Dispersion relation of dusty plasma low frequency waves in ionosphere E- layer Mid latitude with Lennard – Jones potential

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Abstract: For an un magnetized partially ionized three-component dusty plasma contains thermal and non thermal dust particles with Boltzmann distributed electrons and ions,the dispersion relation of dust waves with Lennard-Jones potential have been derived. Also by using the continuity and the momentum equations with Lennard- Jones potential for dust particles the phase velocity of dust-ion- acoustic wave (DIA) obtained, from which we have reached to the dispersion relation for dust wave. More over, for this we assumed that Magneto Hydro Dynamics (MHD) procedure,mean while dust particles in thermal and non thermal position assumed.
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Introduction

A dusty plasma is a multicomponent system consisting of electrons, ions and dust charged mesospheric particles together with neutral atoms or molecules that in this 'unusual' state of matter stems from the ubiquity with which it is found in the laboratory, in space, and in astrophysics [1]. Dusty plasmas may be formed due to the existence of dust particles with sizes ranging from one nanometer to one micrometer having large masses. The charged dust particles in the plasma bring about many significant changes in the behavior of the system including the creation of new waves modes. Many types of waves can exist in the plasma environments that characterize the media that here is ionosphere. In dusty plasmas, then it is well known that the two normal modes of un magnetized, weakly coupled plasmas are of the dust acoustic (DA) and dust ion acoustic (DIA) wave types[2,3]. Dust acoustic waves have been reported theoretically in un magnetized dusty plasma by [Rao et al 1990]. In recent years research on various low-frequency electrostatic waves in dusty plasmas reached to the momentum. [Rao, 1990] have demonstrated theoretically a low-frequency acoustic-like mode in un magnetized dusty plasma, called dust-acoustic (DA) mode.[Shukla &Silin,1992] also have reported another low-frequency acoustic-like mode called dust-ion-acoustic (DIA) mode supported by a dusty plasma with negatively charged stationary dust particles[4]. By using (L-J) potential we can see this potential applied to polar icy water molecules(as dust particles) in the E-layer mid latitude plasma. it is assumed that the medium is free of magnetic field, also we consider dusty plasma system with charged icy dust particles and Boltzmann distributed electron and ions.Then by means of a set of equations together with (L-J) potential at last we reached to dispersion relation of dust particles. More over, the (L-J) potential effects on the phase velocity of dusty waves is considered. It is clear that when the order of the particle separation in (L-J) potential is about(4×10^{-10}m - 7×10^{-10}m), the phase velocity of particle is less than the phase velocity of order of (3×10^{-10} m) particles separation.

It should be noted that in this paper we proceed into 2 parts:

1- In the first part we calculated the dispersion relation of the dusty wave with taking on account the Lenard-Jones potential and thermal velocity of dust particles in the fluid form while in the second part :
2- We calculated the dispersion relation of the dusty plasma wave with taking on account the Lenard-Jones potential without thermal velocity of the dust particles.

Description of ionospheric E-layer plasma model

We assume E- layer plasma as a model with 85−100km height above the earth surface as a multi–component, weakly ionized plasma consisting of plasma particles, with icy dust and neutral particles. Measurements suggest the presence of large amount of dust particles in the Earths lower atmosphere. The visible manifestations of these phenomena are clouds of icy particles called Polar Mesospheric Clouds (PMC) when viewed from space and Noctilucent Clouds (NLC) when viewed by observers on Earth [Vladimirov & Klumov, 2010], further dust is much heavier than the plasma particles, and the mass varies in range m_d (10^{-2} _ 10^{-18} gr). We assumed that the dust are all ice crystals so their temperature is about T_d =150 °K. The number density N_d of dust is up to 10^3 cm^{-3} [5,6]. In addition the various ions containing positive and negative ions viewed in dusty plasma ionosphere so we again assume that the positive ions can play an important role in charging process of icy dust particles.So that when there is no geomagnetic activity during a day,this layer reached to the lowest region of ionosphere[7]. The main component of ions in ionospheric E – layer is O_2^+. This layer is generated by photo ionization of neutral atmospheric species. Permanent interaction in this layer is [8]:

$O_2 + h\nu = O_2^+ + e^-$ (1)

the datas used here all obtained from electron and ion profiles for 90 km altitude above the earth surface of ionospheric E-layer. It can be seen that all emperaturest (° k), converted in to joule.[7, 8,9].

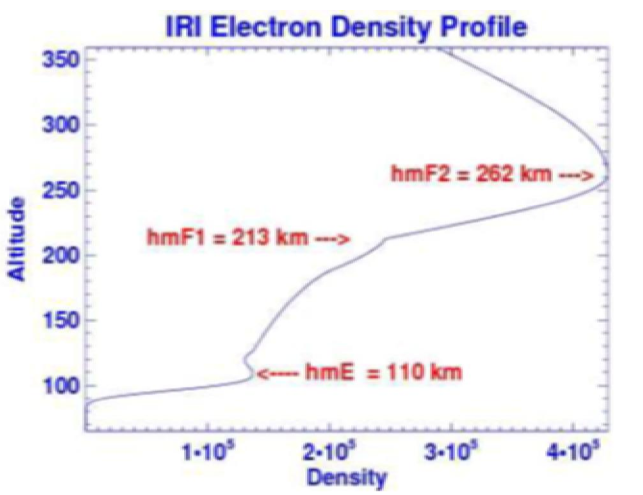


Fig-1.electeron density profile in atmosphere[7]

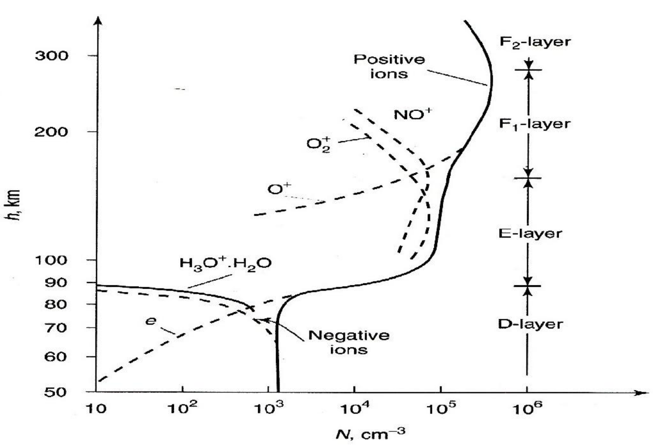
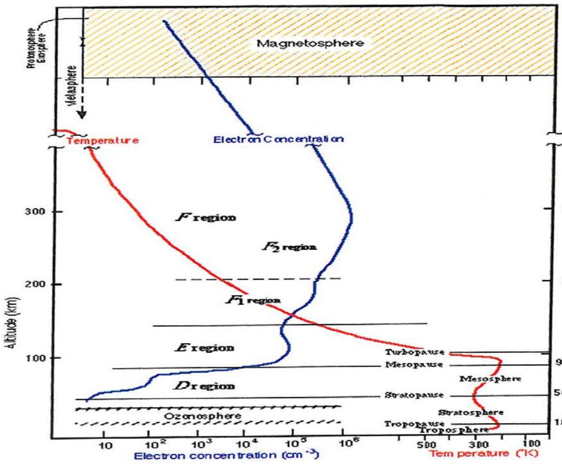


Fig-2. ion density profile in atmosphere[8]

Fig-3. electron density and temperature in atmosphere[9



An Introduction to Lennard-Jones Potential

Lenard-Jones potential (known as, 6-12 or L-J potential) is a physically model that shows the interaction of a pair of atoms or molecules. As it is seen, in this model.the Lennard-Jones potential applied to polar icy, in the form of water molecules.
The most common expressions for the (L-J) potential are:

$$V_{(L-J)} = 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right] \tag{2}$$

Here we considered the (L-J) Potential for two H2O molecules. With the parameters:[10]

$$\begin{bmatrix} \sigma = 0/32 \times 10^{-9}\, m \\ \varepsilon = 1/08 \times 10^{-21}\, J \end{bmatrix} \tag{3}$$

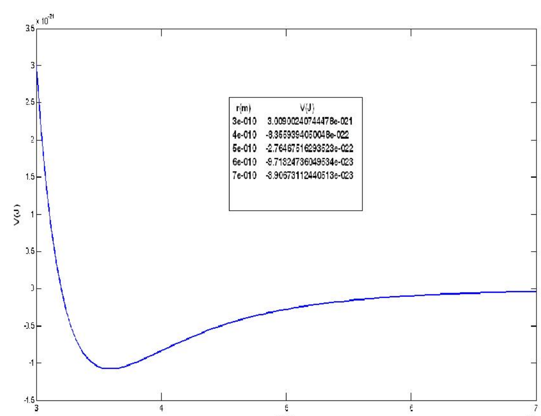


Fig-4.The Lennard-Jones Potential for two H_2O molecules[10]

Material and Method

Part 1, Dispersion relation by using thermal velocity for dusty fluid

let us consider dusty plasma system with charged icy dust particles with Boltzmann distributed electrons and ions. Then the electrostatic field applied to this dusty plasma, and the condition with dust particles thermal in momentum equation is assumed for $\nabla p \neq \circ$.

The basic system of equation governing the dust fluid is given by:

Electron and ions are taken to be Boltzmann distributed [11]:

$$n_i = -n_{io}\frac{e\varphi}{K_B T_i} \qquad n_e = n_{eo}\frac{e\varphi}{K_B T_e} \quad (4)$$

The continuity and the momentum equations in one dimensional form :

The continuity equation:

$$\frac{\partial n_{d_1}}{\partial t} + n_{d_\circ}\frac{\partial V_d}{\partial x} = \circ \quad (5)$$

The momentum equation:

$$\frac{\partial V_d}{\partial t} + V_d\frac{\partial V_d}{\partial x} = \pm\frac{q_{d_\circ} n_{d_\circ}}{m_d n_{d_\circ}}\frac{\partial \varphi}{\partial x} - \frac{\frac{\partial}{\partial x}p}{m_d n_{d_\circ}} - \frac{1}{m_d n_{d_\circ}}\frac{\partial}{\partial x}V_{(L-J)}$$

(6)

$$\frac{\partial V_d}{\partial t} = +\frac{q_d}{m_d}\frac{\partial \varphi}{\partial x} - \frac{3V_{td}^2}{n_{d_\circ}}\frac{\partial n_d}{\partial x} - \frac{1}{m_d n_{d_\circ}}\frac{\partial V_{(L-J)}}{\partial x}$$

Here the convection term ($v_d\frac{\partial vd}{\partial x}$) is neglected.[12]

The Poisson equation also is:

$$\nabla^2\varphi = 4\pi e(n_{e1} \pm Z_{d_\circ} n_{d_1} - n_{i1}) \quad (7)$$

With the Neutrality condition:

$$n_e \pm Z_d n_d - n_i = \circ \quad (8)$$

Here, m_d, n_d, Z_d and v_d, are, the dust mass, number density, charge number and dust fluid velocity respectively, v_{th} *is* dust fluid thermal velocity, $V_{(L-J)}$ is also Lennard – Jones potential, p_d *is* dust pressure φ is electrostatic potential which is related to the ion, electron and dust charge densities through Poisson's equation. Further more T_i, T_e, T_d, refer to the temperature of ion, electron and dust respectively.

Derivation of dispersion relation

By using equations mentioned above in one dimensional form and taking on account the following :

$$\frac{\partial}{\partial x} = ik, \frac{\partial}{\partial t} = -i\omega \quad (9)$$

From the continuity equation:

$$-i\omega n_{d_1} + n_{d_\circ}(ik)V_d = \circ \quad (10)$$

And the momentum equation:

$$-i\omega v_d = \pm\frac{q_d}{m_d}(ik)\varphi - \frac{3V_{td}^2}{n_{d_\circ}}(ik)n_d - \frac{1}{m_d n_{d_\circ}}(ik)V_{(L-J)}$$

(11)

For the continuity equation the value of n_d:

$$n_d = \frac{-q_d n_{d_\circ}\varphi + V_{(L-J)}}{m_d\left[(\frac{\omega}{k})^2 - 3V_{td}^2\right]} \quad (12)$$

And from the momentum equation the value of V_d:

$$V_d = +\frac{q_d}{m_d}(\frac{k}{\omega})\varphi + \frac{3V_{td}^2}{n_{d_\circ}}(\frac{k}{\omega})n_d - \frac{1}{m_d n_{d_\circ}}(\frac{k}{\omega})V_{(L-J)}$$

(13)

By using electrons, ions, dusty fluid densities the dispersion relation will be derived as bellow :

$$n_d = \frac{-q_d n_{d_\circ}\varphi + V_{(L-J)}}{m_d\left[(\frac{\omega}{k})^2 - 3V_{td}^2\right]} \text{ ‘ } n_i = n_{io}(\frac{-e\varphi}{K_B T_i}) \text{ ‘}$$

$$n_e = n_{eo}(\frac{e\varphi}{K_B T_e}) \quad (14)$$

it should be noted that:

$$\varphi = \varphi_\circ \exp(-\frac{r}{\lambda D}) \text{ ‘ } \varphi_\circ = \frac{kB\ Te}{e\ zd} \text{ ‘}$$

$$V_{td}^2 = \frac{T_d}{m_d} \quad (15)$$

Debye length for dusty plasma are taken to be as bellow :[12,13]

$$\lambda D = \frac{\lambda D_e \lambda D_i}{\sqrt{\lambda D_e^2 + \lambda D_i^2}} \text{ ‘}$$

$$\lambda D_e = 69\left[\frac{T_e}{n_e}\right]^{\frac{1}{2}} m \text{ ‘}$$

$$\lambda D_i = 69\left[\frac{T_i}{n_i}\right]^{\frac{1}{2}} m \quad (16)$$

Where λ_D, λ_i, λ_e, are Debye lengths for plasma particles including, dust, electron and ion, respectively.

By using Poisson equation:

$$\nabla^2\varphi = 4\pi e(n_{e1} + Zd_\circ nd_1 - n_{i1}) \quad (17)$$

And for Neutrality condition:

$$n_e \pm Zd_\circ nd_1 - n_{i1} = \circ \quad (18)$$

We reached to the following equation:

$$n_{eo}\left(\frac{e\varphi}{K_B T_e}\right) \pm Zd_\circ\left[\frac{\pm q_d n_d \varphi + V_{(L-J)}}{md\left[\left(\frac{\omega}{K}\right)^2 - 3V_{td}^2\right]}\right] + n_{io}\left(-\frac{e\varphi}{K_B T_e}\right) = \circ \qquad (19)$$

Where: $\varphi = \varphi_\circ \exp\left(-\frac{r}{\lambda D}\right)$ (20)

Finally the dispersion relation would be:

$$\frac{\omega}{K} = \left[\frac{Zd_\circ^{\,2}\left[n_{do} K_B T_e \exp\left(-\frac{r}{\lambda D}\right) + V_{(L-J)}\right]}{m_d\left[n_{eo} + n_{io}\left(\frac{T_e}{T_i}\right)\right]\exp\left(-\frac{r}{\lambda D}\right)} + 3\left(\frac{K_B T_d}{m_d}\right)\right]^{\frac{1}{2}}$$

(21)

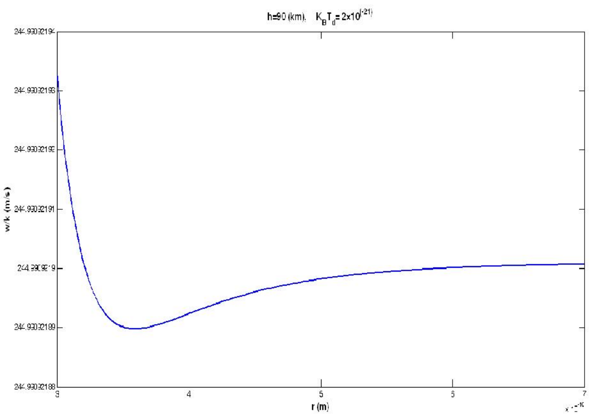


Fig-5. The phase velocity according to (L-J)potential in terms of separation between particles for $K_B T_d = 2 \times 10^{-21} (j)$

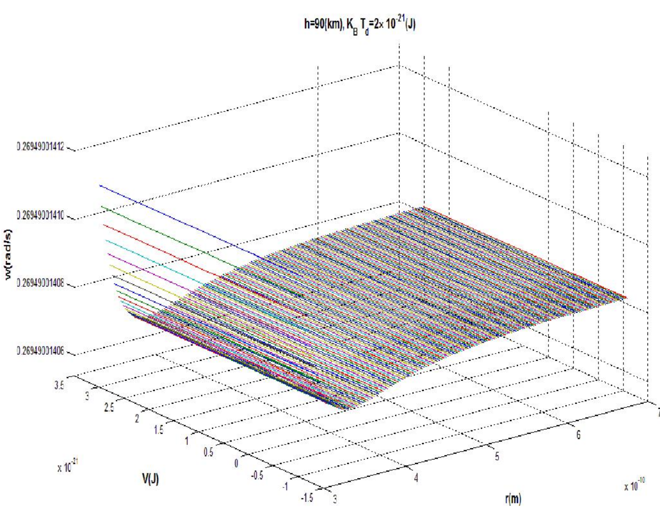


Fig-6.The phase velocity profile according to (L – J) potential in terms of separation between particles, for $K_B T_d = 2 \times 10^{-21} (j)$

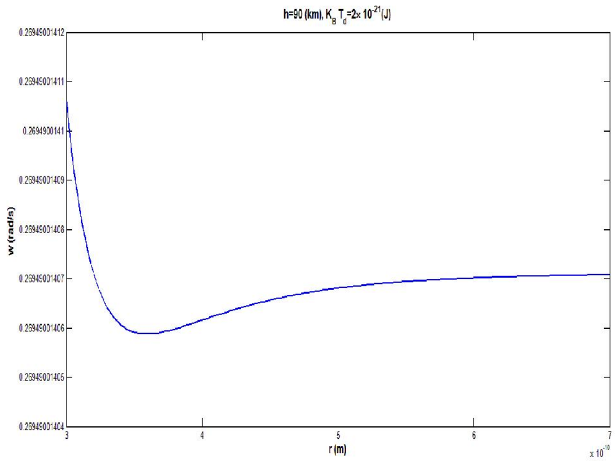


Fig-7. The phase velocity for wave number (k) according to (L-J) potential in terms of separation between particles, $K_B T_d = 2 \times 10^{-21} (j)$,

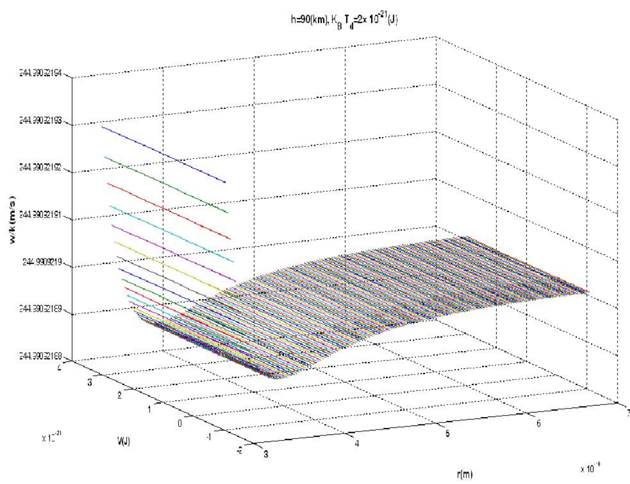


Fig-8. The phase velocity for wave number (k) according to (L-J) potential in terms of separation between particles,

$K_B T_d = 2 \times 10^{-21} (j)$, $K = 1/1 \times 10^{-3} (m^{-1})$

Part 2, Dispersion relation by using non thermal particles for dusty fluid

Now, let us consider dusty plasma system with charged icy dust particles with Boltzmann distributed electrons and ions. Then the electrostatic field applied to this dusty plasma and the condition with dust particles non thermal and momentum equation is assumed for $\nabla p = \circ$.

Then the basic system of equation governing the dust fluid is given by:

Electron and ions are taken to be Boltzmann distributed [14] :

$$n_i = -n_{io}\frac{e\varphi}{K_B T_i} \qquad n_e = n_{eo}\frac{e\varphi}{K_B T_e} \quad (22)$$

The continuity and the momentum equations in one dimensional form :

The continuity equation :

$$\frac{\partial n_{d_1}}{\partial t} + n_{d_\circ}\frac{\partial V_d}{\partial x} = \circ \quad (23)$$

The momentum equation:

(24)

If we consider $\nabla p = \circ$ then the momentum equation would be :

$$\frac{\partial v_d}{\partial t} + v_d\frac{\partial v_d}{\partial x} = \pm\frac{q_{d_\circ} n_{d_\circ}}{m_d n_{d_\circ}}\frac{\partial \varphi}{\partial x} - \frac{1}{m_d n_{d_\circ}}\frac{\partial V_{(L-J)}}{\partial x}$$

(25)

Here the convection term ($v_d \frac{\partial vd}{\partial x}$) is neglected.

Poisson equation also is :

$$\nabla^2 \varphi = 4\pi e(n_{e1} \pm Z_{d_\circ} n_{d_1} - n_{i1}) \quad (26)$$

With the neutrality condition:

$$n_e \pm Z_d n_d - n_i = \circ \quad (27)$$

Here, m_d, n_d, Z_d and v_d are the dust mass, number density, charge number and dust fluid velocity respectively, p_d *is* dust pressure, $V_{(L-J)}$ also is (L-J) potential and φ is electrostatic potential which is related to the ion, electron and dust charge densities through Poisson's equation. Further, T_i, T_e, T_d, referred to the temperature of ion, electron, and dust respectively.

Derivation of dispersion relation

By using equations in one dimensional form and taking on account the following :

$$\frac{\partial}{\partial x} = ik, \quad \frac{\partial}{\partial t} = -i\omega \quad (28)$$

For the continuity equation :

$$-i\omega n_{d_1} + n_{d_\circ}(ik)V_d = \circ \quad (29)$$

And the momentum equation :

$$-i\omega v_d = \frac{\pm q_d}{m_d}(ik)\varphi + \frac{1}{m_d n_{d_\circ}}(ik)V_{(L-J)} \quad (30)$$

For the continuity equation the value of n_d :

$$n_d = \frac{n_{d_\circ} q_d}{m_d}\left(\frac{k}{\omega}\right)^2\varphi + \left(\frac{k}{\omega}\right)^2\frac{1}{m_d}V_{(L-J)} \quad (31)$$

And from the momentum equation for V_d we will have :

$$V_d = \frac{\pm q_d}{m_d}\left(\frac{k}{\omega}\right)\varphi + \frac{1}{m_d n_{d_\circ}}\left(\frac{k}{\omega}\right)V_{(L-J)} \quad (32)$$

By using electrons, ions and dusty fluid densities the dispersion relation will be derived as bellow :

$$n_e = n_{eo}\left(\frac{e\varphi}{K_B T_e}\right),\ n_i = n_{io}\left(-\frac{e\varphi}{K_B T_e}\right),\quad n_d = \frac{-q_d n_d \varphi + V(L-J)}{m_d\left(\frac{\omega}{K}\right)^2}$$

(33)

It should be noted that :

$$\varphi = \varphi_\circ \exp\left(-\frac{r}{\lambda D}\right) \text{ ، } \varphi_\circ = \frac{kB\ Te}{e\ zd} \quad (34)$$

On account of Debye length for dusty plasma :

$$\lambda D = \frac{\lambda D_e \lambda D_i}{\sqrt{\lambda D_e^2 + \lambda D_i^2}}$$

$$\lambda D_e = 69\left[\frac{T_e}{n_e}\right]^{\frac{1}{2}} m$$

$$\text{،}\ \lambda D_i = 69\left[\frac{T_i}{n_i}\right]^{\frac{1}{2}} m \quad (35)$$

Where λ_D, λ_i, λ_e, are Debye lengths for plasma particles,including dust, electron and ion respectively.

By using Poisson equation:

$$\nabla^2\varphi = 4\pi e(n_{e1} + Zd_\circ nd_1 - n_{i1}) \quad (36)$$

And for neutrality condition:

$$n_e \pm Zd_\circ nd_1 - n_{i1} = \circ \quad (37)$$

We reached the following equation:

$$n_{eo}\left(\frac{e\varphi}{K_B T_e}\right) \pm Z_{do}\left[\frac{\mp q_d n_{d_\circ}\varphi + V_{(L-J)}}{m_d\left[\left(\frac{\omega}{k}\right)^2\right]}\right] + n_{io}\left(-\frac{e\varphi}{K_B T_i}\right) = \circ$$

(38)

Where:

$$\varphi = \varphi_\circ \exp\left(-\frac{r}{\lambda D}\right) \quad (39)$$

Finally the dispersion relation would be:

$$\left(\frac{\omega}{K}\right) = \left[\frac{Zd_\circ^{\ 2}\left[n_{do}(K_B T_e)\exp\left(-\frac{r}{\lambda D}\right) + V_{(L-J)}\right]}{m_d\left[n_{eo} + n_{io}\left(\frac{T_e}{T_i}\right)\right]\exp\left(-\frac{r}{\lambda D}\right)}\right]^{\frac{1}{2}}$$

$$\frac{\partial V_d}{\partial t}+V_d\frac{\partial V_d}{\partial x}=\pm\frac{q_{d_\circ}n_{d_\circ}}{m_d n_{d_\circ}}\frac{\partial\varphi}{\partial x}-\frac{\frac{\partial}{\partial x}p}{m_d n_{d_\circ}}-\frac{1}{m_d n_{d_\circ}}\frac{\partial}{\partial x}V_{(L-J)}$$

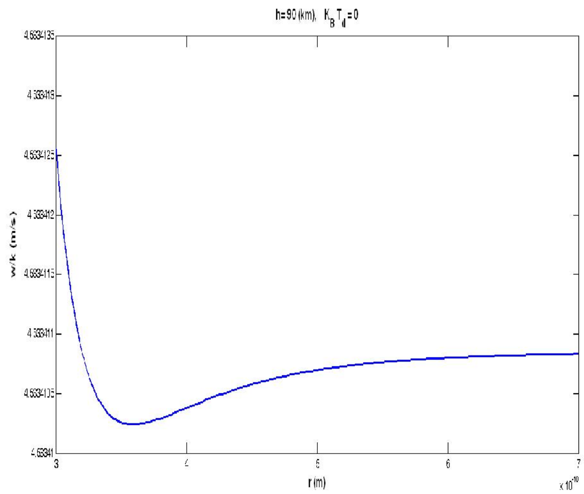


Fig-9. The phase velocity according to(L-J) potentialin terms of separation between particles $K_B T_d = \circ(j)$

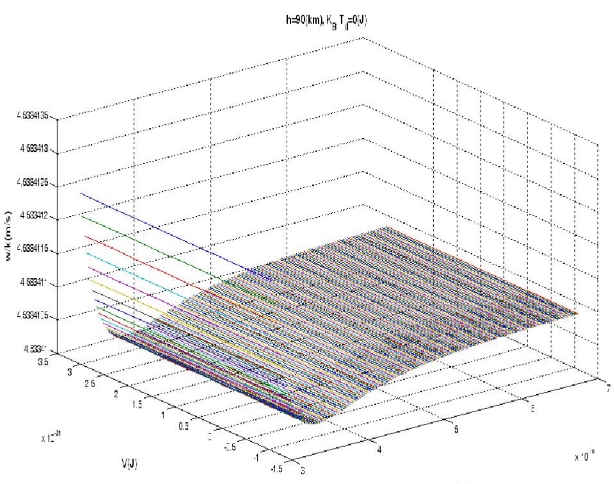


Fig-10.The phase velocity profile, according to (L-J) potential in terms of separation between particles $K_B T_d = \circ(j)$,

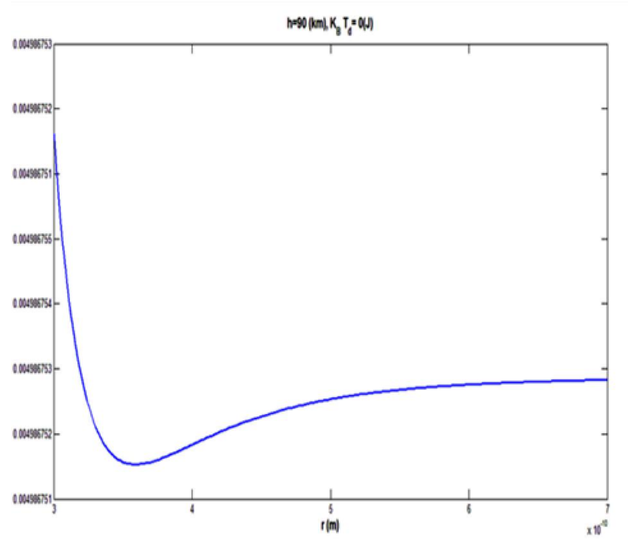


Fig-11.The frequency for wave number (k) according to (L-J) potential in terms of distances between particles, $K_B T_d = \circ(j), K = 1/1\times 10^{-3}(m^{-1})$

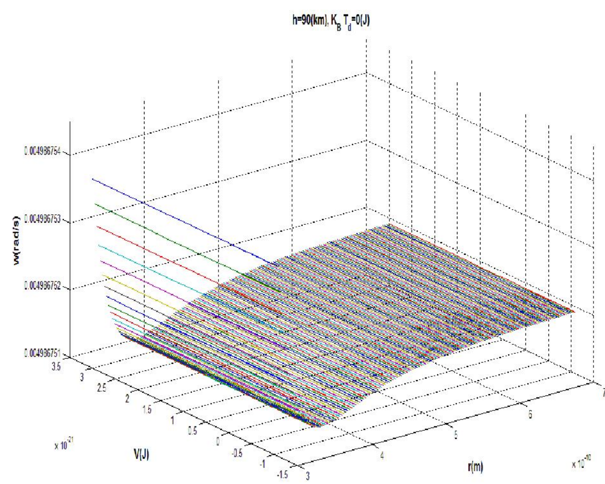


Fig-12.The frequency profile for wave number (k) according to (L-J) potential in terms of separation between particles,

$K_B T_d = \circ(j), K = 1/1\times 10^{-3}(m^{-1})$

Conclusion Remarks

By means of the equations of continuity, momentum and the conservation of charges in plasma the Maxwell equations and dust particles distribution function together with Lennard-Jones potential for dusty fluid with dust particles considered in both thermal and non thermal condition we reached the dispersion relation of dusty waves (DIA) in these 2 positions. From which we calculated also the phase velocity of dust-ion-acoustic waves, our achievement shows the effect of (L-J) potential on the phase velocity of dusty waves. In one hand when the order of the particle separation in (L-J) potential is about (4×10^{-10} m - 7×10^{-10} m) this potential is negative and it apply to the icy particle so the phase velocity of particles is less than that when the order of particles separations are about (3×10^{-10} m).On the other hand we can see the effects of temperature on phase velocity which follows the dust particles temperature.

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