SV is Semivariance, R denotes the time series of portfolio's returns, and  $E[\cdot]$  denotes mean value. The statistical estimation of this measure looks like:

$$\widehat{S}V(R) = \frac{1}{T-1} \sum_{t=1}^{T} \min(R_t - \overline{R}, 0),$$

$$\overline{R} = \frac{1}{T-1} \sum_{t=1}^{T} R_t$$
(5)

Thus, final model looks like:

$$\min_{x} \frac{1}{T-1} \sum_{i=1}^{T} \min\left(\sum_{i=1}^{n} x_{i} \mu_{ii} - \frac{1}{T-1} \sum_{i=1}^{T} \sum_{i=1}^{n} x_{i} \mu_{ii}, 0\right), \\
\sum_{i=1}^{n} x_{i} \mu_{i} \ge R_{const}, \quad (6) \\
\sum_{i=1}^{n} x_{i} = 1, \quad x_{i} > 0.$$

Other risk measure we used is Mean Absolute Deviation (MAD) [4]. This measure calculates the deviation from the expected return. As a result, user gets a linear programming problem. Some researches show that using MAD measure is equal to using the Variance [5]. We are going to test this thesis on Russian stock exchange market. The definition is derived as follows:

$$MAD(R) = E[[R - E[R]]].$$
(7)

The statistical estimation of this measure looks like:

$$\widehat{MAD}(R) = \frac{1}{T-1} \sum_{t=1}^{T} \left( \left| R_t - \overline{R} \right| \right)$$
(8)

Thus, model with MAD risk measure looks like:

$$\min_{x} \frac{1}{T-1} \sum_{t=1}^{T} \left( \left| \sum_{i=1}^{n} x_{i} \mu_{it} - \frac{1}{T-1} \sum_{t=1}^{T} \sum_{i=1}^{n} x_{i} \mu_{it} \right| \right), \\
\sum_{i=1}^{n} x_{i} \mu_{i} \ge R_{const}, \quad (9)$$

$$\sum_{i=1}^{n} x_{i} = 1 \quad x_{i} > 0$$

Value-at-Risk is one of the actively practiceused risk measure. The Value-at-Risk at confidence level  $1-\alpha$  of a portfolio  $(VaR_{\alpha})$  is calculated by

$$P(R \le -VaR_{\alpha}) = \alpha \tag{10}$$

This formula could be interpreted like the minimum amount that investor can loose with a confidence interval of  $1 - \alpha$ . We should make some transformations to get the statistical estimation of this measure. First, let that portfolio's return is normally distributed. Thus:

$$P(R \le -VaR_{\alpha}) = \alpha \Leftrightarrow P\left(\frac{R_{p}-\mu}{\sigma} \le \frac{-VaR_{\alpha}-\mu}{\sigma}\right) = \alpha \Leftrightarrow$$

$$\frac{-VaR_{\alpha}-\mu}{\sigma} = z_{\alpha} \Leftrightarrow VaR_{\alpha} = -\mu - z_{\alpha}\sigma$$
, (11)

where  $\mu$  is the portfolio's mean return,  $\sigma$  is portfolio's standard deviation,  $z_{\alpha}$  is quantile of the standard normal distribution of order  $\alpha$ . For example, VaR with confidence level 0.95 (i.e. when  $\alpha = 0.05$ ) is calculated like  $VaR_{0.05} = -\mu + 1.6449\sigma$ . Thus, portfolio selection model with VaR risk measure looks like:

$$\min_{x} \left[ \left( -\frac{1}{T-1} \sum_{i=1}^{n} x_{i} \mu_{ii} \right) - z_{a} \sqrt{\frac{1}{T-1} \sum_{i=1}^{T} \left( \sum_{i=1}^{n} x_{i} \mu_{ii} - \frac{1}{T-1} \sum_{i=1}^{T} \sum_{i=1}^{n} x_{i} \mu_{ii} \right)^{2}} \right] \\
\sum_{i=1}^{n} x_{i} \mu_{i} \ge R_{const}$$
(12)

$$\sum_{i=1}^{n} x_i = 1$$

$$x_i > 0$$

In cases of alternative risk measures we don't need to calculate covariance matrix. Also, in cases of Semivariance and Mean Absolute Deviation we can replace mean value by a specific return level. For example, we can use some index's return as specific return level. In this case we get model related to index. We also made experiments using Moscow Interbank Currency Exchange Index (MICEX) index's return as specific return level. The main result is that we can get portfolio with return close to index's return but the risk of this portfolio sometimes can be lower.

The Black-Litterman model was designed like a practice-oriented model. To do this, Black and Litterman proposed a theory, which they called «equilibrium» approach [6]. Equilibrium returns are calculated like:

$$\Pi = \lambda V x^{mkt}, \qquad (13)$$

where  $\Pi$  is vector of the equilibrium return,  $\lambda$  is risk aversion coefficient,  $x^{mkt}$  denotes market capitalization portfolio, where every weight  $x_i^{mkt}$  is proportional to asset i market capitalization. Risk aversion coefficient characterizes the investor's willingness to sacrifice the value of the expected portfolio return for reducing the risk expressed by the

dispersion of expected returns.

$$\lambda = \frac{R - r_f}{\sigma^2},\tag{14}$$

where  $r_f$  denotes riskless rate and

$$\sigma^{2} = \sum_{i=1}^{n} \sum_{j=1}^{n} x_{i}^{mkt} x_{j}^{mkt} \sigma_{ij}$$
 is the Variance of

market capitalization portfolio  $x^{mkt}$ . Let k denotes number of expert views. Consider the Black-Litterman formula for aposterior expected return, which allows one to combine equilibrium returns and expert views with given confidence levels:

$$\hat{\mu} = \left[ (\tau V)^{-1} + P' \Omega^{-1} P \right]^{-1} \left[ (\tau V)^{-1} \Pi + P' \Omega^{-1} Q \right], \quad (15)$$

where  $\hat{\mu}_{is new} n \times 1$  aposterior vector of expected returns,  $\tau_{is a}$  scalar coefficient,  $P_{is}$ 

 $k \times n$  matrix identifying the assets, on which the investor has a subjective opinion,  $\Omega$   $k \times k$  is diagonal covariance matrix with the levels of trust for each of the subjective views, Q is  $k \times 1$  vector of subjective views. The uncertainty of the subjective views is reflected in the error vector  $\varepsilon$ , whose elements are normally distributed with zero mean, and the matrix  $\Omega$ . Thus, the final values of subjective opinion are given as  $Q + \varepsilon$ .  $\begin{bmatrix} Q_1 \end{bmatrix} \begin{bmatrix} \varepsilon_1 \end{bmatrix}$ 

$$Q + \varepsilon = \begin{bmatrix} Q_1 \\ \vdots \\ Q_k \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_k \end{bmatrix}$$
(16)

Variations of the elements of the error vector form  $\Omega$  matrix, where  $\Omega$  is a diagonal covariance matrix. The matrix is diagonal indeed, because, according to the prerequisites of the model, subjective opinions are independent of each other.

Variations  $\omega_i$  of the error vector  $\mathcal{E}$  show measure of uncertainty of expert views:

$$\Omega = \begin{bmatrix} \omega_{1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \omega_{k} \end{bmatrix}$$
(17)

There exist several approaches for determining the elements of  $\Omega$  [6, 7]. Finally, model with Black-Litterman approach and VaR risk measure looks like:

$$\max_{x} \sum_{i=1}^{n} x_{i} \hat{\mu}_{i}$$

$$\left(-\frac{1}{T-1} \sum_{i=1}^{n} x_{i} \mu_{i}\right) - \sum_{x_{\alpha} \sqrt{\frac{1}{T-1} \sum_{i=1}^{T} \left(\sum_{i=1}^{n} x_{i} \mu_{i} - \frac{1}{T-1} \sum_{i=1}^{T} \sum_{i=1}^{n} x_{i} \mu_{i}\right)^{2}} \leq VaR_{const} \quad (18)$$

$$\sum_{i=1}^{n} x_{i} = 1 \sum_{i=1}^{n} x_{i} > 0$$

$$K = D$$

where  $VaK_{const}$  denotes the investor desired limit level of Value-at-Risk.

To solve problems (3), (6), (9), (12) and (18) by classical methods is a quite time-consuming task. Sometimes it's more convenient to use genetic algorithms [8] to solve a lot of optimization problems

with nonlinear goal function and complex constraints. Genetic algorithms are a class of the heuristic search algorithms based on the evolution of the approximations used for simulation and optimization problems [9]. Diagram of a base genetic algorithm we used is presented in Figure 1.



Figure 1. Diagram of a base genetic algorithm

Parameters of base genetic algorithm we used to solve portfolio optimization problems are presented in Table 1. One of the main advantages of genetic algorithms is universality. We just changed the fitness-function for each optimization problem when the core of program remained unchanged. We used standard barriers method in fitness-function to take constraints in optimization problems into account.

	rable 1. Farameters of the genetic algorithm				
#	Parameter	Value			
1	Number of chromosomes	7			
2	Number of specimens	1000			
3	Probability of crossover	0.8			
4	The probability of mutation	0.2			
5	Selection type	Tournament selection			
6	Crossover type	Two-point crossover			
7	New generation formation	With descendants			
/	type	only			

Table 1 Parameters of the genetic algorithm

# 3. Results.

In our experiments we used data of Russian stock exchange market on the example of MICEX. Some information about the shares which were chosen is presented in Table 2. Most of those shares are included in MICEX index. The time interval of our experiments is between 01.11.2010 and

01.06.2011. This interval contains all types of trends: growing, falling and sideways.

Table 2. Shares traded at MICEX which were used in experiments

#	Enterprise	Capitalization (billion rubles)	Number of shares
1	Tatneft (TATN)	416.57	2 178 690 700
2	Gazprom (GAZP)	5 627.19	23 673 512 900
3	Rostelekom (RTKM)	124.97	728 696 320
4	VTB (VTB)	1 049.19	10 460 541 337 338
5	Lukoil (LKOH)	1 752.08	850 563 255
6	Polus-Zoloto (PLZL)	321.40	190 627 747
7	Uralsib (URSI)	78.30	292 575 808 568
8	MTS (MTSI)	519.06	1 993 326 138
9	Sberbank (SBER)	2 340.67	21 586 948 000

We used the Sharpe coefficient [10] to estimate the quality of the portfolios. This coefficient is also known as reward-to-variability ratio RVAR :

$$RVAR = \frac{E[R - r_f]}{\sigma} \tag{(19)}$$

where  $\sigma$  is portfolio's standard deviation and  $E[R-r_f]_{\text{is average risk premium.}}$ 

Let's consider the results of the experiments on Russian market. First of all, pay heed to profitability dynamic charts. We compare profitability dynamics of Semivariance model (Figure 2), MAD model (Figure 3), VaR model (Figure 4) and Black-Litterman model (Figure 5) with MICEX index and Markowitz model's profitability dynamics.



Figure 2. Semivariance model's profitability dynamic comparison

# 4. Discussions

As we can see, one of the best results is demonstrated by Black-Litterman model. The result of the Black-Litterman model depends on expert views. For this reason this model is more useful for professional investors. In negative situations models with alternative risk measures (Semivariance, MAD, VaR) demonstrate results better than MICEX index and Markowitz portfolio. In many cases the model with MAD risk measure demonstrates results equal to Markowitz model. The Sharpe ratio shows, that use of alternative risk measures increases the quality of portfolio. In cases of growing trend all portfolios regardless of the structure increase almost identically. In cases of falling and sideways trends the portfolio's structure plays key role. Also we should notice that all models (excluding model with Black-Litterman approach) are optimized historically. It means, that portfolio's structure will be correct if current situation retains in the future. As practice shows, current situation continues not so long. Therefore the structure of portfolio should be periodically recalculated using actual data.



Figure 3. MAD model's profitability dynamic comparison



Figure 4. VaR model's profitability dynamic comparison



Figure 5. Black-Litterman model's profitability dynamic comparison

Portfolio structures for each model are presented below (Figure 6).



Figure 6. Structure for each model

And the Sharpe ratios for each model are presented in Table 3.

#	Model	Average risk premium	Sharpe ratio
1	Markowitz	0.051	1.125
2	Semivariance	0.059	1.188
3	MAD	0.058	1.196
4	VaR	0.091	1.701
5	Black-Litterman	0.113	2.060

Table 3. Sharpe ratios for each model

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