On the Analysis of the Treatment Model for Onchocerciasis infected Host in Tropical Countries

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Abstract: The purpose of this paper is to analyze the mathematical model of Onchocerciasis treatment for stability with respect to the basic reproduction number R_c with control measures in place. The basic reproduction number with control measures R_c is obtained to be stable at the disease free equilibrium (DFE) on one hand. On the other hand, we make use of the homotopy decomposition methods to derive the effect of treatment with Ivermectin/Mectizan drugs in the given population infected with Onchocerciasis. The numerical results are presented to test the efficiency and the accuracy of the treatment with Ivermectin/Mectizan drugs. The numerical solutions in both cases display the biological behavior of the real life situation.

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Introduction

Onchocerciasis, also known as river blindness and Robles disease is a parasitic disease caused by *Onchocerca volvulus*, a nematode (roundworm) and it is endemic in tropical countries. The present estimates had suggested that about 7-10 million Nigerians which is just one of the tropical countries are infected with *Onchocerca volvulus*, approximately 40 million are at risk of the disease [1], and 120,000 cases of Onchocerciasis related blindness [2], with many thousands suffering from disabling complications of the disease [3]. It is termed as 'river blindness' because, it is spread by the bite of the blackfly vector that breads near oxygen-high fast- flowing streams and rivers people rely on for washing, drinking and farming which results in depopulation of the fertile river valleys. The lifespan of Black Flies is short, lasting only 2-3 weeks. The disease affects rural communities in Nigeria [11] and is the major cause of blindness and skin disease in endemic areas with serious socio-economic effects. It has been identified by the World Health Organization as one of the neglected tropical disease (NTD) [4-5]. 'Neglected' because they are not mentioned in the millennium Development Goals. This implied that they are not usually included among the important development discussions, and as such do not receive adequate attention or funding. We assume a population where some individuals are already infected with the disease Onchocerciasis and there are no presence of the vector Black flies for the transmission of the disease and re-infections within the given population. We consider the Onchocerciasis treatment model in Figure 1 below, and suppose that those infectious individuals are treated with Ivermectin/Mectizan drugs

Mathematical formulation of the Onchocerciasis transmission Model

In this section, we focus on the latent and infectious compartments where the effect of Onchocerciasis in humans who are bitten by black flies and the transmission of the parasite *Onchocerca volvulus* have taken place. Individuals are categorized based on the state of the infection. We say that a compartment is infected, if we can find individuals who have the disease Onchocerciasis, but do not show any symptoms yet and also, individuals who already show all the symptoms associated to the disease. Assume there are $n-$Onchocerciasis disease compartments and $m-$ nononchocerciasis disease compartments and let $u \in \mathbb{R}^n$ and $v \in \mathbb{R}^m$ be the two subdivisions of the total population in each of these compartments. Let $\mathcal{F}_i$ be the rate of increase in the secondary infection in the i^{th} disease compartment and $\mathcal{K}_i$ be the rate of decrease in the progression, death and recovery from the disease in the i^{th} compartment. We therefore reduce the compartmental model above as follows [7]:

$$\frac{du_i}{dt} = \mathcal{F}_i(u,v) - \mathcal{K}_i(u,v), \quad i = 1,\dots,n \qquad (1)$$

$$\frac{dv_i}{dt} = g_j(u,v), \quad j = 1,\dots, \qquad (2)$$

Assumptions

$\mathcal{F}_i(0,v) = 0$ and $\mathcal{K}_i(0,v) = 0 \forall v \geq 0, i = 1,\dots,n$

Suppose that those infectious individuals are treated with Ivermectin/Mectizan drugs at the rate $x_2 H_I$, bearing in mind that the treatment is partially effective. We assume that the fraction α of the

infectious individual treated with Ivermectin/Mectizan drugs recover without a complete immunity and the impaired sight with the skin deformation does not return back to normal completely and a fraction $\lambda = 1 - \alpha$ move back to the latent stage of infection. $x_1 H_L$ is rate of the treatment of individuals in the latently infected compartment; b is the inflow (birth) rate into the population; H_s is the compartment of susceptible individuals; H_L is the compartment of individuals latent to the disease; H_I is the compartment of individuals that are infected with Onchocerciasis and are also infectious; H_T is the compartment of individuals that are treated with Ivermectin/Mectizan drugs; ρH_T is the rate of reinfection of the treated individual suppose a Black flie made its way in the environment within that period; μ is the natural death rate. Assume that the disease does not kill the individual directly after contact. $N = H_s + H_L + H_I + H_T$. F and K are the $n \times n$ matrices with entries [6]. In Figure 1 below, we present the treatment model for Onchocerciasis as follows:

Model Diagram

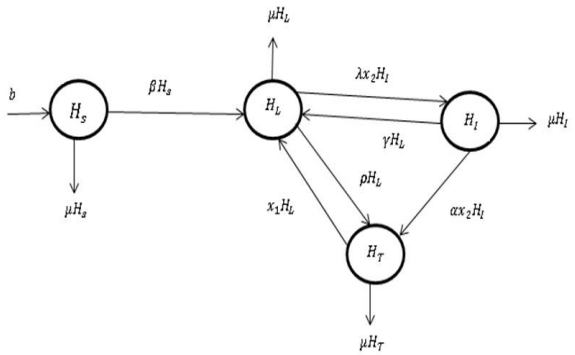

Figure 1: Onchocerciasis Transmission Model

$$\frac{dH_s}{dt} = b(N) - (\beta + \mu)H_s \quad (3)$$

$$\frac{dH_L}{dt} = \beta H_s + \rho H_T + \lambda x_2 H_I - (\gamma + x_1 + \mu)H_L \quad (4)$$

$$\frac{dH_I}{dt} = \gamma H_L - (x_2 + \mu)H_I \quad (5)$$

$$\frac{dH_T}{dt} = x_1 H_L + \alpha x_2 H_I - \rho H_T - \mu H_T \quad (6)$$

Here, the system above has a remarkable disease free equilibrium (DFE) with $H_s(0) = \frac{b}{\mu}$, taking the infected compartment as H_L and H_I. Hence, we have an equilibrium state with $H_L = H_I = H_T = 0$ and $H_s = H_{s0}$, which is locally stable, where, H_{s0} is any positive solution of $b(H_{s0}) = \mu H_{s0}$.

From equation (2), the treatment failure denoted by $\lambda x_2 H_I$ is a part of the progression of an infected individual through the disease compartments rather than a new infection by *Onchocerca volvulus*. Following this interpretation, we have that

$$\mathcal{F} = \begin{pmatrix} \beta H_s + \rho H_T \\ 0 \end{pmatrix},$$

and $\mathcal{K} = \begin{pmatrix} (\gamma + x_1 + \mu)H_L - \alpha x_2 H_I \\ -\gamma H_L + (x_2 + \mu)H_I \end{pmatrix}$

We evaluate the derivatives of $\mathcal{F}$ and $\mathcal{K}$ at $H_s = H_{s0}$, for $H_L = H_I = H_T = 0$ and obtain the following expression for $\mathcal{F}$ and $\mathcal{K}$

$$\mathcal{F} = \begin{pmatrix} 0 & \beta \\ 0 & 0 \end{pmatrix},$$

$$\mathcal{K} = \begin{pmatrix} \gamma + x_1 + \mu & -\alpha x_2 \\ -\gamma & x_2 + \mu \end{pmatrix}$$

According to a heuristic derivation of R_c given in [7], therefore, the reproduction number with control measure in place R_c is given as

$$R_c = \frac{\gamma\beta}{(\gamma + x_1 + \mu)(x_2 + \mu) - \gamma\alpha x_2} < 1 \quad (5)$$

Re-writing R_c as a geometric series $(\varpi_1 + \varpi_1^2\varpi_2 + \cdots)\frac{\beta}{x_2+\mu}$, where $\varpi_1 = \frac{\gamma}{\gamma + x_1 + \mu}$ is the fraction of individuals leaving compartment H_L into H_I; the fraction of individuals moving back to compartment H_L from H_I is given as $\varpi_2 = \frac{\lambda x_2}{x_2+\mu}$; Since we have a population assumed to be free from the presence of Black flies which is the vector that transmits Onchocerciasis disease, we therefore have that the fraction of latent individuals that move to the H_I compartment p-times is $\varpi_1^p \varpi_2^{p-1}$, were $\varpi_1 + \varpi_1^2\varpi_2 + \cdots$ is the number of times a latent individual move through the H_I compartments.

Considering the treatment failure as a new infection with *Onchocerca volvulus*, we have $\mathcal{F}$ and $\mathcal{K}$ as

$$\mathcal{F} = \begin{pmatrix} \beta H_s + \rho H_T + \lambda x_2 H_I \\ 0 \end{pmatrix}$$

$$\mathcal{K} = \begin{pmatrix} (\gamma + x_1 + \mu)H_L \\ -\gamma H_L + (x_2 + \mu)H_I \end{pmatrix}$$

Evaluating the derivative at $H_s = H_{s0}$, for $H_L = H_I = H_T = 0$, we have for F and K as

$$F = \begin{pmatrix} 0 & \beta + \lambda x_2 \\ 0 & 0 \end{pmatrix}$$

$$K = \begin{pmatrix} \gamma + x_1 + \mu & 0 \\ -\gamma & x_2 + \mu \end{pmatrix}$$

and the spectral radius

$$\xi(FK^{-1}) = \frac{\gamma(\beta + \lambda x_2)}{(\gamma + x_1 + \mu)(x_2 + \mu)} < 1$$

Here, the infection rate with Onchocerciasis $\beta + \lambda x_2$ and individuals latent to Onchocerciasis is expected to spend $\frac{\gamma}{(\gamma + x_1 + \mu)(x_2 + \mu)}$ time in the H_I compartment.

Therefore, we have that both R_c and $\xi(FK^{-1})$ are stable.

Application of the Homotopy Decomposition Method (HDM)

In this section, we apply the (HDM) [10] to solve these mathematical set of equations

representing the Onchocerciasis disease model as follows:

$$H_s(t) - H_s(0) = \int_0^t \big(b - (\beta + \mu)H_s(\tau)\big)d\tau$$

$$H_L(t) - H_L(0) = \int_0^t (\beta Hs(\tau) + \rho H_T(\tau) - (\gamma + \mu + x_1)H_L(\tau) + \lambda x_2 H_I(\tau))d\tau$$

$$H_I(t) - H_I(0) = \int_0^t \big(\gamma H_L(\tau) - (\mu + x_2)H_I(\tau)\big)d\tau$$

$$H_T(t) - H_T(0) = \int_0^t \big(x_1 H_L(\tau) + \alpha x_2 H_I(\tau) - (\rho + \mu)H_T(\tau)\big)d\tau$$

We then assume that the solutions of the above integral equations can be put in the following form, for $P \in (0,1)$

$$H_s(t) = \sum_{n=0}^{\infty} P^n H_{sn}(t)$$

$$H_L(t) = \sum_{n=0}^{\infty} P^n H_{Ln}(t)$$

$$H_I(t) = \sum_{n=0}^{\infty} P^n H_{In}(t)$$

$$H_T(t) = \sum_{n=0}^{\infty} P^n H_{Tn}(t)$$

Replacing the above expressions in the integral equations, we obtain the following expressions

$$\sum_{n=0}^{\infty} P^n H_{sn}(t) - H_s(0) = P\int_0^t \left[\left(b - (\beta + \mu)\sum_{n=0}^{\infty} P^n H_{sn}(\tau)\right)d\tau\right]$$

$$\sum_{n=0}^{\infty} \mathrm{P^n H_{Ln}(t)} - \mathrm{H_L(0)} = \mathrm{P}\int_0^t \left[\left(\beta\sum_{n=0}^{\infty} \mathrm{P^n H_{sn}(\tau)} + \rho\sum_{n=0}^{\infty} \mathrm{P^n H_{Tn}(\tau)} - (\gamma + \mu + x_1)\sum_{n=0}^{\infty} \mathrm{P^n H_{Ln}(\tau)} + \lambda x_2 \sum_{n=0}^{\infty} \mathrm{P^n\, H_{In}(t)}\right)d\tau\right]$$

$$\sum_{n=0}^{\infty} P^n\, H_{In}(t) - H_I(0) = P\int_0^t \left[\left(\gamma\sum_{n=0}^{\infty} P^n H_{Ln}(\tau) - (\mu + x_2)\sum_{n=0}^{\infty} P^n\, H_{In}(\tau)\right)d\tau\right]$$

$$\sum_{n=0}^{\infty} P^n H_{Tn}(t) - H_T(0) = P\int_0^t \left[\left(x_1\sum_{n=0}^{\infty} P^n H_{Ln}(\tau) + \alpha x_2 \sum_{n=0}^{\infty} P^n\, H_{In}(\tau) - (\rho + \mu)\sum_{n=0}^{\infty} P^n H_{Tn}(\tau)\right)d\tau\right]$$

we compared the terms of the same power of P and obtain the following

$$P^0: H_{s0}(t) = H_s(0)$$

$$P^0: H_{L0}(t) = H_L(0)$$

$$P^0: H_{I0}(t) = H_I(0)$$

$$P^0: H_{T0}(t) = H_T(0)$$

$$P^1: H_{s1}(t) = \int_0^t \big(b - (\beta + \mu)H_{s0}(\tau)\big)d\tau$$

$$P^1: H_{L1}(t) = \int_0^t (\beta H_{s0}(\tau) + \rho H_{T0}(\tau) - (\gamma + \mu + x_1)H_{L0}(\tau) + \lambda x_2 H_{I0})d\tau$$

$$P^1: H_{I1}(t) = \int_0^t \big(\gamma H_{L0}(\tau) - (\mu + x_2)H_{I0}(\tau)\big)d\tau$$

$$P^1: H_{T1}(t) = \int_0^t \big(x_1 H_{L0}(\tau) + \alpha x_2 H_{I0}(\tau) - (\rho + \mu)H_{T0}(\tau)\big)d\tau$$

Continuing for $n \geq 2$, we have that

$$H_{sn}(t) = \int_0^t \Big(b - (\beta + \mu)H_{s(n-1)}(\tau)\Big)\, d\tau$$

$$H_{Ln}(t) = \int_0^t \big(\beta H_{s(n-1)}(\tau) + \rho H_{T(n-1)}(\tau) - (\gamma + \mu + x_1)H_{L(n-1)}(\tau) + \lambda x_2 H_{I(n-1)}\big)d\tau$$

$$H_{In}(t) = \int_0^t \Big(\gamma H_{L(n-1)}(\tau) - (\mu + x_2)H_{I(n-1)}(\tau)\Big)\, d\tau$$

$$H_{Tn}(t) = \int_0^t \Big(x_1 H_{L(n-1)}(\tau) + \alpha x_2 H_{I(n-1)}(\tau) - (\rho + \mu)H_{T(n-1)}(\tau)\Big)\, d\tau$$

We now integrate the above equations to obtain the following at the disease free equilibrium (DFE)

$$H_{s0}(t) = N$$
$$H_{L0}(t) = 0$$
$$H_{I0}(t) = 0$$
$$H_{T0}(t) = 0$$
$$H_{I1}(t) = 0$$
$$H_{s1}[t] = t(b - N(\beta + \mu))$$
$$H_{L1}(t) = Nt\beta$$
$$H_{T1}[t] = 0$$
$$H_{I2}(t) = \frac{1}{2}Nt^2\beta\gamma$$
$$H_{L2}(t) = \frac{1}{2}bt^2\beta - \frac{1}{2}Nt^2\beta^2 - \frac{1}{2}Nt^2\beta\gamma - Nt^2\beta\mu - \frac{1}{2}Nt^2\beta x_1$$
$$H_{s2}[t] = bt - \frac{1}{2}t^2(\beta + \mu)(b - N(\beta + \mu))$$
$$H_{T2}[t] = \frac{1}{2}Nt^2\beta x_1$$
$$H_{I3}(t) = \frac{1}{6}bt^3\beta\gamma - \frac{1}{6}Nt^3\beta^2\gamma - \frac{1}{6}Nt^3\beta\gamma^2 - \frac{1}{2}Nt^3\beta\gamma\mu - \frac{1}{6}Nt^3\beta\gamma x_1 - \frac{1}{6}Nt^3\beta\gamma x_2$$
$$H_{L3}(t) = \frac{1}{2}bt^2\beta - \frac{1}{6}bt^3\beta^2 + \frac{1}{6}Nt^3\beta^3 - \frac{1}{6}bt^3\beta\gamma + \frac{1}{6}Nt^3\beta^2\gamma + \frac{1}{6}Nt^3\beta\gamma^2 - \frac{1}{3}bt^3\beta\mu + \frac{1}{2}Nt^3\beta^2\mu + \frac{1}{2}Nt^3\beta\gamma\mu + \frac{1}{2}Nt^3\beta\mu^2 - \frac{1}{6}bt^3\beta x_1 + \frac{1}{6}Nt^3\beta^2 x_1 + \frac{1}{3}Nt^3\beta\gamma x_1 + \frac{1}{2}Nt^3\beta\mu x_1 + \frac{1}{6}Nt^3\beta\rho x_1 + \frac{1}{6}Nt^3\beta x_1^2 + \frac{1}{6}Nt^3\beta\gamma\lambda x_2$$
$$H_{s3}[t] = bt - \frac{1}{6}t^2(\beta + \mu)(Nt(\beta + \mu)^2 - b(-3 + t(\beta + \mu)))$$
$$H_{T3}[t]\frac{1}{6}bt^3\beta x_1 - \frac{1}{6}Nt^3\beta^2 x_1 - \frac{1}{6}Nt^3\beta\gamma x_1 - \frac{1}{2}Nt^3\beta\mu x_1 - \frac{1}{6}Nt^3\beta\rho x_1 - \frac{1}{6}Nt^3\beta x_1^2 + \frac{1}{6}Nt^3\alpha\beta\gamma x_2 H_{s4}[t] = bNt - \frac{1}{2}bNt^2\beta + \frac{1}{6}bNt^3\beta^2 - \frac{1}{24}bNt^4\beta^3 + \frac{1}{24}Nt^4\beta^4 - \frac{1}{2}bNt^2\mu + \frac{1}{3}bNt^3\beta\mu - \frac{1}{8}bNt^4\beta^2\mu + \frac{1}{6}Nt^4\beta^3\mu + \frac{1}{6}bNt^3\mu^2 - \frac{1}{8}bNt^4\beta\mu^2 + \frac{1}{4}Nt^4\beta^2\mu^2 - \frac{1}{24}bNt^4\mu^3 + \frac{1}{6}Nt^4\beta\mu^3 + \frac{1}{24}Nt^4\mu^4$$
$$H_{I4}(t) = \frac{1}{6}bt^3\beta\gamma - \frac{1}{24}bt^4\beta^2\gamma + \frac{1}{24}Nt^4\beta^3\gamma - \frac{1}{24}bt^4\beta\gamma^2 + \frac{1}{24}Nt^4\beta^2\gamma^2 + \frac{1}{24}Nt^4\beta\gamma^3 - \frac{1}{8}bt^4\beta\gamma\mu + \frac{1}{6}Nt^4\beta^2\gamma\mu + \frac{1}{6}Nt^4\beta\gamma^2\mu + \frac{1}{4}Nt^4\beta\gamma\mu^2 - \frac{1}{24}bt^4\beta\gamma x_1 + \frac{1}{24}Nt^4\beta^2\gamma x_1 + \frac{1}{12}Nt^4\beta\gamma^2 x_1 + \frac{1}{6}Nt^4\beta\gamma\mu x_1 + \frac{1}{24}Nt^4\beta\gamma\rho x_1 + \frac{1}{24}Nt^4\beta\gamma x_1^2 - \frac{1}{24}bt^4\beta\gamma x_2 + \frac{1}{24}Nt^4\beta^2\gamma x_2 + \frac{1}{24}Nt^4\beta\gamma^2 x_2 + \frac{1}{24}Nt^4\beta\gamma^2\lambda x_2 + \frac{1}{6}Nt^4\beta\gamma\mu x_2 + \frac{1}{24}Nt^4\beta\gamma x_1 x_2 + \frac{1}{24}Nt^4\beta\gamma x_2^2$$
$$H_{s4}[t] = bt + \frac{1}{24}t^2(\beta + \mu)(Nt^2(\beta + \mu)^3 - b(12 + t(\beta + \mu)(-4 + t(\beta + \mu))))$$

$$H_{L4}(t) = \frac{1}{2}bt^2\beta - \frac{1}{6}bt^3\beta^2 + \frac{1}{24}bt^4\beta^3 - \frac{1}{24}Nt^4\beta^4 - \frac{1}{6}bt^3\beta\gamma + \frac{1}{24}bt^4\beta^2\gamma - \frac{1}{24}Nt^4\beta^3\gamma + \frac{1}{24}bt^4\beta\gamma^2 - \frac{1}{24}Nt^4\beta^2\gamma^2 - \frac{1}{24}Nt^4\beta\gamma^3 - \frac{1}{3}bt^3\beta\mu + \frac{1}{8}bt^4\beta^2\mu - \frac{1}{6}Nt^4\beta^3\mu + \frac{1}{8}bt^4\beta\gamma\mu - \frac{1}{6}Nt^4\beta^2\gamma\mu - \frac{1}{6}Nt^4\beta\gamma^2\mu + \frac{1}{8}bt^4\beta\mu^2 - \frac{1}{4}Nt^4\beta^2\mu^2 - \frac{1}{4}Nt^4\beta\gamma\mu^2 - \frac{1}{6}Nt^4\beta\mu^3 - \frac{1}{6}bt^3\beta x_1 + \frac{1}{24}bt^4\beta^2 x_1 - \frac{1}{24}Nt^4\beta^3 x_1 + \frac{1}{12}bt^4\beta\gamma x_1 - \frac{1}{12}Nt^4\beta^2\gamma x_1 - \frac{1}{8}Nt^4\beta\gamma^2 x_1 + \frac{1}{8}bt^4\beta\mu x_1 - \frac{1}{6}Nt^4\beta^2\mu x_1 - \frac{1}{3}Nt^4\beta\gamma\mu x_1 - \frac{1}{4}Nt^4\beta\mu^2 x_1 + \frac{1}{24}bt^4\beta\rho x_1 - \frac{1}{24}Nt^4\beta^2\rho x_1 - \frac{1}{12}Nt^4\beta\gamma\rho x_1 - \frac{1}{6}Nt^4\beta\mu\rho x_1 - \frac{1}{24}Nt^4\beta\rho^2 x_1 + \frac{1}{24}bt^4\beta x_1^2 - \frac{1}{24}Nt^4\beta^2 x_1^2 - \frac{1}{8}Nt^4\beta\gamma x_1^2 - \frac{1}{6}Nt^4\beta\mu x_1^2 - \frac{1}{12}Nt^4\beta\rho x_1^2 - \frac{1}{24}Nt^4\beta x_1^3 + \frac{1}{24}bt^4\beta\gamma\lambda x_2 - \frac{1}{24}Nt^4\beta^2\gamma\lambda x_2 - \frac{1}{12}Nt^4\beta\gamma^2\lambda x_2 - \frac{1}{6}Nt^4\beta\gamma\lambda\mu x_2 + \frac{1}{24}Nt^4\alpha\beta\gamma\rho x_2 - \frac{1}{12}Nt^4\beta\gamma\lambda x_1 x_2 - \frac{1}{24}Nt^4\beta\gamma\lambda x_2^2$$

$$H_{T4}[t] = \frac{1}{6}bt^3\beta x_1 - \frac{1}{24}bt^4\beta^2 x_1 + \frac{1}{24}Nt^4\beta^3 x_1 - \frac{1}{24}bt^4\beta\gamma x_1 + \frac{1}{24}Nt^4\beta^2\gamma x_1 + \frac{1}{24}Nt^4\beta\gamma^2 x_1 - \frac{1}{8}bt^4\beta\mu x_1 + \frac{1}{6}Nt^4\beta^2\mu x_1 + \frac{1}{6}Nt^4\beta\gamma\mu x_1 + \frac{1}{4}Nt^4\beta\mu^2 x_1 - \frac{1}{24}bt^4\beta\rho x_1 + \frac{1}{24}Nt^4\beta^2\rho x_1 + \frac{1}{24}Nt^4\beta\gamma\rho x_1 + \frac{1}{6}Nt^4\beta\mu\rho x_1 + \frac{1}{24}Nt^4\beta\rho^2 x_1 - \frac{1}{24}bt^4\beta x_1^2 + \frac{1}{24}Nt^4\beta^2 x_1^2 + \frac{1}{12}Nt^4\beta\gamma x_1^2 + \frac{1}{6}Nt^4\beta\mu x_1^2 + \frac{1}{12}Nt^4\beta\rho x_1^2 + \frac{1}{24}Nt^4\beta x_1^3 + \frac{1}{24}bt^4\alpha\beta\gamma x_2 - \frac{1}{24}Nt^4\alpha\beta^2\gamma x_2 - \frac{1}{24}Nt^4\alpha\beta\gamma^2 x_2 - \frac{1}{6}Nt^4\alpha\beta\gamma\mu x_2 - \frac{1}{24}Nt^4\alpha\beta\gamma\rho x_2 - \frac{1}{24}Nt^4\alpha\beta\gamma x_1 x_2 + \frac{1}{24}Nt^4\beta\gamma\lambda x_1 x_2 - \frac{1}{24}Nt^4\alpha\beta\gamma x_2^2$$

In general, by the repeated application of the above procedure, we have that

$$H_{sn} = \frac{t^n}{n!}a_{1n,}\ \ H_{Ln} = \frac{t^n}{n!}a_{2n,}\ \ H_{In} = \frac{t^n}{n!}a_{3n,}\ \ H_{Tn} = \frac{t^n}{n!}a_{4n}$$

Where $a_{1n,}a_{2n,}a_{3n,}a_{4n}$ depends on fixed set of empirical parameters. Hence, the approximate solution of the model above are given as

$$H_s(t) = \sum_{n=0}^{\infty}\frac{t^n}{n!}a_{1n}$$

$$H_L(t) = \sum_{n=0}^{\infty}\frac{t^n}{n!}a_{2n}$$

$$H_I(t) = \sum_{n=0}^{\infty}\frac{t^n}{n!}a_{3n}$$

$$H_T(t) = \sum_{n=0}^{\infty}\frac{t^n}{n!}a_{4n}$$

We therefore use the following theoretical parameters $\beta = 0.5$, $x_1 = 0.5$, $x_2 = 0.3$, $\alpha = 0.3$, $b = 0.5$, $\mu = 0.5$, $N = 100$, $\rho = 0.6$, $\gamma = 0.5$, $\lambda = 0.7$ to examine the behavior of the numerical solutions in the system within the latent, infectious

and treatment compartments. The figures 2-4 below show the behavior of the numerical solution:

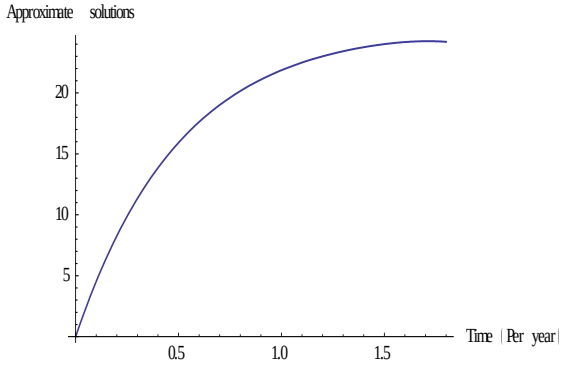

Figure 2: Latent compartment

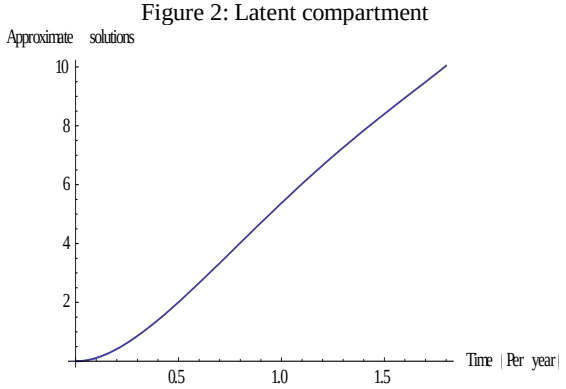

Figure 3: Infectious Compartment

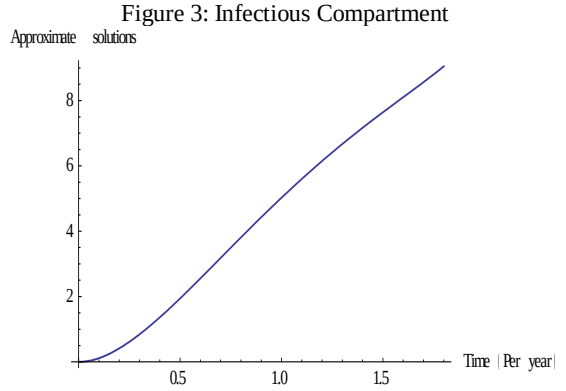

Figure 4: Treatment with Ivermectin/Mectizan drug

Conclusion

We analyzed the mathematical model of Onchocerciasis treatment for stability with respect to the basic reproduction number R_c with control measures in place. The basic reproduction number with control measures R_c was obtained to be stable at the disease free equilibrium (DFE) on one hand. On the other hand, we made use of the homotopy decomposition methods to derive the effect of treatment with Ivermectin/Mectizan drugs in the given population infected with Onchocerciasis. The numerical results were presented to test the efficiency and the accuracy of the treatment with Ivermectin/Mectizan drugs. The numerical solutions confirmed that the more re-infection cases we have, the more need for the use of Ivermectin/Mectizan drugs to get infected people controlled back into the latent stage and possibly prevent the rate of infection. Therefore, in order to build a healthy and a sustainable population, treatment with Ivermectin/ Mectizan drugs and a more hygienic environment that does not allow the breeding and survival of Black flies could lead to a total eradication of the disease.

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