Proposed Conditions to Select Best Technique for 0νββ Decay Mode of ^{128,130}Te

M. H. Sidky

Dept. of Eng. Math. And Physics, Faculty of Engineering, Cairo University, Egypt
Sidkym@yahoo.com

Abstract: This work presents new distributions of three nuclear parameters versus the strength of the particle-particle interaction g_{pp} for the 0νββ decay mode of $^{128,\,130}$Te. These parameters are: (1) RM0 which is the ratio between the nuclear matrix elements of ^{128,130}Te. (2) DR0 which is the ratio between the total ββ–decay rates of $^{128,\,130}$Te. (3) neutrino mass m_ν. This is carried out by using pn-QRPA, pn-RQRPA, full-RQRPA and SQRPA techniques with small and large basis of Hilbert space. Conditions are proposed to select the best technique. It is found that pn-QRPA and SQRPA techniques should not be applied to the 0νββ decay mode of $^{128,\,130}$Te. The other techniques are accepted to operate the 0νββ decay mode of $^{128,\,130}$Te with small basis of Hilbert space as follows: pn-RQRPA and full-RQRPA techniques may be used within $0.8 \le g_{pp} \le 0.85$, $0.85 \le g_{pp} \le 0.9$ respectively. These ranges are determined such that the distributions of RM0, DR0, m_ν which belong to the accepted techniques verify the criterion RM0 ≈ 1 and agree with the available experimental limits $3.41 \times 10^{-4} \le DR0 \le 3.85 \times 10^{-4}$, $0.21\ eV \le m_\nu \le 0.27$ eV.

[M. H. Sidky. **Proposed Conditions to Select Best Technique for 0νββ Decay Mode of ^{128,130}Te.** *Life Sci J* 2013;10(4):3509-3515]. (ISSN:1097-8135). http://www.lifesciencesite.com. 467

Keywords: neutrino mass, pn-RQRPA technique, Hilbert space, strength of the particle-particle interaction

1.Introduction

Double beta decay (ββ) has been and continues to be a popular topic. It is a powerful tool for the study of lepton conservation in general and of neutrino properties in particular. Because the lifetimes of ββ decay are so long the experimental study of double beta decay is particularly challenging and has a spawned a whole field of experiments requiring very low background [1,2].

The ββ decay has been detected by the geochemical technique [3]. Such technique does not determine the decay rate of the 2νββ decay mode (two neutrinos emission) nor the decay rate of the 0νββ decay mode (no neutrinos emission) it determines only the total decay rate. The measurement of the total decay rate is affected by many possible sources of systematic errors like errors in the ore age and sample concentration. However the ratio DR0 of the total decay rates of tellurium isotopes $^{128,\,130}$Te is not affected by these errors because they cancel in ratio. There are many factors affecting the calculation of the ββ decay rate. One of these factors is the nuclear matrix element. Previously [4] it was shown that the uncertainty in the estimation of the nuclear matrix element produces an overall error of ± 100 years in the calculated decay rate of mode. It has been pointed out that [5] the situation is improved if the ratio of decay mode rates of the same tellurium isotopes is considered because the ratio RM0 of their respective relevant nuclear matrix element factors should be nearly unity.

This work shows graphically the variation of RM0 versus the strength of the particle-particle interaction g_{pp}. Also a new theoretical results for DR0 are presented and compared with experimental results obtained from geochemical technique. A new distribution of the neutrino mass versus g_{pp} is shown graphically. This has been done by using different nuclear techniques.

ββ–decay rates of $^{128,\,130}$Te

The total rate λ^T of a ββ decay is the sum of the decay rates $\lambda^{2\nu}$ and $\lambda^{0\nu}$ of the 2νββ and 0νββ modes. This relation can be written for $^{128,\,130}$Te isotopes as follows:

$$(\lambda^T)_{128} = (\lambda^{2\nu})_{128} + (\lambda^{0\nu})_{128} \quad (1)$$

$$(\lambda^T)_{130} = (\lambda^{2\nu})_{130} + (\lambda^{0\nu})_{130} \quad (2)$$

The total ββ–decay rates ratio for $^{128,\,130}$Te is defined as: $DR0 = (\lambda^T)_{128} / (\lambda^T)_{130}$. It can be expressed by using eqns. (1), (2) as follows:

$$DR0 = [(\lambda^{0\nu})_{128} / (\lambda^{0\nu})_{130}]\ \{1 + [(\lambda^{2\nu})_{128} / (\lambda^{0\nu})_{128}]\} / \{1 + [(\lambda^{2\nu})_{130} / (\lambda^{0\nu})_{130}]\} \quad (3)$$

According to the equations which calculate the decay rates $\lambda^{2\nu}$ and $\lambda^{0\nu}$ [6], eqn. (3) can be written as:

$$DR0 = RF0\ (RM0)^2\ [1 + (RF28)\ (M228 / M028)^2] / [1 + (RF30)\ (M230 / M030)^2] \quad (4)$$

RF0 is the ratio between the phase space factor of 128Te to that of ^{130}Te for the 0νββ decay mode. RF28 is the ratio between the phase space factor of 2νββ decay mode to that of 0νββ for ^{128}Te. RF30 is the ratio between the phase space factor of 2νββ decay mode to that of 0νββ for ^{130}Te. Previously [7] the phase space factors of 128, ^{130}Te were calculated by using its integral form for the 2νββ and 0νββ decay modes to give:

RF0 = 0.04, RF28 = 1.337 x 10^5, RF30 = 3.027 x 10^7

M228 and M028 are the nuclear matrix elements of 2νββ and 0νββ modes for ^{128}Te.

M230 and M030 have the same definitions of M228 and M028 but for ^{130}Te. RM0 is the ratio between the nuclear matrix element of ^{128}Te to that for ^{130}Te for the 0νββ mode. It has been reported that [8] the value of RM0 should be approximately unity, since the nuclear structure of $^{128, 130}$Te are nearly the same.

RM0≈1 (5)

The above equation may be considered as a good criterion to test the calculated values of the nuclear matrix element of the 0νββ mode for $^{128, 130}$Te.

In an earlier work [9], the matrix elements of 2νββ and 0νββ decay modes has been studied for $^{128, 130}$Te with pn-QRPA, pn-RQRPA, full-RQRPA, SQRPA techniques.

The proton-neutron quasi particle random phase approximation (pn-QRPA) have clarified that the particle-particle interaction, which is the counterpart of the particle-hole interaction, enhances the spin-isospin correlations in the ground-state wave functions. The SQRPA technique uses the boson expansion for the phonon and β operators associated with pn-QRPA technique [10]. An alternative approach for extending pn-QRPA is based on the idea of partial restoration of the Pauli exclusion operator involved in the derivation of the pn-QRPA equations [9]. The commutator is replaced by its expectation value in the RPA (correlated) g.s and this leads to a renormalization of the relevant operators and of the forward and backward going QRPA amplitudes as well. This technique is called pn-RQRPA. It has been extensively used for both 2νββ and 0νββ decay modes and for transition to g.s. and excited states and for different nuclei [11,12].

The extension of this technique when the proton-neutron pairing interactions, besides the proton-proton and neutron-neutron ones, are also included was called the full-RQRPA [13]. Each one of the nuclear matrix elements M228, M028,M230 and M030 is allowed to change with the strength of the particle-particle interaction g_{pp} in two cases. These cases are small (S) and large (L) basis which can be considered as as two different options of the size of Hilbert space [9]. These variations are shown graphically [9] for $^{128, 130}$Te by using pn-QRPA, pn-RQRPA, full-RQRPA, SQRPA techniques. Such variations have been utilized in this work as well as eqn. (4) to generate a new distributions of the parameters RM0, DR0 versus g_{pp}.

Distribution of RM0 versus g_{pp}

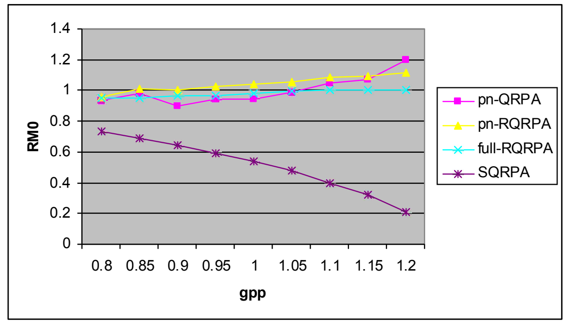

Fig. (1) Variation of the nuclear matrix elements ratio of ^{128,130}Te RM0 with g_{pp} within the range $0.8 \le g_{pp} \le 1.2$ for pn-QRPA, pn-RQRPA, full-RQRPA, SQRPA techniques with small basis of Hilbert space

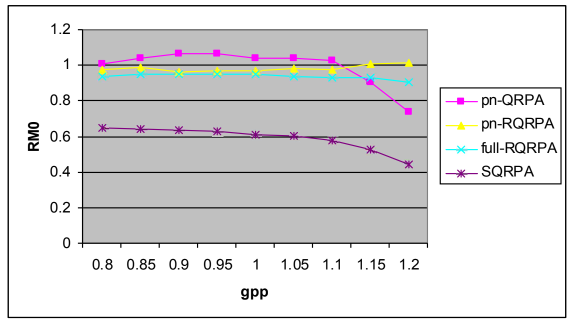

Fig.(2) Variation of the nuclear matrix elements ratio of ^{128,130}Te RM0 with g_{pp} within the range $0.8 \le g_{pp} \le 1.2$ for pn-QRPA, pn-RQRPA, full-RQRPA, SQRPA techniques with large basis of Hilbert space

Figs. (1,2) present the distributions of RM0 versus g_{pp} for pn-QRPA, pn-RQRPA, full-RQRPA, SQRPA techniques with small and large basis of Hilbert space. The absolute difference between a value of RM0 on these distributions and RM0 = 1 is the deviation D. The values of RM0, D are confined within certain ranges as presented in tables (1,2). The comparison between the results presented in figs. (1,2) and tables (1,2) indicate that the deviation D from criterion RM0 = 1 has:

(1) largest value by using SQRPA technique with small and large basis of Hilbert space.

(2) smallest value with small basis of Hilbert space by using full-RQRPA technique.

(3) smallest value with large basis of Hilbert space by using pn-RQRPA technique.

It is clear that the SQRPA technique fail to satisfy the criterion described by eqn. (5). On the other hand the full-RQRPA and pn-RQRPA techniques with small and large basis of Hilbert space are succeeding in verifying this criterion more accurately than the others.

This means that the nuclear structure of the isotopes $^{128, 130}$Te may be accurately described during 0νββ mode by:

(i) using the renormalization of the relevant operators and of the forward and backward going QRPA amplitudes in pn-QRPA technique with large basisof Hilbert space [9].

(ii) including the proton-neutron pairing interactions, besides the proton-proton and neutron-neutron ones, in full-RQRPA technique with small basis [13].

Table (1) The range of the nuclear matrix element ratio RM0 and deviation D for different techniques with small basis of Hilbert space.

Technique	RM0	D
pn – QRPA	0.889 ≤ RM0 ≤ 1.195	0 ≤ D ≤ 0.195
Pn – RQRPA	0.960 ≤ RM0 ≤ 1.113	0 ≤ D ≤ 0.113
full – RQRPA	0.950 ≤ RM0 ≤ 1.005	0 ≤ D ≤ 0.010
SQRPA	0.208 ≤ RM0 ≤ 0.735	0.265 ≤ D ≤ 0.792

Table (2) The range of the nuclear matrix element ratio RM0 and deviation D for different techniques with large basis of Hilbert space.

Technique	RM0	D
pn – QRPA	0.735 ≤ RM0 ≤ 1.067	0 ≤ D ≤ 0.265
Pn – RQRPA	0.962 ≤ RM0 ≤ 1.012	0 ≤ D ≤ 0.038
full – RQRPA	0.906 ≤ RM0 ≤ 0.950	0.050 ≤ D ≤ 0.094
SQRPA	0.444 ≤ RM0 ≤ 0.650	0.350 ≤ D ≤ 0.556

Distribution of DR0 versus g_{pp}

Figs. (3-6) show the distribution of theoretical DR0 versus g_{pp} for pn-QRPA, pn-RQRPA, full-RQRPA, SQRPA techniques with small and large basis of Hilbert space.

The parameter DR0 has been determined experimentally by different laboratories [14,15] as presented in table (3). The lower and upper limits of the experimental data shown in table (3) are 3.41 x 10^{-4}, 3.85 x 10^{-4} respectively. Such limits are shown in figs. (3-6) as a horizontal lines. The distributions of theoretical DR0 which lie outside the experimental range 3.41 x 10^{-4} ≤ DR0 ≤ 3.85 x 10^{-4} are shown in figs. (3,5,6) They are: pn-QRPA, SQRPA techniques with small and large basis of Hilbert space and full-RQRPA technique with small basis of Hilbert space. Thus the theoretical results obtained from these techniques disagree with the available experimental data shown in table (3).

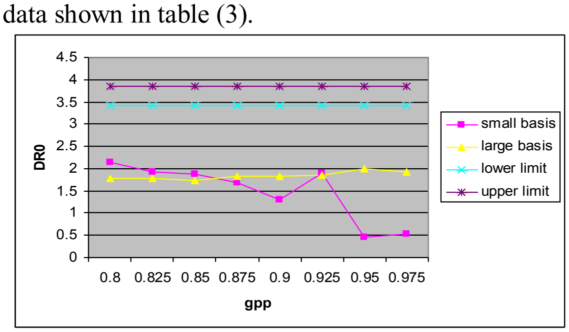

Fig.(3) Variation of the total decay rates ratio of ^{128,130}Te DR0 x 10^{-4} with g_{pp} within the range 0.8 ≤ g_{pp} ≤ 0.975 for small and large basis of Hilbert space of pn-QRPA technique. The lower and upper limits of DR0: 3.41 x 10^{-4}, 3.85 x 10^{-4} are shown as horizontal lines

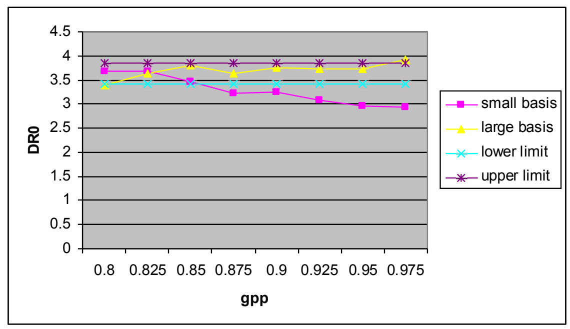

Fig.(4) Variation of the total decay rates ratio of ^{128,130}Te DR0 x 10^{-4} with g_{pp} within the range 0.8 ≤ g_{pp} ≤ 0.975 for small and large basis of Hilbert space of pn-RQRPA technique. The lower and upper limits of DR0: 3.41 x 10^{-4}, 3.85 x 10^{-4} are shown as horizontal lines

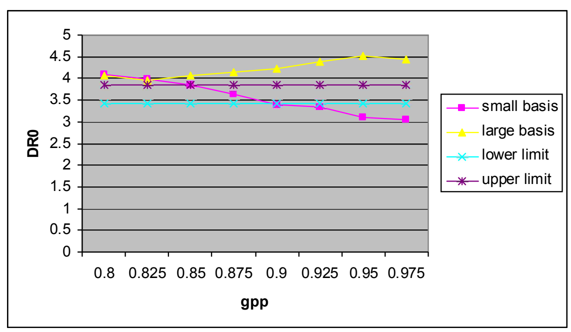

Fig.(5) Variation of the decay rates ratio of ^{128,130}Te DR0 x 10^{-4} with g_{pp} within the range 0.8 ≤ g_{pp} ≤ 0.975 for small and large basis of Hilbert space of full-RQRPA technique. The lower and upper limits of DR0: 3.41 x 10^{-4}, 3.85 x 10^{-4} are shown as horizontal lines

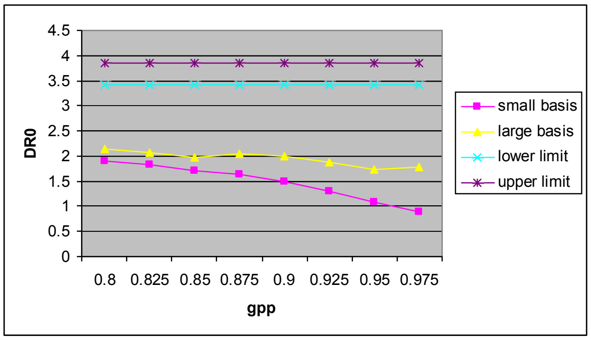

Fig.(6) Variation of the decay rates ratio of ^{128,130}Te DR0 x 10^{-4} with g_{pp} within the range $0.8 \le g_{pp} \le 0.975$ for small and large basis of Hilbert space of SQRPA technique. The lower and upper limits of DR0: 3.41 x 10^{-4}, 3.85 x 10^{-4} are shown as horizontal lines

Table (3) Experimental determinations of DR0 collected from different laboratories

DR0	Reference
$(3.52 \pm 0.11)\ 10^{-4}$	[14]
$(3.74 \pm 0.11)\ 10^{-4}$	[15]

In figs. (4,5) there are some distributions lie partially within the experimental limits 3.41 x $10^{-4} \le$ DR0 $\le$ 3.85 x10^{-4}. These distributions belong to pn-RQRPA technique with small and large basis of Hilbert space and full-RQRPA technique with large basis of Hilbert space. They agree with the available experimental data shown in table (3) within a certain range of g_{pp} [see table (4)]. The ratio between the width of this range and the width of the whole available range of g_{pp} is denoted by R [see table (4)].

Operating range of g_{pp}

Table (4) presents different determinations of g_{pp} and R for pn-RQRPA and full-RQRPA techniques with small and large basis of Hilbert space. There are two options for the operation of the 0νββ for ^{128,130}Te:

(1) $0.8 \le g_{pp} \le 0.85$ for pn-RQRPA technique with small basis of Hilbert space and

$0.85 \le g_{pp} \le 0.9$ for full-RQRPA technique with small basis of Hilbert space.

(2) $0.85 \le g_{pp} \le 0.975$ for pn-RQRPA technique with large basis of Hilbert space.

It should be noticed that the techniques used in these options have an acceptable deviations D from the criterion given by eqn. (5) [see tables (1,2)].

Table (4) New determinations of g_{pp}, R of $^{128,\ 130}$Te to make consistency between theoretical and experimental results of DR0.

Technique	Base	g_{pp}	R
Pn- RQRPA	Small	$0.8 \le g_{pp} \le 0.85$	12.5 %
Pn- RQRPA	Large	$0.8 \le g_{pp} \le 0.975$	43.75 %
Full- RQRPA	Small	$0.85 \le g_{pp} \le 0.9$	12.5%

Neutrino mass

Previously [16] the neutrino mass limits are determined experimentally to be:

$$m_v \pm \delta\ m_v = 0.24 \pm 0.03 \text{ eV} \quad (6)$$

The neutrino mass m_v is one of a set of parameters which have been used to calculate the decay rates $(\lambda^{0v})_{128}$, $(\lambda^{0v})_{130}$ of the (0νββ) decay mode [6]. It should be mentioned that considerable attention has been noted for the (0νββ) decay mode [17, 18, 19, 20].

$$(\lambda^{0v})_{128} = F028\ (M028\ m_v)^2 \quad (7)$$

$$(\lambda^{0v})_{130} = F030\ (M030\ m_v)^2 \quad (8)$$

The phase space factors F028, F030 of ^{128,130}Te were calculated [7] by using their integral forms for the 0νββ decay modes to give:

$F028 = 0.0636\ x10^{-25}\ (y^{-1}eV^{-2})$

$F030 = 1.5900\ x10^{-25}\ (y^{-1}eV^{-2})$

In another work [9] the variations of M028, M030 with g_{pp} have been shown graphically. Such variations are used in this work to generate a new distribution of m_v versus g_{pp} by using eqns. (7), (8) keeping $(\lambda^{0v})_{128}$, $(\lambda^{0v})_{130}$ constant. This is carried out for $^{128,\ 130}$Te with the techniques which are succeeding in making agreement between theoretical and experimental results of DR0 [see table (4)]. The results are shown in figs. (7-12). In each one of these figures there are two distributions of neutrino mass. The first one starts at the lower limit m_v = 0.21 eV and ends before the upper limit m_v = 0.27 eV [see eqn. (6)] such that the corresponding decay rate is the lower limit of $(\lambda^{0v})_{128}$, $(\lambda^{0v})_{130}$ [see tables (5,6)]. The second one starts after lower limit m_v = 0.21 eV and ends at the upper limit m_v = 0.27 eV [see eqn. (6)]. such that the corresponding decay rate is the upper limit of $(\lambda^{0v})_{128}$, $(\lambda^{0v})_{130}$ [see tables (5,6)]. These two distributions are the boundaries of a region which contains the theoretical values of neutrino mass which agree with $m_v \pm \delta\ m_v = 0.24 \pm 0.03$ eV [16].

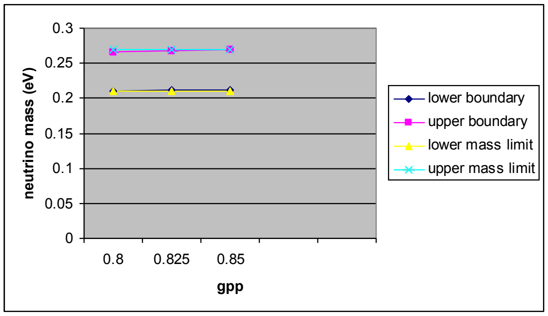

Fig.(7) Two different distributions of neutrino mass with g_{pp} within the range $0.8 \leq g_{pp} \leq 0.85$. The lower and upper boundaries correspond to $(\lambda^{0\nu})_{128} = 5.576$ x$10^{-27}yr^{-1}$, 9 x$10^{-27}yr^{-1}$ of $(0\nu\beta\beta)$ mode in ^{128}Te. pn-RQRPA technique is used with small basis of Hilbert space. The lower and upper limits of neutrino mass = 0.21eV, 0.27 eV are shown as horizontal lines.

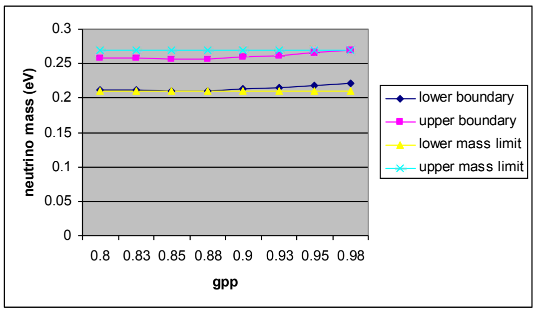

Fig.(8) Two different distributions of neutrino mass with g_{pp} within the range $0.8 \leq g_{pp} \leq 0.85$. The lower and upper boundaries correspond to $(\lambda^{0\nu})_{128} = 3$ x$10^{-27}yr^{-1}$, 4.55 x$10^{-27}yr^{-1}$ of $(0\nu\beta\beta)$ mode in ^{128}Te. pn-RQRPA technique is used with large basis of Hilbert space. The lower and upper limits of neutrino mass = 0.21 eV, 0.27 eV are shown as horizontal lines.

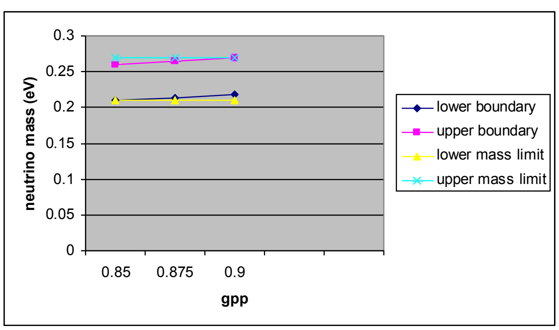

Fig.(9) Two different distributions of neutrino mass with g_{pp} within the range $0.8 \leq g_{pp} \leq 0.85$. The lower and upper boundaries correspond to $(\lambda^{0\nu})_{128} = 5.88$ x$10^{-27}yr^{-1}$ 9.024 x$10^{-27}yr^{-1}$ of $(0\nu\beta\beta)$ mode in ^{128}Te. full- RQRPA technique is used with small basis of Hilbert space. The lower and upper limits of neutrino mass = 0.21eV, 0.27 eV are shown as horizontal lines.

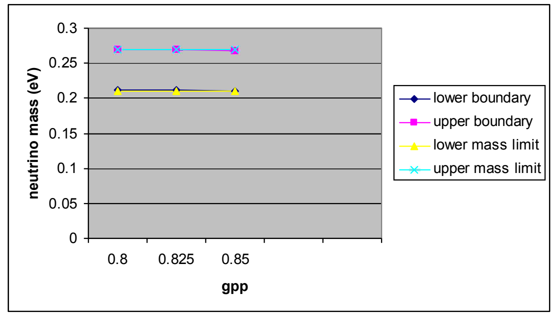

Fig.(10) Two different distributions of neutrino mass with g_{pp} within the range $0.8 \leq g_{pp} \leq 0.85$. The lower and upper boundaries correspond to $(\lambda^{0\nu})_{130} = 0.131$ x$10^{-24}yr^{-1}$ 0.212 x$10^{-24}yr^{-1}$ of $(0\nu\beta\beta)$ mode in ^{130}Te. pn- RQRPA technique is used with small basis of Hilbert space. The lower and upper limits of neutrino mass = 0.21 eV, 0.27 eV are shown as horizontal lines.

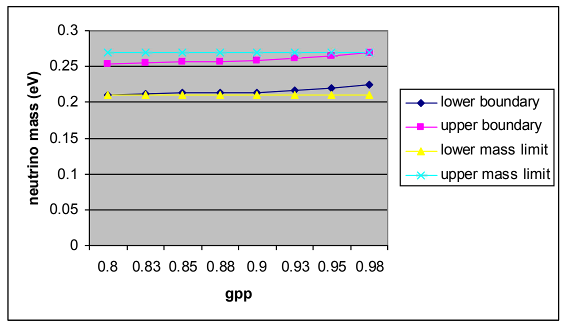

Fig.(11) Two different distributions of neutrino mass with g_{pp} within the range $0.8 \leq g_{pp} \leq 0.85$. The lower and upper boundaries correspond to $(\lambda^{0\nu})_{130} = 0.070$ x$10^{-24}yr^{-1}$, 0.102 x$10^{-24}yr^{-1}$ of $(0\nu\beta\beta)$ mode in ^{130}Te. pn- RQRPA technique is used with large basis of Hilbert space. The lower and upper limits of neutrino mass = 0.21eV, 0.27 eV are shown as horizontal lines.

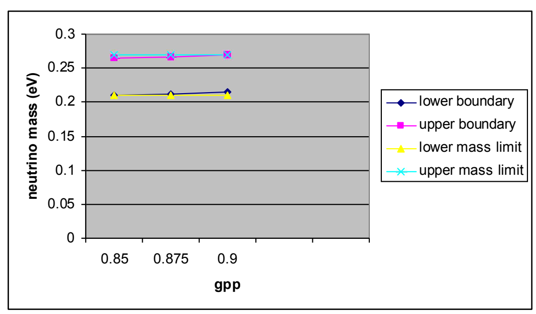

Fig.(12) Two different distributions of neutrino mass with g_{pp} within the range $0.8 \leq g_{pp} \leq 0.85$. The lower and upper boundaries correspond to $(\lambda^{0\nu})_{130} = 0.134$ x$10^{-24}yr^{-1}$, 0.211 x$10^{-24}yr^{-1}$of $(0\nu\beta\beta)$ mode in ^{130}Te. full- RQRPA technique is used with small basis of Hilbert space. The lower and upper limits of neutrino mass = 0.21eV, 0.27 eV are shown as horizontal lines.

Probability of the 0νββ for ^{128,130}Te

The probabilities $(P^{0\nu})_{128}$, $(P^{0\nu})_{130}$ of the 0νββ mode for the isotopes ^{128}Te, ^{130}Te are defined as: $(\lambda^{0\nu})_{128} / (\lambda^{T})_{128}$, $(\lambda^{0\nu})_{130} / (\lambda^{T})_{130}$ respectively. Tables (5,6) show that the limits of $(\lambda^{0\nu})_{128}$, $(\lambda^{0\nu})_{130}$ for full-RQRPA with small basis are nearly twice those for pn-RQRPA with large basis. This means that the probabilities $(P^{0\nu})_{128}$, $(P^{0\nu})_{130}$ of the 0νββ mode for the isotopes ^{128}Te, ^{130}Te are improved by a factor of about two by:

(1) using small basis rather than large basis.

(2) including the proton-neutron pairing interactions, besides the proton-proton and neutron-neutron ones, in full-RQRPA technique [13].

Table (5) limits of $(\lambda^{0\nu})_{128}$ obtained from different techniques

Technique	base	lower limitx10^{-27}yr^{-1}	Upper limitx10^{-27}yr^{-1}
pn-RQRPA	Small	5.576	9
pn-RQRPA	Large	3	4.550
full-RQRPA	Small	5.880	9.024

Table (6) limits of $(\lambda^{0\nu})_{130}$ obtained from different techniques

Technique	Base	Lower limitx10^{-24}yr^{-1}	Upper limitx10^{-24}yr^{-1}
pn-RQRPA	Small	0.131	0.212
pn-RQRPA	Large	0.070	0.102
full-RQRPA	Small	0.134	0.211

Selection of the best nuclear technique

Table (7) comparison between different nuclear techniques

technique	RM0 ≈ 1	$3.41\times10^{-4} \le$ DR0 $\le 3.85\times10^{-4}$
pn-QRPA	verify with maximum deviation: 19.5 %, 26.5 % with small, large basis of Hilbert space	Disagreement
pn-RQRPA	verify with maximum deviation: 11.3 %, 3.8 % with small, large basis of Hilbert space	agree within $0.8 \le g_{pp} \le 0.85$, $0.8 \le g_{pp} \le 0.975$ which are 12.5 %, 43.75% of the available range of g_{pp} for small, large basis of Hilbert space
full-RQRPA	verify with maximum deviation: 1 %, 9.4 % with small, large basis of Hilbert space	agree within $0.85 \le g_{pp} \le 0.9$ which is 12.5 % of the available range of g_{pp} for small basis of Hilbert space
SQRPA	violate with maximum deviation:79.2 %, 55.6 % with small, large basis of Hilbert space	Disagreement

Table (7) presents a comparison between the results obtained from different techniques used in this work. It can be noticed that full-RQRPA with small basis of Hilbert space succeeds in verifying the criterion RM0 ≈ 1 with a good accuracy (maximum deviation 1%) and agree with the limits $3.41\times10^{-4} \le$ DR0 $\le 3.85\times10^{-4}$ within a small range of g_{pp} (12.5 %). On the other hand pn-RQRPA with large basis of Hilbert space agree with the limits $3.41\times10^{-4} \le$ DR0 $\le 3.85\times10^{-4}$ within large range of g_{pp} (43.75 %) and verify the criterion RM0 ≈ 1 with a good accuracy (maximum deviation 3.8%) however it has a lowest probability 0νββ decay mode of $^{128,\ 130}$Te [see tables (7,8)]. A compromise has to be done to select the best technique. According to the above results this work suggests that the 0νββ decay mode of $^{128,\ 130}$Te may be proceeded on two stages. The first one has $0.8 \le g_{pp} \le 0.85$ and pn-RQRPA is applied with small basis of Hilbert space. The next one has $0.85 \le g_{pp} \le 0.9$ and full-RQRPA is used with small basis of Hilbert space

Conclusion

The variations of the nuclear matrix elements versus the strength of the particle-particle interaction g_{pp} have been shown graphically in a previous work for the 0νββ decay mode of $^{128,\ 130}$Te. This has been utilized in this work to generate a new distributions of three nuclear parameters RM0 (nuclear matrix elements ratio), DR0 (total decay rates ratio), m_ν (neutrino mass) versus g_{pp} for the 0νββ decay mode of $^{128,\ 130}$Te. This is carried out by using pn-QRPA, pn-RQRPA, full-RQRPA and SQRPA techniques with small and large basis of Hilbert space. The parameters RM0, DR0, m_ν should: (1)verify the criterion RM0 ≈ 1 (2) the experimental limits: 3.41 x $10^{-4} \le$ DR0 $\le$ 3.85 x10^{-4}, 0.21 eV $\le m_\nu \le$ 0.27 eV. The results obtained from pn-QRPA and SQRPA techniques disagree with the experimental range 3.41 x $10^{-4} \leq DR0 \leq 3.85 \times 10^{-4}$. Both techniques are not recommended to be used the $0\nu\beta\beta$ decay mode of ^{128,130}Te. The results obtained from pn-RQRPA and full-RQRPA techniques are succeeding in: (1) verifying the criterion RM0 ≈ 1 with a good accuracy (2) agreement with the experimental limits 3.41 x $10^{-4} \leq DR0 \leq 3.85 \times 10^{-4}$, 0.21 eV $\leq m_\nu \leq 0.27$ eV. A comparison has been done to select the best technique to operate the $0\nu\beta\beta$ decay mode of ^{128,130}Te.It is found that the $0\nu\beta\beta$ decay mode of ^{128,130}Te may be proceeded with small basis of Hilbert space as follows: pn-RQRPA within $0.8 \leq g_{pp} \leq 0.85$ and full-RQRPA within $0.85 \leq g_{pp} \leq 0.9$.

Acknowledgement

Thanks for Prof. Dr. H.M.Hussein for his assistance

References

1. Andreotti E., Amaboldi C., Avignone F.T., *et al.* "^{130}Te Neutrinoless Doouble-Beta Decay with CUORICINO" Astrop. Phys. 34 (2011) 822.
2. Kamland-Zen collaboration, "Measurement of the Double-Beta Decay Half Life of ^{136}Xe With the KamLAND-Zen Experiment" Phys. Rev. C85, (2012) 045504
3. Manuel O.K. "Geochemical Measurements of Double-Beta Decay" Proc. Int. Symp. On Nuclear Beta Decays and Neutrino ed. T. Kotani, N.Ejiri and E.Takasugi, (1986).
4. Rosen P.S. and Primakoff H., "Double β Decay, in Alpha, Beta, Gamma-Ray Spectroscopy Volume 2 (1965) 1499.
5. Pontecorvo B., " Superweak Interactions and Double Betay DecayPhys. Lett. 826 (1968) 630.
6. Suhonen J. and Civitarese C., "Effects of Orbital Occupancies and Spin-Orbit Partners on $0\nu\beta\beta$ Decay Rates" Nucl. Phys. A 847 (2010) 207.
7. Toivanen J. and J.Suhonen, "Study of Several Double-Beta-Decaying Nuclei using the Renormalized Proton-Neutron Quasi Random Phase Approximation" Phys. Rev. C55
8. Kristen T., Richter H. and Jessberger E., " Rejection of Evidence for Nonezero Neutrino Rest Mass from Double Beta Decay" *et al.* Phys. Rev. 50 (1983) 474.
9. Stoica S.and Klapdor-Kleingrothaus H.V. "Critical View On Double-Beta Decay Matrix Elements Within Quasi Random Phase Approximation –based methods" Nucl. Phys. A 694 (2001) 269.
10. Raduta A.A., Faessler A. and Stoica S, "The Decay Rate Within a Boson Expansion Formalism" Nucl. Phys. A 534 (1991) 149.
11. Schwieger J., Simkovic F. and Amand Faessler, "Double β Decay to Excited States of Several Medium- Heavy Nuclei Within the Renormalized Quasiparticle Random Phase us Approximation" *et al.* Phys. Rev. C57 (1998)1738.
12. Suhonen J. and O.Civitarese, "Weak Interaction and Nuclear-Structure Aspects of Nuclear Double Beta Decay" Phys. Rep. 300, (1998) 123.
13. Simkovic F., Schwieger, Veselsky *et al.*, "Non Collapsing Renormalized QRPA with Proton-Neutron Pairing for Neutrinoless Double Beta Decay" Phys. Lett. 393 (1997)267
14. Bernatowicz T., Brannon J., Brazzle R., " Neutrino Mass Limits from a Precise Determination of ββ Decay Rates of ^{128}Te and ^{130}Te., Phys. Rev. Lett. 69 (1992) 2341.
15. Meshik A.P., Hohenberg C.M., " ^{130}Te and ^{128}Te Double Beta Decay Half Lives" Nucl. Phys. A 809 (2008) 275.
16. Faessler A., "Nuclear Structure, Double Beta Decay and Neutrino mass" Journal of Physics Conference Series 267 (2011) 012059.
17. Barabash A.S., "Double Beta Decay: Historical Review of 75 years of Research" Phys. Atom. Nucl. 74, (2011) 603.
18. Gomez-Cadenas J.J., Martin-Albo J., Mezzetto M. *et al.*, "The Search for Neutrinoless Double Beta Decay" Riv Nuovo Cim 35, (2012) 2998.
19. Werner Rodejohann J., "Neutrino less Double Beta Decay and Neutrino Physics" Phys. G39, (2012) 124008.
20. Bilenky S.M. and C.Giunti, "Neutrinoless Double Beta Decay A Brief Review" Mod. Phys. Lett. A27, (2012).

12/23/2013