Electron Beam Instability in Magneto-Active Inhomogeneous Cold Plasma

Khaled Hamed El-Shorbagy^{1,2}, Yahia. Swilem^{3,4} and Bassam Mohamad Dakhel⁵

^{1.} Math. Dept., Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia.

² Plasma Physics and Nuclear Fusion Department, Nuclear Research Center, Atomic Energy Authority, Cairo,

Egypt.

^{3.} Physics Department, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia.

⁴ Physics Department, Faculty of Science, Zagazig University, 44519 Sharkia, Egypt.

⁵ General Required Courses Department, Jeddah Community College King Abdulaziz University, Jeddah, Saudi

Arabia

drkhalede@yahoo.com

Abstract: This paper reports on the interaction of an electron beam with magneto-active inhomogeneous cold plasma that results in an increase in the beam-plasma instability. We show that the variation in the plasma density has a profound effect on the instability of the spatial beam of plasma. The application of an external static magnetic field leads to enhancement if the power absorption from the electron beam, and accordingly to plasma heating in beam-plasma system.

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1. Introduction

The problem of electron beam linear interaction with cold unmagnetized plasma has been studied intensively (Amein, 1975, Bohmer, 1973, Gupta, 1988, Thode, 1977, Frigo, 2005, El-Sharif, 2009, Silin, 2007, Zaki, 1999 and Kuzelev, 2012). In these reports, a beam-plasma interaction takes the form of amplification of waves by the electron beam. It can be concluded that due to the resonance rise of the wave field with the plasma dielectric permeability being reduced to zero, the power absorbed by the plasma is finite and independent of the value of the dissipation. In this case, the beam not only amplifies waves in the plasma, but also provides effective absorption of these waves by the plasma. Investigation of the electron beam-plasma interaction is of great interest for the development of effective methods for plasma stability, amplification and generation of electromagnetic waves, acceleration of charged particles in plasma, high frequency heating of plasma (Bohmer, 1973, Ivanov, 1984, Kuzelev, 1990, Kuzelev, 1995 and Amein, 1994), etc..

In this paper we investigate the influence of both variable cold plasma under the effect of an external static magnetic field $(\vec{H}_{ext} = H_0 \vec{e}_z)$ directed along z-direction and an electron beam (under the condition that the waves have small phase velocity compared with the beam velocity) on the quenching of the beam-plasma instability. We suppose also that the electron beam propagates along the direction of the magnetic field $\vec{V}_b = (0,0,V_b)$.

We also consider a semi-infinite beam plasma system $(x \ge 0)$, in which two models are used for the unperturbed plasma density $n_0(x)$, which is an arbitrary function of x; one is $(n_0(x) = N_0(1-x); N_0$ is a constant) and the other is more realistic $\omega_p^2(x) = \omega_{p1}^2(x)(1+\varepsilon(x/L)); (L\ge x\ge 0; \varepsilon>-1)$. It is assumed that ions are at rest and that the electron beam is cold and homogeneous.

2. Fundamental waves

The initial linearized set of equations (the equation of motion and the continuity equation) describing the oscillations in 1-D, for the electron beam, which travels along the magnetic field, are:

$$\frac{\partial V_b}{\partial t} + (\vec{V}_b \cdot \vec{\nabla})\vec{V}_b = -e\vec{E}; \quad \vec{V}_b = \vec{V}_{0b} + \vec{V}_{1b} \quad , \quad \vec{V}_{0b} = V_{0b} \quad \vec{e}_z$$
(1)

$$\frac{\partial N_b}{\partial t} + \vec{\nabla} \cdot (N_b \vec{V}_b) = 0; \ N_b = n_{0b} + n_{1b}$$
(2)

where; V_{1b} is the component of beam velocity in the z-direction.

The initial linearized set of equations (the equation of motion and the continuity equation) describing the oscillations in 1-D, for inhomogeneous plasma electrons in the oscillating electric field and a static magnetic field $\vec{H}_{ext.}$ perpendicular to the plasma density gradient are given by: -

$$\frac{\partial V_p}{\partial t} + (\vec{V}_p \cdot \vec{\nabla}) \vec{V}_p = -\frac{e}{m} [\vec{E} + \frac{1}{C} (\vec{V}_p \times \vec{H}_{ext.})] - v \vec{V}_p \quad (3)$$

$$\frac{\partial N_P}{\partial t} + \vec{\nabla} \cdot (N_P \vec{V}_P) = 0; \ N_P = n_{0P} + n_{1P} \quad (4)$$

In equations (1) – (4), n_{oP} , n_{1P} and \vec{V}_{1P} are the unperturbed and perturbed density of the plasma and the velocity of plasma, respectively, and ν is the collision coefficient of plasma electrons with plasma ions, while \vec{V}_{0b} and n_{0b} are the unperturbed velocity and density of the beam, respectively. In the case of weak nonlinearity $(|n_{1b}| << n_{0b}, |V_{1b}| << V_{0b})$,

we have $\left| \vec{V}_{1p} \frac{\partial}{\partial x} \right| < \left| \frac{\partial}{\partial t} \right|.$

By using the Poisson's equation

$$\frac{dE}{dx} = -4\pi \, e \, (n_{1p} + n_{1b}) \tag{5}$$

one can reduce equations (1) - (5) to a single second order differential equation,

$$\frac{d^2 F}{dx^2} + \chi^2(x) F(x) = 0$$
 (6)

where

$$F(x) = \chi^{-2}(x) E(x) e^{-\frac{i\omega}{v_{0b}}x}$$

$$\widetilde{\omega} = \left((\omega + i\nu)^2 - \omega_c^2\right)^{\frac{1}{2}}, \quad \omega_c = \frac{eH_0}{mc},$$

$$\chi^2(x) = \frac{\omega_b^2}{V_{0b}^2} \frac{\omega\widetilde{\omega}}{(\omega\widetilde{\omega} - \omega_p^2)},$$

$$\omega_{p,b}^2 = \frac{4\pi e^2 n_{0(p,b)}}{m}$$

$$(7)$$

It is clear that the wave number $\chi(x)$ contains the effect of the static magnetic field, i.e., it depends on H_0 . An equation similar to (6) was obtained previously by many authors (Bohmer, 1973 and Ivanov, 1984), however the static magnetic field effect H_0 , which is of importance for the analysis of plasma instability and heating, was neglected.

The solution of equation (6) in the region $x \le 0$ gives the following spatially growing modes (upstream):

$$E_1(x,t) = E_1(x) e^{i(k_1x - \omega t)}$$
, $\text{Im} k_1 < 0$

where $k_1 = (\frac{\omega}{V_{0b}}) + \chi_1$, χ_1 is given by relation (7) in the region $x \le 0$.

The most important mode is that where $|\text{Im} \chi_1(\omega)|$ is a maximum. Providing that the discontinuity at x = 0 has no influence on the solution in the region x < 0, we can determine the following solutions of equation (6) in the regions $x \le 0$ and $x \ge 0$ as

$$\begin{split} F_1 &= A_1 \, e^{i\chi_1(x) \, x} \; ; \quad x \leq 0 \\ F_2 &= A_2 \; e^{i\chi_2(x) \, x} + A_3 \; e^{-i\chi_2(x) \, x} \; ; \quad x \geq 0 \end{split}$$

where both Im χ_1 and Im χ_2 are negative. The constants of integration A_i (*i*=1-3) are determined from the boundary conditions that both *F* and $\frac{dF}{dx}$ are continuous at x = 0. Hence,

$$A_{2} = \frac{1}{2} \frac{\chi_{1} + \chi_{2}}{\chi_{2}} A_{1} ; A_{3} = \frac{1}{2} \frac{\chi_{2} - \chi_{1}}{\chi_{2}} A_{1}$$

Using definition (7), the electric field $E_2(x)$ is determined in terms of $E_1(0)$ as:

$$E_{2}(x) = \left\{ \left(\frac{E_{1}(0)\chi_{2}}{2\chi_{1}^{2}} \right) \left[(\chi_{1} + \chi_{2})e^{i\chi_{2}x} + (\chi_{2} - \chi_{1})e^{-i\chi_{1}x} \right] e^{\frac{i\omega}{v_{0b}}x}$$
(8)

 E_2 yields a power of the form:

$$|E_{2}(x)|^{2} = \left(\frac{|E_{1}(0)|^{2}|\chi_{2}|^{2}}{4|\chi_{1}|^{4}}\right) \begin{bmatrix} |\chi_{1} + \chi_{2}|^{2} e^{i(\chi_{1} - \chi_{1}^{*})x} + \\ |\chi_{2} - \chi_{1}|^{2} e^{-i(\chi_{2} - \chi_{2}^{*})x} + \\ 2(|\chi_{2}|^{2} - |\chi_{1}|^{2}) \\ \cos(\chi_{2} + \chi_{2}^{*})x - \\ 2i(\chi_{2}\chi_{1}^{*} - \chi_{1}\chi_{2}^{*}) \\ \sin(\chi_{2} + \chi_{2}^{*})x \end{bmatrix}$$
(9)

The 3^{rd} and the 4^{th} order terms on the R.H.S. of equation (9) are due to the mixing (spatial beats) between the growing and decreasing modes in the region $x \ge 0$. It is clear that the field power is

strongly affected by both the mixing and the effect of an external static magnetic field, i.e., the power of the electric field at $H_0 \neq 0$ is greater than the power when $H_0 = 0$. The mixing produces a noticeable effect on $|E_2(x)|^2$ under the conditions that $\chi_1 \neq \chi_2$ and $|\operatorname{Re} \chi_2| >> |\operatorname{Im} \chi_2|$, which are necessary in order for the trigonometric terms in (9) to vary rapidly compared with the exponential growth terms. The (*) quantities represent the complex conjugate values.

From equation (9), we get

$$\left|E_{2}(0)\right|^{2} = \left|\frac{\chi_{2}}{\chi_{1}}\right|^{4} \left|E_{1}(0)\right|^{2}$$
(10)

so that the electric field is discontinuous at x = 0.

Let us now analyze the solution (8) for a realistic plasma model, i.e. inhomogeneous plasma with a finite gradient in $n_o(x)$. For this it is assumed that:

$$\begin{array}{c} \omega_p^{2}(x) = \omega_{p1}^{2}(x) \left(1 + \varepsilon(x/L)\right); \\ (L \ge x \ge 0; \varepsilon > -1) \end{array} \right\}$$
(11)

corresponds to a constant density gradient in the transition region. It can be shown that the linear approximation is valid in this case provided the

$$1 >> |\varepsilon| (\omega_b / \omega_p)^2 (\omega_p / \nu) (V_{0b} / \omega_p L)$$
(12)

This indeed requires that $L \neq 0$.

In order to prove that expression (8) is essentially correct, a solution of the wave equation (6) using the density profile (11) yields the equation:

$$\frac{d^2 F}{d\xi^2} + b^{-2}\xi^{-1}F = 0 \tag{13}$$

where,

$$\xi = a - b(\frac{\omega_b}{V_{0b}}x), \ a = \frac{\omega \widetilde{\omega} - \omega_{p1}^2}{\omega \widetilde{\omega}}; \ b = \frac{\varepsilon}{L} \frac{V_{0b} \omega_{p1}^2}{\omega \widetilde{\omega} \omega_b}.$$

Solution of equation (13) is

$$F(z) = A z J_1(z) + B z N_1(z); \quad 0 \le x \le L$$
 (14)

where; $z = 2\xi^{-1/2}/b$ and $J_1(z)$; $N_1(z)$ are the Bessel function of the first and second kind, respectively.

$$F(x) = \sum_{\pm} A_{\pm} e^{\pm i\chi_2(x-L)}; \qquad x \ge L$$
(15)

With

$$A_{\pm} = \frac{\pi}{4} z_{1} F(0) \begin{cases} \left[\begin{pmatrix} N_{0}(z_{0}) J_{1}(z_{1}) - \\ J_{0}(z_{0}) N_{1}(z_{1}) \end{pmatrix} \pm \\ \begin{pmatrix} N_{0}(z_{1}) J_{1}(z_{0}) - \\ J_{0}(z_{1}) N_{1}(z_{0}) \end{pmatrix} \\ i & \begin{bmatrix} (N_{1}(z_{0}) J_{1}(z_{1}) - \\ J_{1}(z_{0}) N_{1}(z_{1})) \pm \\ (N_{0}(z_{0}) J_{0}(z_{1}) - \\ J_{0}(z_{0}) N_{0}(z_{1}) \end{pmatrix} \end{bmatrix} \end{cases}$$
(16)

such that

$$z_{0} = \frac{2\omega_{b}}{bV_{0b}} \frac{1}{\chi_{1}} ; \quad z_{1} = \frac{2\omega_{b}}{bV_{0b}} \frac{1}{\chi_{2}} \text{ Correspond} \quad \text{to}$$

(x = 0, x = L) and:
$$\chi_{1} = \frac{\omega_{b}\omega}{V_{0b}\sqrt{\omega\tilde{\omega} - \omega_{p1}^{2}}} , \quad \chi_{2} = \frac{\omega_{b}\omega}{V_{0b}\sqrt{\omega\tilde{\omega} - \omega_{p1}^{2}(1+\varepsilon)}} .$$

Equation (16) can be rewritten as:

$$A_{\pm} = \left(\frac{\omega_{b}}{bV_{0b}}\right)^{2} \frac{F(0)}{\chi_{1}\chi_{2}^{2}} \left[(\chi_{1} \mp \chi_{2}) \pm i \frac{V_{0b}}{\omega_{b}} b \chi_{1} \chi_{2} \right] \ln \frac{\chi_{2}}{\chi_{1}} .$$

The case of interest is when \mathcal{E} is not too small and (L/λ) is not very large [large rapid changes in $n_0(x)$] which is the opposite extreme from the WKB situation. From the definition following equation (13), we note that

$$\max (v / \omega_p; \varepsilon) > |\xi| \text{ and } |b| \approx \varepsilon (\omega_p / \omega_b) (\lambda / L)$$

where $\lambda = v_{0b} / \omega_p$. Therefore, if (L / λ) is not too
large, and ε is not too small, b is large and $|\xi|$ will
be fairly small. Consequently, z is small in this case
and the Bessel function in equation (16) may be
expanded for a small argument. When this is done,
one finally obtains the approximate result

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$$E_{2}(x) = \sum_{\pm} A_{\pm} e^{\frac{\pm i\chi_{2}(x-L) + \frac{i\omega}{V_{0b}}x}} = \left(\frac{\omega_{b}}{bV_{0b}}\right)^{2} \frac{F(0)}{\chi_{1}\chi_{2}^{2}} \left[\frac{(\chi_{1} \mp \chi_{2}) \pm}{i\frac{V_{0b}}{\omega_{b}}b\chi_{1}\chi_{2}}\right]$$
(17)
$$\ln \frac{\chi_{2}}{\chi_{1}} e^{\frac{\pm i\chi_{2}(x-L) + \frac{i\omega}{V_{0b}}\chi}}$$

where $x \ge L$.

Note that the effect of the external static magnetic field leads to an increase in the electric field intensity. Equation (17) agrees with the equation of Bohmer et al 1973 in unmagnetized plasma, i.e. at $H_0 = 0$. The result (17) may be compared with the result (8) for the simple discontinuous model. It can be seen that provided b is large and (13) is satisfied (L not too large and not too small), equation (8) is a good approximation to equation (17).

3. Conclusions

We investigated the beam-plasma heating due to an electron beam under the effect of an external static magnetic field. We considered a longitudinal 1-D oscillation in plasma, which is inhomogeneous and bounded in the direction of the beam motion. The imposition of an external static magnetic field leads to wave amplification and accordingly to plasma heating in a beam-plasma system. The power absorbed from the beam into the plasma is strongly affected by both mixing and the static magnetic field. The variation in the plasma density does have a profound effect on the spatial beam-plasma instability. This effect indicates that the resulting drop in intensity of electric field is a sensitive function of the plasma discontinuity. Growing modes can be observed only if the plasma density decreases.

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Corresponding Author:

Prof. Dr. Khaled Hamed El-Shorbagy Department of Math. Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia. E-mail: <u>drkhalede@yahoo.com</u>

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