Variational principle and the problems dynamics

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Abstract. The ability of variational approach in physical science is demonstrated. The local variational principle (LVP) is formulated, and the matter of the V-function method is revealed, the method is used for the continuation of the optical-mechanical analogy. The properties of the wave nature of electron motion in the hydrogen-like atom and the object (particle) motion for the harmonic oscillator are investigated. It is shown that the hydrogen-like atom energy levels obtained by the V-function method are the same ones available in the quantum physics classical results.
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Introduction

The variational principles determine the laws in physical science. The idea that the nature effects by means of easier and more accessible ways led famous French mathematician Fermat [1] to the formulation of the first variational principle - the principle of the shortest time of light propagation. The I.Bernoulli formulation of the brachistochrone problem and its solution initiated the calculus of variations branch of mathematics and were the launching point for the formulation of the principle of least action. The complex concept of action itself was introduced to the mechanics by Leibniz [2] as the product of the particle weight with its velocity, and with the path traversed by the particle at this velocity, i.e. as $m\vartheta s$. The fundamental principle of least action was formulated by P. Maupertuis [3]. Later on, the principle of least action was affected by the researchers Euler [4], Lagrange [5]. But, the more smart and convenient principle in the theoretical physics appeared to be the principle of least (stationary) action in the form which is established by Hamilton [6]. He was the first who accomplished the formulation of the optical-mechanical analogy and solved it at the level of geometrical optics. The Hamilton principle as compared to Lagrange principle is applicable for the non-stationary mechanic systems as well, remaining also invariant in regard to the co-ordinates transformations at that. It is the Hamilton principle that allowed obtaining of the important thermodynamics equations, electrodynamics equations, and compact characterization of the continuous fields.

The Hamilton principle achieved a brilliant success when it was implemented in Einstein theory of relativity [7]. The reason is that the Hamilton action value is invariant relative to Lorentz transformation. Plank emphasizes the fundamental role of the Hamilton principle in modern physics saying that the principle has major significance for the characterization of a natural process than the law of conservation of energy [8]. The Hamilton principle had played supreme part both in quantum mechanics and in the construction of quantum field theory [9-10].

Only the path variations are present in all of those variational principles. Therefore, the optical-mechanical analogy can be drawn at the basis of the existing variational principles. The analogy is a kind of fundamental synthesis of wave and corpuscular aspects of movement, but at the level of geometrical optics. Following the idea of deep identity of the principle of least action to the Fermat principle, Louis de Broglie established his famous equation relating the momentum of particle to the wavelength in his studies [11]. Thus, each particle is correlated with the indissolubly associated wave process.

The idea that particle motion conceals undulating movement became especially productive for physics. The Louis de Broglie studies were the basis for the formulation of the Schrödinger [12] wave equation. Basing on the Hamilton-Jacobi mechanics and on the results of the geometrical optics development Schrödinger proceeded from the Hamilton analogy.

The wave-particle duality is left unsolved on the basis of the existing variational principles. This work offers a new approach on this way. The approach is based on the wave-particle monism for the explanation of the particle (electron) nature. Namely, the theory developed below uses the description of physical reality where the existence of electron paths is taken into account. The paths are the evidence of the fact of the particle existence. It is also taken that the electron motion is determined by physical wave V(x,t). Such a statement could be made on the basis of process-state concept which is introduced for the description of the entity and the way of existence of electron. This concept is initially based on the ontology of dynamism strategy [13] where the motion (process) is *the entity* of the reality, and the path (state) is the reality *way of existence*.

The offered theory is developed using new continuation of the optical-mechanical analogy for the characterization of the particle path and wave behavior. The basic provisions of the wave-particle monism are presented and their physical sense is explained in the beginning of the article. The explanation is based on the variational approach, namely on the utilizing of the local variational principle (LVP) [14]. This theory is used later on for the characterization of the object (electron) motion in free space, in the stationary Coulomb field of hydrogen-like atom - one of the known test objects of quantum theory, and for the one-dimensional harmonic oscillator as well.

Local variational principle and the V(x,t)-function method

Let us determine the matter of the local variational principle (LVP). Let's specify the object path motion by the system of the differential equations of classical physics.

$$\tfrac{d}{dt} x = f(x), \qquad (1)$$

where the particle phase coordinates vector $x(t) = (x_1, x_2, ..., x_n)^T$ is specified in *n*-dimensional Euclidean space ($x \in R^n$), t – time ($t \in T$). Let's say that the equations (1) characterizing the object path motion determine the state of the object under study.

Along with the equations system (1) we introduce the wave function *V(x,t)* as well. It's changing rate for the system under study (1) would be determined by the expression $\tfrac{d}{dt} V = \tfrac{\partial}{\partial t} V + \tfrac{\partial}{\partial x} V^T f$. Let's consider the isochronous variation of the wave function rate of change $\delta\left(\tfrac{d}{dt} V\right) = \tfrac{\partial}{\partial t}\delta V + \tfrac{\partial}{\partial x}\delta V^T f + \tfrac{\partial}{\partial x} V^T \delta f$, (where $\delta V = \tfrac{\partial}{\partial x} V^T \delta x$, $\delta f = \tfrac{\partial}{\partial x} f \delta x$). Postulate that with the variation of the wave function rate of change $\delta\left(\tfrac{d}{dt} V\right)$ the object comes from some initial state to the state notable for the new space coordinate $x + \delta x$. Let's name such a transition the object wave transition, the value δV sets the eventual wave transition from the initial state to a new state while δx determines the path variations. The space variation takes the form of the realized-in-space displacement $\delta x = dx = \dot{x} dt$ with the wave transition.

Let's formulate the LVP: *Among all possible transitions to a new state that transition is realized for which the V(x,t) wave function rate of change takes stationary value at every point of time*

$$\delta\left(\tfrac{d}{dt} V\right) = 0 \qquad (2)$$

Assuming that (2) is true, it is proved [14] that complying of the wave function to the additional condition of *full variation of the V(x,t) wave function rate of change* is necessary and sufficient for the wave transition to a new state:

$$\Delta\left(\tfrac{d}{dt} V\right) = 0, \qquad (3)$$

where $\Delta(.) = \delta(.) + \tfrac{d}{dt}(.)\Delta t$

Having classical equations (1) and conditions (2), (3), we determine the wave equation for *V(x,t)* taking into account the wave transition ($\delta x = dx = \dot{x} dt$) at (2) and (3):

$$\Delta\left(\frac{dV}{dt}\right) = \left\{\frac{\partial^2 V}{\partial t^2} + 3\frac{\partial^2 V}{\partial t \partial x}^T f + 2 f^T W f + 2\frac{\partial V}{\partial x}^T \frac{df}{dt}\right\} dt =$$

$$= 3\delta\left(\frac{dV}{dt}\right) + \left(\frac{\partial^2 V}{\partial t^2} - f^T W f - \frac{\partial V}{\partial x}^T \frac{df}{dt}\right) dt = 0 \rightarrow$$

$$\frac{\partial^2 V}{\partial t^2} - f^T W f - \frac{\partial V}{\partial x}^T \frac{df}{dt} = 0 , \qquad (4)$$

where *V(x,t)* – the piecewise continuous, finite, single valued function, $W = [\partial^2_{x_i x_j} V(x,t)]$ - matrix of functions. Let's show that the following equality holds

$$\frac{\partial V}{\partial x}^T \frac{d}{dt} \dot{x} = 0 . \qquad (4.1)$$

According to the *V*-function method the particle motion goes so that the particle rate is directed along the wave function gradient, i.e. $\frac{\partial}{\partial x} V^T \dot{x} = \left|\frac{\partial}{\partial x} V\right| \left|\dot{x}\right|$. Hence we have $\partial V / \partial x = k_2(x)\dot{x}$. Below we suppose that the field of velocities in *n*-dimensional space and the corresponding gradient field are congruent. That is true when $k_2(x) = k_2$. Accordingly, we have the equation

$$\partial V / \partial x = k_2 \dot{x} , \qquad (4.2)$$

In the case when the wave transition is realized the equation (2) takes the following form:

$$\frac{d}{dt}\left(\frac{\partial V^T}{\partial x}\delta x\right)=\frac{d}{dt}\left(\frac{\partial V^T}{\partial x}\dot{x}dt\right)=0\Rightarrow\frac{\partial V^T}{\partial x}\dot{x}=const\ . \quad (4.3)$$

Taking into account (4.2) and (4.3) the equality (4.1) holds, i.e. $\frac{\partial V^T}{\partial x}\frac{d}{dt}\dot{x}=k_2\dot{x}^T\frac{d}{dt}\dot{x}=\frac{k_2}{2}\frac{d}{dt}(\dot{x}^T\dot{x})=\frac{1}{2}\frac{d}{dt}(\frac{\partial V^T}{\partial x}\dot{x})=0$. As a result, taking into account (4.1) the equation (4) takes the following form

$$\frac{\partial^2 V}{\partial t^2}-\dot{x}^T W\dot{x}=0 \quad (5)$$

Furthermore, if the condition [15] $\dot{x}_j=\lambda_i\frac{\partial \dot{x}_i}{\partial x_j}\ (i,j=\overline{1,n})$ is true, the equation (5) is expressed in the following form:

$$\frac{\partial^2 V}{\partial t^2}-\vartheta^2\nabla^2 V=0\ , \quad (6)$$

where $\vartheta^2=\sum_{i=1}^{n}\dot{x}_i^{\ 2}=\dot{x}^T\dot{x}$ and $\nabla^2=\sum_{i=1}^{n}\frac{\partial^2}{\partial x_i^2}$

Suggested approach to the characterization of the particle behavior contains the system of the path equation (1) and the wave equation (5), or (6). It is necessary to know the boundary conditions to find a solution for the system of equations. It should be noted to compare that the particle dynamics characterization is limited by the equation (1) in classical physics, where the initial coordinate and the particle rate at some fixed point of time are set. Below we find the boundary conditions for the wave *V(x,t)* at the particle path based on the *V*-function method.

If the wave and the path are related, and the *V(x,t)* wave amplitude equals to zero in the particle location point, we would have: $V(x=x_M,t)=0,\ V(x,t=0)=0$.

Having in our account that the wave transition is realized in (2) we obtain

$$\frac{\partial}{\partial x}V^T\dot{x}=const\ . \quad (7)$$

Using condition (7) for the full variation (3) we in turn obtain the equation $\frac{d}{dt}\left(\frac{\partial}{\partial t}V\right)=0$, then we find the following condition for the wave behavior at the particle path $\frac{\partial}{\partial t}V=k_1$.

From the equation (4.2) we have the boundary condition for the wave at a point $x=x_M$ of particle path $\partial V/\partial x\big|_{x=x_M}=k_2\dot{x}\big|_{x=x_M}$, where $k_{1,2}$ - some constants.

Thus, summing we write the common system of the equations of the particle path-wave motion according to the V-function method.

$$\frac{d}{dt}x=f(x)\ , \quad (8.1)$$

$$\frac{\partial^2}{\partial t^2}V-f^T Wf=0\ , \quad (8.2)$$

augmented with the ratios for the particle path and wave

$$V(x,t)\big|_{t=0}=V(x,0)=0 \quad (9.1)$$

$$V(x,t)\big|_{x=x_M}=V(x_M,t)=0 \quad (9.2)$$

$$\partial V/\partial x\big|_{x=x_M}=k_2\dot{x}\big|_{x=x_M} \quad (10)$$

$$\frac{\partial}{\partial t}V=k_1 \quad (11)$$

It should be noted that the condition (10) is a special case of (4.2). It is introduced as a boundary condition in order to use the available information of the particle rate in some part (or at the boundary) of the space ($x\subset x_M$). Let's in turn note that (9.1) and (9.2) are the supplementary to (10) conditions of the existence of the particle path.

Based on the local variational principle we can realize a new formulation of ***the direct problem and the inverse problem of dynamics.***

The direct problem of dynamics:

We have the differential equations characterizing the object motion path (8.1).

It is required to determine the wave function *V(x,t)* complying to the equation (8.2) and to the boundary conditions (9)-(11).

The inverse problem of dynamics is formulated based on the V-function method as follows:

It is required to determine the object motion path equations (8.1) for the given wave function *V(x,t)* complying to the equation (8.2), and which is to be considered in the form of (5).

If the wave function is given, the solution of the inverse problem of dynamics follows just from (4.3):

$$\dot{x}_i=k\frac{\partial V}{\partial x_i} \quad (12)$$

It should be noted that since the the wave motion comes to the object path motion, if the equality (12) holds, the ratio (4.3) takes the following form:

$$\frac{\partial}{\partial x}V^T\dot{x} = k_2\dot{x}^T\dot{x} = k_2\dot{s}^2 = 2T = const\,. \quad (13)$$

Solving the inverse problem we have got not only the motion equations (12) with right parts depending on the V-function definition method, but the approach to H.Hertz Principles of Mechanics [16], that follows from (13).

Continuation of the Optical-Mechanical analogy

Consider the free rectilinear motion of the object (particle) with a constant rate. Then the equations (8.1) and (8.2) are changed as follows

$$\dot{x} = \vartheta \quad (14)$$

$$\frac{\partial^2 V(x,t)}{\partial t^2} - \frac{\partial^2 V(x,t)}{\partial x^2}\dot{x}^2 = 0 \quad (15)$$

Let's solve the direct problem of dynamics. As follows from (14) and (15) the path motion of the particle corresponds to the wave motion complying to the classic wave equation

$$\frac{\partial^2 V}{\partial t^2} - \vartheta^2\frac{\partial^2 V(x,t)}{\partial x^2} = 0, \quad (16)$$

for which the conditions (9.1)-(9.2) stay the same, but (10)-(11) take the following form:

$$\left.\frac{\partial V(x,t)}{\partial t}\right|_{t=0} = \frac{\partial V(x,0)}{\partial t} = \tilde{C}_1,$$

$$\left.\frac{\partial V(x,t)}{\partial x}\right|_{x=0} = \frac{\partial V(0,t)}{\partial x} = \tilde{C}_2\,. \quad (17)$$

For the solution of the equation (16) with the initial and boundary conditions (9.1),(9.2),(17) we apply the method of separation of variables

$$V(x,t) = \varphi(t)\psi(x)$$

$$\frac{\ddot{\varphi}(t)}{\varphi(t)} = \vartheta^2\frac{\psi''(x)}{\psi(x)} = -\omega^2 \quad \textbf{(18)}$$

Then, considering the wave propagation in the direction of the particle motion we've got the solution of the direct problem in the following form:

$$V(x,t) = Ae^{\pm i(\omega t - \frac{\omega}{\vartheta}x)} \quad (19)$$

Let the wave function is given in the form of the equation of monochromatic plane-wave (19). Then if the equality (14) holds, (19) will satisfy (15) as a solution for the inverse problem of dynamics.

From the equality (11) where the wave function is specified in the form of (19) we have the following

$$\frac{\partial V(x,t)}{\partial t} = \mp iA\,\omega e^{\pm i(\frac{\omega}{\vartheta}x - \omega t)} = k_1\,. \quad (20)$$

The complex coefficient in (20) can be excluded if the phase takes the values:

$$(\frac{\omega}{\vartheta}x - \omega t) = \frac{\pi}{2} + \pi n\,, \; (n=0,1,2,3....) \quad (21)$$

It follows that considering (14) the equality (20) takes only discrete values, i.e.

$$|A|\omega = |A|\omega_0(1/2+n) = k_1 \quad (22)$$

And from the equality (4.2)

$$\frac{\partial V(x,t)}{\partial x} = \pm i\frac{A\omega}{\vartheta}e^{\pm i(\frac{\omega}{\vartheta}x - \omega t)} = k_2\vartheta\,, \quad (23)$$

taking into account (21) we have

$$|A|\omega = k_2\vartheta^2\,. \quad (24)$$

Let in (19) $A = \frac{h}{2\pi} = \hbar\,,$ (27)

where h - Planck's constant. Then from the solution of the inverse problem of dynamics (22) we have the same Rule of Energy Quantization as in the case of linear oscillator of Schrödinger. From the equality (24) and taking into account (25) we have

$$k_2 = \frac{\hbar\omega}{\vartheta^2}\,. \quad (26)$$

Hence, based on the action dimensions $[кг][м/с][м]$ it follows that

$$k_2 = m \quad (27)$$

where m - the particle mass. Using the obtained results we can find the following correspondence between the wave and the particle

$$\vartheta = \vartheta; \quad \omega = \frac{m\vartheta^2}{\hbar} = \frac{2E}{\hbar};$$
$$\lambda = \frac{h}{m\vartheta}; \quad A = \hbar \quad (28)$$

It is the main in the relations (28) that the wave phase velocity and the particle velocity are equal. While particle velocity equals to the group-velocity of L. de Broglie waves in quantum mechanics. The condition of Energy Quantization (22) is naturally a result of the solution of the inverse problem. According to the second relation of (28) the energy transfers by the particle. In turn, according to the third relation of (28) the particle impulse defines the wave length, that match the well-known L. de Broglie's formula. Upon physical interpretation the wave $V(x,t)$ characterizes the properties of the action showing up in the particle motion. Thus, the wave node is bound to the particle location. It directs the particle. At the same time the particle (path) generates the wave propagating with it.

Harmonic Oscillator

Let's consider linear harmonic oscillator. In this case the equation of the object (particle) path motion allows applying of the first integral. Hence, we have the expression for the squared velocity

$$\dot{x}^2 = \frac{2E - kx^2}{m}. \quad (29)$$

The particle path motion corresponds to the wave motion that takes the form (15). If we substitute (29) to the equation (15) we obtain:

$$\frac{\partial^2 V}{\partial t^2} - \left(\frac{2E - kx^2}{m}\right)\frac{\partial^2 V}{\partial x^2} = 0 \quad (30)$$

The wave function $V(x,t)$ is found in the following form $V(x,t) = \psi(x)\varphi(t)$. Dividing the variables of the equation (30) we get the following stationary equation:

$$\psi'' + \frac{m\omega^2}{2E - kx^2}\psi = 0. \quad (31)$$

From the boundary conditions (9.2), (10) for the V-function we get the initial conditions for the function $\psi(x)$, which are written in the following form:

$$\psi(x)\big|_{x=0} = \psi(0) = 0,$$
$$\psi'(x)\big|_{x=0} = \psi'(0) = C_1; \quad (32)$$

As follows from the equation (31) the finite at the interval and the only solution $\psi(x)$ should meet the natural condition: $\psi\left(x = \sqrt{\frac{2E}{k}}\right) = 0$, that is possible for certain discrete values of the equation natural frequencies only. Let's introduce the non-dimensional value $\xi = \frac{x}{\sqrt{\frac{2E}{k}}}$, than the equation (31) transfers to

$$\psi'' + \frac{\eta^2}{1 - \xi^2}\psi = 0, \quad (33)$$

where $\eta^2 = \frac{m\omega^2}{k} = \frac{\omega^2}{\omega_0^2}$

The frequencies were defined in an analytical and numerical way from the equation (33) with the initial conditions (32), i.e.

$$\eta_1^2 = \frac{\hbar^2\omega_1^2}{\hbar^2\omega_0^2} = 6, \qquad \eta_2^2 = \frac{\hbar^2\omega_2^2}{\hbar^2\omega_0^2} = 20,$$
$$\eta_3^2 = \frac{\hbar^2\omega_3^2}{\hbar^2\omega_0^2} = 42, \quad \eta_4^2 = \frac{\hbar^2\omega_4^2}{\hbar^2\omega_0^2} = 72, \ldots$$

Taking into account the results of the optical-mechanical analogy $2E = \hbar\omega$, we get the following rule for the harmonic oscillator energy quantization

$$E_{n+2}^2 - 2E_{n+1}^2 + E_n^2 = \Delta\Delta E_n^2 = 2\hbar^2\omega_0^2 \quad (34)$$

So, when the object path is bound directly to the wave motion the harmonic oscillator energy can have certain discrete values only: $E_1^2 = 6\hbar^2\omega_0^2$, $E_2^2 = 20\hbar^2\omega_0^2$, $E_3^2 = 42\hbar^2\omega_0^2$, $E_4^2 = 72\hbar^2\omega_0^2 \ldots$. If we substitute Schrödinger's results for harmonic oscillator $E_n = \left(n + \frac{1}{2}\right)\hbar\omega_0$ to the equation (34) we would get $\left(\left(n+2+\frac{1}{2}\right)^2 - 2\left(n+1+\frac{1}{2}\right)^2 + \left(n+\frac{1}{2}\right)^2\right)\hbar^2\omega_0^2 = 2\hbar^2\omega_0^2$, i.e. we obtain the identity. As it is well-known, in the real microscopic oscillators based on light source the transitions occur between neighbour levels only. That is totally coincides with our results emerging from the equality (34).

Electron motion in a hydrogen-like atom

As it is well-known, the first integral is applicable for the electron path motion in a hydrogen-like atom. Thus, taking into account the potential energy of the hydrogen-like atom we have the squared velocity of the particle (electron)

$$\vartheta^2 = \frac{2(E + Ze^2/r)}{m} \quad (35)$$

The wave motion corresponds to the path motion. It is characterized by the equation (6) and taking into account (35) takes the following form:

$$\frac{\partial^2 V}{\partial t^2} - \frac{2(E + Ze^2/r)}{m}\nabla^2 V = 0, \quad (36)$$

Using the method of variables separation in the equation (36) $\left(V = X(x,y,z)T(t)\right)$, we get the following steady-state equation.

$$\left(-\beta_0^2+\frac{\alpha}{r}\right)\nabla^2 X+\omega^2 X=0, \quad (37)$$

where $\beta_0^2=-\frac{2E}{m}$, $\alpha=\frac{2Ze^2}{m}$.

Using a spherical coordinate system in the equation (37) let's consider the spherically symmetric solutions only $(X=R(r))$ making the following substitution $R=u/r$:

$$\frac{d^2u}{dr^2}+\left(\frac{k_0^2\alpha}{\alpha-\beta_0^2 r}-k_0^2\right)u=0, \quad (38)$$

where $k_0^2=\frac{\omega^2}{\beta_0^2}=-\frac{\omega^2 m}{2E}$.

Considering an asymptotic solution of the equation (38) ($r\to\infty$) let's write it's general solution as follows $u=c_1u_-(r)+c_2u_+(r)=e^{-k_0r}f_-(r)+e^{k_0r}f_+(r)$. Substituting it to (38) we get the following equations:

$$f_\pm''(r)\pm 2k_0 f_\pm'(r)+\frac{\beta_1}{r_0-r}f_\pm(r)=0, \quad (39)$$

where $\beta_1=\mathrm{k}_0^2\alpha/\beta_0^2=\frac{1}{2}\mathrm{Ze}^2\omega^2\mathrm{m_e}/\mathrm{E}^2$.

The equation (39) solution we are looking for in the form of power series $f_\pm(r)=\sum_{m=0}^{\infty}a_m^{(\pm)}(r_0-r)^m$. After the substitution the equation (39) takes the following form

$$\sum_{n=0}^{\infty}[(n+1)na_{n+1}^{(\pm)}\mp 2k_0na_n^{(\pm)}+\beta_1a_n^{(\pm)}](r_0-r)^{n-1}=0, \quad (40)$$

It means that $a_0=0$. And the coefficients $a_{n+1}^{(\pm)}$ meet the recurrent ratio

$$a_{n+1}^{(\pm)}=\frac{\pm 2k_0n-\beta_1}{(n+1)n}a_n^{(\pm)} \quad (41)$$

Following the inverse problem of dynamics we are searching for the particle (electron) path. And the relations (9.2) and (10) should be satisfied. It occurs if $\beta_1=2k_0n$ ($\beta_1=\frac{1}{2}Ze^2\omega^2m_e/E^2$, $k_0^2=-\frac{\omega^2m}{2E}$, $\beta_1>0, k_0>0$). (42)

This condition is satisfied when the series $f_+(r)=\sum_{m=1}^{\infty}a_m^{(+)}(r_0-r)^m$ terminates, i.e. $a_m^{(+)}=0$ when $m\geq n+1$ that leads to the following solution

$$u_{+,n}(r)=C\exp\{k_{0,n}r\}\sum_{m=1}^{n}a_m^{(+)}(r_{0,n}-r)^m, \quad (43)$$

where C – a constant,

From the equality (42) that is expressed in the form $E^3/\omega^2=-\frac{1}{8}Z^2e^4m_e/n^2$ taking into account the connection between frequency and energy $2E=\hbar\omega$, following from the drawn optical-mechanical analogy we find the energy value of the electron n-state

$$E_n=-\frac{Z^2e^4m_e}{2\hbar^2}\frac{1}{n^2}. \quad (44)$$

Let's note that the energy of n-state exactly matches to the solution obtained in Bohr's model [17] or based on Schrödinger steady-state equation [12].

In order to find the second linearly independent solution of the equation of second order decreasing exponentially by the distance $u_{-,n}(r\to\infty)\sim\exp\{-k_{o,n}r\}$ we use Louisville's formula. The formula takes the form $u_-(r)=Cu_+(r)\int\frac{1}{(u_+(r))^2}dr$ for the equation (38). Considering the (43) solution we get the required solution, i.e.

$$u_{-,n}(r)=Ce^{k_{0,n}r}\sum_{m=1}^{n}a_m^{(+)}(r_{0,n}-r)^m\int\frac{e^{-2k_{0,n}r}}{\left(\sum_{m=1}^{n}a_m^{(+)}(r_{0,n}-r)^m\right)^2}dr, \quad (45)$$

As far as $R_{-,n}=\frac{u_{-,n}}{r}$ in accordance with (45) we get the solutions for $R_{-,n}$.

Conclusion

Thus, the foundations of the path-wave mechanics have been laid down by means of the variational approach based on the formulated local variational principle and new statement of the direct and the inverse problems of dynamics. The possibilities of the path-wave mechanics have been demonstrated for the specific modelling of the physical objects' motion.

The conducted research shows that L. De Broglie's intention to overcome the wave-particle dualism is justified here through the continuation of the optical-mechanical analogy that is solved at the level of wave optics. It becomes evident that the resolving power of classical quantum physics is insufficient for the detection of the energy quantization for constant motion at a constant velocity.

When modelling the electron motion in Coulomb field the *V*-function method allows determination of the rule of energy quantization of hydrogen-like atom. The rule exactly matches to Schrödinger's and Bohr's classic results. The research is very close to B.N. Rodimov's investigations [18]. The lumpy energy occurs from meeting the conditions based on the *V*-function method at that. For this approach, the electron stationary behaviour at nth steady state is characterized by the wave R_n decreasing exponentially to zero when $r \to \infty$, and the wave amplitude crosses zero at the sphere with Bohr's radius $r_{o,n}$. It means the existing of the electron path at the sphere of this radius [19]. The predicted electron paths in Hydrogen atom appear to be equally distributed along the sphere of the fixed radius, not in the cloud of Schrödinger's wave function. This is a fundamental difference between the picture of Hydrogen atom discussed here and the well-known results of Schrödinger's quantum theory. We consider that the verification of the predicted *equal distribution of the electron density along the spheres of the steady quantum states of hydrogen-like atom* should be done first in order to test the suggested theory. Using the existing possibilities of modern experimental physics this experiment has a chance to be conducted. Particularly, the methods of scanning tunnel and force microscopy [20, 21] are the advanced tools for the detailed analysis of the spatial peculiarities of the inter-atomic electron motion.

It should be noted that in this work we use the specific approach for getting the knowledge of electron nature and its properties. The approach is based on the acknowledgement of the electron unified physical nature, which contains it's wave entity and the corpuscular (path) way of existence without contradictions. The approach is not specifically based on the capabilities of the existing methods of measurement.

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References

1. Fermat, P., 1891. Senthesis ad Refractiones. Oeuvres, v.1. Paris, pp: 173-179.
2. Leibniz, G., 1860. Mathematische Schriften, Hrsg, von C.J. Gerhardt, v.III.
3. Maupertuis, P., 1746. Les lois du movement et du repos, deduites d'un principe metaphysique, Mem. de l'Acad. d.Sci.de Berlin.
4. Euler, L., 1750. Reflexions sur quelques lois generals de la nature qui s'observent dans les effets des forces quelconques, Mem. de l'Acad. d. Sci. de Berlin. v. 4, pp: 189-219.
5. Lagrange, J., 1892. Application de la method, exposee dans le memoire precedent a la solution de different problems de dynamique. Oeuvres, v.1. Paris, pp: 365.
6. Hamilton, W.R., 1834. On a General Method in Dynamics. Philos. Trans.
7. Einstein, A., 1916. Hamiltonsches Prinzip und allgemeint Relativitatstheorie. "Sitzungsberichte der Preuss. Arademie der Wissenschaften".
8. Planck, M., 1915. Das Prinzip der kleinsten Wirkung. "Die Kultur der Gegenwart". V. 1. Physik.
9. Dirak, P., 1979. The principles of quantum mechanics. Moscow: Nauka.
10. Feynman, R. and A. Hibs, 1968. Quantum mechanics and path integrals. Moscow: Nauka.
11. De Broglie, L., 1925. Recherches sur la theorie des quanta. Ann. de Phys. V.3.
12. Schrödinger, E., 1926. Quantisierung als Eigenwertproblem (I Mitt) Annalen der Physik, Bd 79.
13. Valishin, F.T., 1992. The problem of methodology in the concept of dynamism. Methodological concepts and schools in the USSR. Novosibirsk, pp: 151-154.
14. Valishin, N.T., 1998. Local variational principle: to the new statement of the direct and the inverse problems of dynamics. Thesis of candidate of phys.-math. sciences, KSTU named after A.N.Tupolev, Kazan, pp: 111.
15. Valishin, N.T., 2005. The V-function method for measuring waves in mathematical modelling. Bulletin of KSTU named after A.N.Tupolev, 1: 26-28
16. Hertz, H., 1952. Principles of Mechanics in new edition. Moscow.
17. Bohr, N., 1913. On the constitution of atoms and molecules. Philosophical Magazine, v. 26: 1-25, 476-502, 857-875.
18. Rodimov, B.N., 2010. Auto-oscillation quantum mechanics. Physical-mathematical heritage: physics (quantum mechanics). Moscow, pp: 416.
19. Valishin, N.T, F.T. Valishin and S.A. Moiseev, 2011. Trajectory-Wave Approach to Electron Dynamics in Hydrogen Atom. Cornell University Library.
20. Seo1, Y. and W. Jhe, 2008. Atomic force microscopy and spectroscopy, Rep. Prog. Phys., 71, 016101.
21. Gross, L., F. Mohn, N. Moll, P. Liljeroth and G. Meyer, 2009. The Chemical Structure of a Molecule Resolved by Atomic Force Microscopy, Science, 325, 1110.

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