Treatment of Inconsistent Linear Systems of Equations Based on the Simplex Algorithm

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Abstract. In this paper, we introduce an algorithm for treating an in-consistent linear algebraic system $Ax = b$ of the form $\sum_{j=1}^{n} a_{ij} x_j = b_i$, $i = 1, 2, 3, \ldots, m$, where the matrix $A = \left[a_{ij}\right] \in R^{m \times n}$ and $b \in R^m$. Our algorithm is based on phase I of the simplex method where we check whether the linear system is consistent or not. In case of inconsistency the algorithm will decompose it to a finite number of consistent linear systems of equations and introduce a solution of each system. An example is added to illustrate our algorithm.

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1. Introduction

Mathematical modeling of real life-problems, is often not simple nor unambiguous. In in order to make optimal decisions, the first step in the process of gaining the necessary insights is converting this type of problems from a natural language to a mathematical system. Linear programming or constraint programming is a powerful tool for understanding problems and constructing suitable abstractions in order to help us arrive at the best decisions.

When a mathematical system is inconsistent and we are left without an applicable solution, we can restructure the mathematical model with a view to feasibility. As discussed in (Chakravarti, 1994, Chinneck, 1997, Greenberg, 1993 – Guieu and Chinneck, 1999 and Roodman, 1979), the information relevant to infeasibility can be gained by analyzing the model. It may be of interest to study some theoretical aspects concerning duality concepts for inconsistent systems, see (Eremin 1981, Vatolin, 1986 and 1998), as well as to evaluate the distance to feasibility in the case of ill-posed problems, see (Pena, 2000 and Renegar, 1994).

Rectifying the model is also possible through removing constraints. A method to remove constraints in order to achieve a feasible set, based on a hierarchical classification of constraints, is proposed by these authors, (Holzbaur, Menezes and Barahona, 1996). Removing a minimal set of constraints is another possibility.

The generation of irreducible inconsistent systems, as seen in (Chinneck, 1997 and 1991, and Tamiz, Mardle and Jones, 1996), enables the implementation of a heuristic approach. A set of inconsistent constraints for which every proper subsystem is consistent, is called an IIS. Several algorithms for finding an IIS exist, for more detail see (Holzbaur, Menezes and Barahona, 1996).

Many commercial solvers like CPLEX and LINDO can be used efficiently for finding IISs. Iteratively generating an IIS, removing one constraint from it and then repeating this process, gives us a feasible set of constraints. Sometimes, we can reintroduce some constraints that no longer need to be removed to achieve feasibility. When the deleted restrictions are soft constraints, this approach might be acceptable, even though it ignores some of the inherent relations between the decision variables. On the other hand, this procedure can be completely inadequate when we have a situation where it is preferable to derive a feasible model that essentially retains the parent constraints, but due to parameter approximation it is admissible to perturb those parameters somewhat, see (Amaral, Fernandes, Judice and Sherali, 2009).

In this paper we introduce a new approach to treating the inconsistent systems of linear equations. This approach decomposes the inconsistent system to a finite number of consistent linear systems and introduces a solution for each system. The approach is based on the idea introduced in phase I of the Simplex Method. The Simplex Method is a systematic procedure for generating and testing candidate vortex solutions for a linear program. It begins an arbitrary corner of the solution set. The Simplex Method selects the variable that will produce the largest change towards the minimum (or maximum) solution, see (Belegundu and Chandrupatia, 2011).

The Simplex Method contains two phases and here we use phase I which helps us to determine whether the original system is feasible or not. Then, if it is not feasible, we construct an initial basic feasible solution whenever the original system allows it. Towards this end, we consider a so-called auxiliary linear system, see (Hiebert, 1980 and Duy, 2011).

The following example is an attempt to illustrate our idea. Consider the following systems of linear equations

$$x + y + z = 3,\ \ 2x - y + z = 1,\ \ x + y + z = 4,$$

$$x + y + 2z = 6 \ \text{ and } \ 2y + z = 4.$$

The previous system has no solution (inconsistent), but if we omit the third and fourth equations, the system is solvable and the solution is $x = 0,\ y = 1$ and $z = 2$.

In the following section we introduce the problem formulation and numerical solution procedure. In section three we give a detailed discussion of the algorithm to decompose an inconsistent system of linear equations into a finite number of consistent systems of linear equations. Furthermore, we then discuss in detail the test problems with all their possible solutions in section four. Finally, the conclusion is discussed in section five, and we come to acknowledgments in section six.

2. Problem Formulation and Numerical Solution Procedure

Our problem concerns a system of linear equations $Ax = b$,

where $A \in R^{m\times n}$, $x \in R^{n}$ and $b \in R^{m}$, (2.1)

where we have to determine whether it is consistent and we can find the solution, or it is inconsistent and we have to further divide it into subsystems of linear equations until each of them is consistent. Our treatment is based on the Simplex Method, let $x = x^{\cdot} - x^{\cdot\cdot}$

so system (2.1) can be formulated as follows:

$$A(x^{\cdot} - x^{\cdot\cdot}) = b$$

Such that

$$A \in R^{m\times n},\ x^{\cdot}, x^{\cdot\cdot} \in R^{n}, x^{\cdot}, x^{\cdot\cdot} \geq 0 \text{ and } b \in R^{m}. \tag{2.2}$$

For $b \geq 0$, add the artificial variable η, where $\eta = (\eta_1, \eta_2, ..., \eta_m)$. Therefore, we can replace system (2.2) with the following system

$$A(x^{\cdot} - x^{\cdot\cdot}) + \eta = b,$$

$$\text{Such that } \ x^{\cdot}, x^{\cdot\cdot}, \eta \geq 0. \tag{2.3}$$

If $\eta = (0, 0, ..., 0)$ then it is simply the original system (2.2). Thus, if we can find values for the ordinary variables $x^{\cdot}, x^{\cdot\cdot}$ we can say that the system (2.1) is consistent.

Obviously, we would like to exclude non-zero artificial variables. This can be done by writing this linear programming problem:

$$\text{minimize } \ \sum_{i=1}^{m} \eta_i$$

$$\text{subject to } \ A(x^{\cdot} - x^{\cdot\cdot}) + I_m \eta = b,$$

$$x^{\cdot}, x^{\cdot\cdot}, \eta \geq 0, \tag{2-4}$$

where, I_m is an identity matrix $m \times m$. We can see that the artificial variables should have zero value because they have no effect on the system (2.2), but we introduce them artificially to help in the treatment of our problem and to help identify an initial, basic, feasible solution of the system (2.4).

Definition 2.1. Basic Feasible Solution

Any solution of a linear system of equations $A(x^{\cdot} - x^{\cdot\cdot}) + I_m \eta = b$ is called basic feasible solution if the number of zero variables is at least $(n - m)$ and the other variable is greater than zero.

Lemma 2.2. The optimum objective function value in system (2.4) is bounded by zero. Furthermore, if the optimal solution of system (2.4) has $\eta_i = 0$, where $i = 1, 2, ..., m$, then the variables $x^{\cdot}, x^{\cdot\cdot}$ have known values in the system of linear equations (2.2), and therefore, the original system of linear equations (2.1) will have its own solution, since $x = x^{\cdot} - x^{\cdot\cdot}$.

Proof. Clearly, setting $\eta_i = 0$ where $i = 1, 2, ..., m$ will produce an objective function value of zero. We cannot obtain a smaller objective function value, since $\eta_i \geq 0$ where $i = 1, 2, ..., m$. However, if at optimality we have $\eta_i = 0$ where $i = 1, 2, ..., m$, then we have found a basic feasible solution of system (2.4), which means a solution of our original system of linear equations (2.1).

Theorem 2.3. Let $x^{\cdot}, x^{\cdot\cdot}$ and η_i be an optimal solution of the system of linear equations (2.4), then system (2.1) is consistent if and only if, $\eta_i = 0$.

Proof. In lemma (2.2) we have proven that if $\eta_i = 0$, then the variables $x^{\cdot}$ and $x^{\cdot\cdot}$ are a basic feasible solution of system (2.4) and thus $x = x^{\cdot} - x^{\cdot\cdot}$ is a solution of the system of linear equations (2.1). Hence, system (2.1) is consistent. On the other hand, suppose that the system of linear equations (2.1) is consistent, then it has a least one solution, so the value of η_i must be zero and $x^{\cdot}, x^{\cdot\cdot}$ are the solution of the system of linear equations (2.2) and then $x = x^{\cdot} - x^{\cdot\cdot}$ is the solution of the system of linear equations (2.1). This is clearly an optimal solution of system (2.4), because it forces the objective function value to its lower boundary (zero).

3. Main Algorithm Outline

In this section we introduce and explain an algorithm to decompose an inconsistent system of linear equations into a finite number of consistent systems of linear equations. Below, we will present the basic steps of the procedure we use in our algorithm. In the beginning, we prepare the equations to ensure that the right hand side of all equations is positive and replace each variable by the difference of two variables, such that $x_j = x_j^{\cdot} - x_j^{\cdot\cdot}, x_j^{\cdot}, x_j^{\cdot\cdot} \geq 0,$ and $j = 1,2,...,n.$ Second, we test the first and second equations for consistency using phase I of the simplex method using the LINGO software package. Third, if they are consistent we add the next equation and apply the same procedure until we arrive at one equation that is inconsistent with all the previous consistent ones . Now we separate this equation from the system and put it into a newly created subsystem. We continue to apply the same procedure until we have tested all equations in the system for consistency. Fourth, we now use the same procedure on all separate subsystems. Finally, we will obtain a finite number of consistent systems with their solutions.

Algorithm 3.1. (The Main Algorithm)

(Initialization)

For $Ax = b$, $x_j = x_j^{\cdot} - x_j^{\cdot\cdot}, x_j^{\cdot}, x_j^{\cdot\cdot} \geq 0$ *and* $j = 1,2,...,n.$ *Set* $B_L =$ *the number of generated subsystems, where* $L = 1,2,...$ $, I$ *is an identity matrix.* η_i *are artificial variables, where* $\eta_i \geq 0,$ *and* $i = 1,2,..., m.$

INPUT: integers m, n *and* s *; matrices* (a_{ij}) *and* (I_{ii}) *; vectors* x_j, b_i *and* η_i *for each* $i = 1,2,...,m$ *and* $j = 1,2,\ldots,n.$

OUTPUT: all the linear subsystems set which are consistent with their solutions.

Step 1.

Set $\sum_{j=1}^{n} a_{ij} x_j = b_i$ *in* B_L

Step 2.

For each $b_i < 0$ *, multiply the equation by (−1)*

Step 3. (Add η_i *for each equation in the system)*

Set $m = m_L$

Set $\sum_{j=1}^{n} a_{ij} x_j + I_{ii}\eta_i = b_i$, *in* B_L

Step 4. (Solve and compute a consistent linear system)

Set $i = 1$

Step 5.

for $s = i = 1,2,..., i+1$

Set $z_s = \min \sum_{s=1}^{i+1} \eta_s$,

Subject to,

$\sum_{j=1}^{n} a_{ij} x_j^{\cdot} - x_j^{\cdot\cdot}) + I_{ii}\eta_i = b_i, i = 1,2,..., i+1$

where $x_j^{\cdot}, x_j^{\cdot\cdot}, \eta_s \geq 0$

If $i + 1 = m_L$

then go to step (8)

If $z_s = 0$

then go to step (6)

Else

insert equation ($a_{i+1,j} x_j + \eta_{i+1} = b_{i+1}$*) into sub-system* B_{L+1}

Step 6. (Update)

Set $i = i+1$

Set $s = 1,2,..., i+1$

Step 7.

go to step (5)

Step 8.

Set $x_j = x_j^{\cdot} - x_j^{\cdot\cdot}$

OUTPUT x_j, B_L *(*B_L *is a linear consistent subsystem)*

Step 9.

If $\sum_{i=1}^{L} m_i = m$

go to step (11)

Step 10.

Set $L = L + 1$

go to step (3)

Step 11.

OUTPUT $(B_1, ..., B_L)$

Step 12.
Stop. (The procedure is complete.)

4. Numerical Application

Here we give examples of linear systems of equations to clarify our algorithm for testing the consistency of the system and decomposing it into a finite number of consistent linear equations. Below is a detailed outline of the basic steps involved in solving our problem using LINGO software.

4.1. Example 1

Consider the following system of linear equations (I):

$$x_1+2x_2-x_3+x_4=0, \quad (4.1)$$
$$x_1-x_2+x_3-x_4=2, \quad (4.2)$$
$$2x_1+x_2+4x_3-x_4=3, \quad (4.3)$$
$$x_1-x_2+x_3+x_4=0, \quad (4.4)$$
$$3x_1+3x_2+4x_3-2x_4=-1, \quad (4.5)$$
$$4x_1-4x_2+4x_3=4, \quad (4.6)$$
$$-2x_1+2x_2-2x_3+x_4=-3, \quad (4.7)$$

In the above system (I) the first step is to prepare the equations by multiplying equations (4.5) and (4.7) by (-1). Then we replace each x_j with $x_j=x^{\cdot}_j-x^{\cdot\cdot}_j$ such that $x^{\cdot}_j, x^{\cdot\cdot}_j \geq 0$ so that now system (I) takes the following form:

$$x^{\cdot}_1-x^{\cdot\cdot}_1+2x^{\cdot}_2-2x^{\cdot\cdot}_2-x^{\cdot}_3+x^{\cdot\cdot}_3+x^{\cdot}_4-x^{\cdot\cdot}_4=0, \quad (4.8)$$
$$x^{\cdot}_1-x^{\cdot\cdot}_1-x^{\cdot}_2+x^{\cdot\cdot}_2+x^{\cdot}_3-x^{\cdot\cdot}_3-x^{\cdot}_4+x^{\cdot\cdot}_4=2, \quad (4.9)$$
$$2x^{\cdot}_1-2x^{\cdot\cdot}_1+x^{\cdot}_2-x^{\cdot\cdot}_2+4x^{\cdot}_3-4x^{\cdot\cdot}_3-x^{\cdot}_4+x^{\cdot\cdot}_4=3, \quad (4.10)$$
$$x^{\cdot}_1-x^{\cdot\cdot}_1-x^{\cdot}_2+x^{\cdot\cdot}_2+x^{\cdot}_3-x^{\cdot\cdot}_3+x^{\cdot}_4-x^{\cdot\cdot}_4=0, \quad (4.11)$$
$$-3x^{\cdot}_1+3x^{\cdot\cdot}_1-3x^{\cdot}_2+3x^{\cdot\cdot}_2-4x^{\cdot}_3+4x^{\cdot\cdot}_3+2x^{\cdot}_4-2x^{\cdot\cdot}_4=1, \quad (4.12)$$
$$4x^{\cdot}_1-4x^{\cdot\cdot}_1-4x^{\cdot}_2+4x^{\cdot\cdot}_2+4x^{\cdot}_3-4x^{\cdot\cdot}_3=4, \quad (4.13)$$
$$2x^{\cdot}_1-2x^{\cdot\cdot}_1-2x^{\cdot}_2+2x^{\cdot\cdot}_2+2x^{\cdot}_3-2x^{\cdot\cdot}_3+x^{\cdot}_4-x^{\cdot\cdot}_4=3, \quad (4.14)$$
$$\text{where} \quad x^{\cdot}_1,x^{\cdot\cdot}_1,x^{\cdot}_2,x^{\cdot\cdot}_2,x^{\cdot}_3,x^{\cdot\cdot}_3,x^{\cdot}_4,x^{\cdot\cdot}_4 \geq 0.$$

This form will be denoted as system (II). In the second step, we add the artificial variables $\eta_i, i=1,2,3,4,5,6,7, \eta_i \geq 0$ to every equation in system (II), as seen in the following form:

$$x^{\cdot}_1-x^{\cdot\cdot}_1+2x^{\cdot}_2-2x^{\cdot\cdot}_2-x^{\cdot}_3+x^{\cdot\cdot}_3+x^{\cdot}_4-x^{\cdot\cdot}_4+\eta_1=0, \quad (4.15)$$
$$x^{\cdot}_1-x^{\cdot\cdot}_1-x^{\cdot}_2+x^{\cdot\cdot}_2+x^{\cdot}_3-x^{\cdot\cdot}_3-x^{\cdot}_4+x^{\cdot\cdot}_4+\eta_2=2, \quad (4.16)$$
$$2x^{\cdot}_1-2x^{\cdot\cdot}_1+x^{\cdot}_2-x^{\cdot\cdot}_2+4x^{\cdot}_3-4x^{\cdot\cdot}_3-x^{\cdot}_4+x^{\cdot\cdot}_4+\eta_3=3, \quad (4.17)$$
$$x^{\cdot}_1-x^{\cdot\cdot}_1-x^{\cdot}_2+x^{\cdot\cdot}_2+x^{\cdot}_3-x^{\cdot\cdot}_3+x^{\cdot}_4-x^{\cdot\cdot}_4+\eta_4=0, \quad (4.18)$$
$$-3x^{\cdot}_1+3x^{\cdot\cdot}_1-3x^{\cdot}_2+3x^{\cdot\cdot}_2-4x^{\cdot}_3+4x^{\cdot\cdot}_3+2x^{\cdot}_4-2x^{\cdot\cdot}_4+\eta_5=1, \quad (4.19)$$
$$4x^{\cdot}_1-4x^{\cdot\cdot}_1-4x^{\cdot}_2+4x^{\cdot\cdot}_2+4x^{\cdot}_3-4x^{\cdot\cdot}_3+\eta_6=4, \quad (4.20)$$
$$2x^{\cdot}_1-2x^{\cdot\cdot}_1-2x^{\cdot}_2+2x^{\cdot\cdot}_2+2x^{\cdot}_3-2x^{\cdot\cdot}_3+x^{\cdot}_4-x^{\cdot\cdot}_4+\eta_7=3, \quad (4.21)$$
$$\text{where} \quad x^{\cdot}_1,x^{\cdot\cdot}_1,x^{\cdot}_2,x^{\cdot\cdot}_2,x^{\cdot}_3,x^{\cdot\cdot}_3,x^{\cdot}_4,x^{\cdot\cdot}_4,\eta_1,\eta_2,\eta_3,\eta_4,\eta_5,\eta_6,\eta_7 \geq 0.$$

After preparing the equations at step one and two, at step three, we now begin to obtain the solutions, starting from equation (4.15)

We construct the following linear programming problem to minimize the summation of η_1 and η_2 that is

$$\text{minimize } (\eta_1+\eta_2)$$
$$\textit{subject to}:$$
$$x^{\cdot}_1-x^{\cdot\cdot}_1+2x^{\cdot}_2-2x^{\cdot\cdot}_2-x^{\cdot}_3+x^{\cdot\cdot}_3+x^{\cdot}_4-x^{\cdot\cdot}_4+\eta_1=0,$$
$$x^{\cdot}_1-x^{\cdot\cdot}_1-x^{\cdot}_2+x^{\cdot\cdot}_2+x^{\cdot}_3-x^{\cdot\cdot}_3-x^{\cdot}_4+x^{\cdot\cdot}_4+\eta_2=2,$$
$$\text{where } x^{\cdot}_1,x^{\cdot\cdot}_1,x^{\cdot}_2,x^{\cdot\cdot}_2,x^{\cdot}_3,x^{\cdot\cdot}_3,x^{\cdot}_4,x^{\cdot\cdot}_4,\eta_1,\eta_2 \geq 0.$$

When solving this system, we find that $\eta_1=\eta_2=0$ which means that equation (4.15) and equation (4.16) are consistent.

In step four we add equation (4.17) to equations (4.15) and (4.16) and construct the following linear programming problem to minimize the summation of $\eta_1, \eta_2 \, and \, \eta_3$.

After solving this system, we find that $\eta_1=\eta_2=\eta_3=0$ which means that equations (4.15), (4.16) and (4.17) are consistent.

Following the same procedure for equation (4.18) we find that $\eta_1=\eta_2=\eta_3=\eta_4=0$ which means that equation (4.18) is consistent with equations(4.15), (4.16) and (4.17).

However, when we apply this procedure to equation (4.19) it results in $\eta_1=\eta_2=\eta_4=\eta_5=0,$ but $\eta_3=4.5,$ showing that equation (4.19) is inconsistent with the first four equations. Now we have to go to step five, meaning the creation of a separate subsystem to accommodate inconsistent equations. After extracting equation (4.19) from our system we continue to solve the remaining equations in the previous manner, isolating inconsistent equations in the subsystem.

After that, when solving equation (4.20) with equations (4.15), (4.16), (4.17), (4.18) we observe that

$\eta_1 = \eta_2 = \eta_3 = \eta_4 = \eta_6 = 0$, meaning they are consistent.

Solving the final equation (4.21) in the main system we find that equation (4.21) is inconsistent with the previous five consistent equations which leads us to extract it into the subsystem.

Now, in the main system, after finding equations (4.15), (4.16), (4.17),(4.18) and (4.20) to be consistent, one of the solutions is $\dot{x}_1 = 1, \ddot{x}_1 = 0, \dot{x}_2 = 0, \ddot{x}_2 = 0, \dot{x}_3 = 0, \ddot{x}_3 = 0, \dot{x}_4 = 1, \ddot{x}_4 = 0.$

In step six, in order to solve the subsystem and using the same procedure as in the main system we check equations (4.19) and (4.21) for consistency and find that $\eta_5 = \eta_7 = 0$, which means they are consistent with this solution $\dot{x}_1 = 0.58333, \ddot{x}_1 = 0, \dot{x}_2 = 0, \ddot{x}_2 = 0.916667,$ $\dot{x}_3 = 0, \ddot{x}_3 = 0, \dot{x}_4 = 0, \ddot{x}_4 = 0.$

In conclusion, we can say that our method has proven to be useful in treating inconsistent linear systems of equations. To obtain the solution of our problem, we have created two consistent linear systems. The first one contains equations (4.15), (4.16), (4.17), (4.18) and (4.20) where the solution is $x_1 = 1, x_2 = 0, x_3 = 0$ and $x_4 = 1$. The second one contains equations (4.19) and (4.21) with their solution $x_1 = 0.58333, x_2 = -0.916667, x_3 = 0$ and $x_4 = 0.$

4.2. Example 2

Consider the following system of linear equations:

$$x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 + 2x_6 = 10, \quad (4.22)$$

$$-2x_1 + x_2 - 2x_3 + x_4 - 2x_5 - x_6 = 8, \quad (4.23)$$

$$2x_1 - 2x_2 - x_3 - x_4 - 2x_5 + x_6 = 6, \quad (4.24)$$

$$x_1 + 8x_2 + 4x_3 - x_4 - 5x_5 + 2x_6 = 12, \quad (4.25)$$

$$-3x_1 + 2x_2 - 4x_3 + 2x_4 + 5x_5 + 2x_6 = 14, \quad (4.26)$$

$$3x_1 - 34x_2 - 3x_3 + 2x_4 + 2x_5 + x_6 = 100, \quad (4.27)$$

$$-x_1 + 4x_2 - 3x_3 + 2x_4 + x_5 + 2x_6 = 10, \quad (4.28)$$

$$5x_1 + 2x_2 - x_3 + x_4 + 2x_5 + 6x_6 = 50, \quad (4.29)$$

$$x_1 + 2x_2 + 2x_3 + x_4 + 4x_5 - 5x_6 = 8, \quad (4.30)$$

$$-4x_1 + x_2 - 4x_3 + x_4 - 4x_5 + 4x_6 = 16, \quad (4.31)$$

To further demonstrate the usefulness of our proposed algorithm, we follow the same procedure as in example 1, and arrive at the following solutions, where the above system is decomposed into two subsystems. The first one consists of equations (4.22), (4.23), (4.24), (4.25), (4.26), (4.27) and (4.28) and its solution is

$x_1 = 20.72410, x_2 = 5.46509, x_3 = -6.10360,$

$x_4 = 22.95209,\ x_5 = 12.56869$ and $x_6 = -33.96059,$

while the second one, which consists of equations (4.29), (4.30) and (4.31), has the following solution

$x_1 = 4.153846, x_2 = 0,\ x_3 = 0,$

$x_4 = 15.38462,\ x_5 = 0$ and $x_6 = 2.30792.$

This shows the suitability of our lgorithm for solving any consistency problems in similar systems.

5. Conclusion

In conclusion, our purpose for the work we have presented here is to introduce an algorithm to treat inconsistent linear algebraic systems of equations. Briefly, we propose an algorithm which is using phase I of the Simplex Method and which we have introduced to decompose an inconsistent system of equations into a finite number of consistent systems and which has the ability to find accurate solutions for the consistent subsystems it creates. Using this algorithm in our two test problems has proven it to be most useful and efficient. Furthermore, the algorithmic procedure we have introduced can be easily coded in any programming language. This approach could be developed further because of its great potential for handling general systems of linear equations and inequality. Therefore it could be an important contribution to solving problems of inconsistent systems in real life application.

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