Generalized tanh method extended with the Riccati equation for solving the some of nonlinear equations

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Abstract: In this paper, we find new exact traveling wave solutions of the Benjamin-Bona-Mahony equation, Lax's fifth-order KdV equation and Drinfeld-Sokolov-Wilson equation system by using generalized tanh method. The main idea of this method is to take full advantage of the Riccati equation which has more new solutions.
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1. Introduction

In recent years, there are many ways for solving the nonlinear equations [1 - 6]. We extend the generalized tanh method [7] to special types of nonlinear equations for constructing their multiple travelling wave solutions [8 - 10]. The key idea is to use the solution of a Riccati equation to replace the tanh function in the tanh method.The efficiency of the method can be demonstrated for a large variety of special equations. For example, the travelling wave solutions of some equations such as Benjamin-Bona-Mahony equation, Lax's fifth-order KdV equation and Drinfeld-Sokolov-Wilson equation system.

Recently, much work has been concentrated on the various extensions and applications of the method [2 - 21]. We simply describe this method as follows.

2. The generalized tanh function method

The main idea of our method is to take full advantage of the Riccati equation that tanh function satisfies and use its solutions F to replace $tan\xi$. The desired Riccati equation reads

$$F' = AF^2 + BF + C \qquad (1)$$

where $' = \frac{d}{d\xi}$, $\xi = \xi(x,t) = \alpha x + q(t)$ and $A; B; C$ are constants.

1. if $A = C = 1$, then (1) has solution $\tan\xi$.
2. if $A = C = -1$, then (1) has solution $\cot\xi$.
3. if $A = 1,\ C = -1$, then (1) has solution $\tanh\xi,\ \coth\xi$.
4. if $A = C = \frac{1}{2}$, then (1) has solution $\tan\xi \pm \sec\xi,\ \csc\xi - \cot\xi,\ \frac{\tan\xi}{1 \pm \sec\xi}$.
5. if $A = C = -\frac{1}{2}$, then (1) has solution $\cot\xi \pm \csc\xi,\ \sec\xi - \tan\xi,\ \frac{\cot\xi}{1 \pm \csc\xi}$.
6. if $A = \frac{1}{2},\quad C = -\frac{1}{2}$, then (1) has solution $\coth\xi \pm \csc h\xi,\quad \tanh\xi \pm i\sec h\xi$ $(i^2 = -1),\ \frac{\tanh\xi}{1 \pm \sec h\xi},\ \frac{\coth\xi}{1 \pm i\csc h\xi}$.
7. if $A = 1,\quad B = -2,\quad C = 2$, then (1) has solution $\frac{\tan\xi}{1 + \tan\xi}$.
8. if $A = 1,\ B = 2,\ C = 2$, then (1) has solution $\frac{\tan\xi}{1 - \tan\xi}$.
9. if $A = -1,\ B = 2,\ C = -2$, then (1) has solution $\frac{\cot\xi}{1 + \cot\xi}$
10. if $A = -1,\quad B = -2,\quad C = -2$, then (1) has solution $\frac{\cot\xi}{1 - \cot\xi}$
11. if $A = B = 0$, $C \neq 0$, then (1) has solution $\frac{-1}{C\xi + c_0}$.

12. if $C=0$, $B\neq 0$, then (1) has solution $\frac{\exp(B\xi)-A}{B}$.

For a given PDE with two variables general form of nonlinear PDE

$$\varphi(u,u_t,u_x,u_{xx},...)=0. \qquad (2)$$

The solution of Equation (2) we are looking for is expressed in the form as a finite series of tanh functions

$$u(x,t)=\sum_{i=0}^{n}a_i(x,t)F^i(\xi), \qquad (3)$$

where n is a positive integer that can be determined by balancing the highest order derivate and with the highest nonlinear terms in equation, $a_0(x,t), a_1(x,t),\ldots,a_n(x,t)$ and $\xi(x,t)$ can be determined. Substituting solution (3) into Eq. (2) yields a set of algebraic equations for $F^i(\xi)$, then, all coefficients of $F^i(\xi)$ have to vanish. After this separated algebraic equation, we could found coefficients $a_0(x,t), a_1(x,t),\ldots,a_n(x,t)$ and $\xi(x,t)$.

In the following we illustrate the method by considering the Benjamin-Bona-Mahony equation, Lax's fifth-order KdV equation and Drinfeld-Sokolov-Wilson equation system.

3. Results

Example 1. Consider the Benjamin-Bona-Mahony equation

$$u_t-u_x+uu_x-u_{xxt}=0. \qquad (4)$$

When balancing uu_x with u_{xxt} then gives $m=2$. Therefore, we may choose

$$u=f(t)+g(t)F(\xi)+h(t)F^2(\xi) \qquad (5)$$

where $\xi=\xi(x,t)=\alpha x+q(t)$.

Substituting (5) into Eq. (4) yields a set of algebraic equations for $f(t), g(t), h(t)$ and $\xi(x,t)$. These equations are finding as

$$\begin{aligned}
&A\alpha g+BF(\xi)\alpha g+CF^2(\xi)\alpha g+A\alpha fg+BF(\xi)\alpha fg\\
&+CF^2(\xi)\alpha fg+AF(\xi)\alpha g^2+BF^2(\xi)\alpha g^2+CF^3(\xi)\alpha g^2\\
&+2AF(\xi)\alpha h+2BF^2(\xi)\alpha h+2CF^3(\xi)\alpha h+2AF(\xi)\alpha fh\\
&+2BF^2(\xi)\alpha fh+2CF^3(\xi)\alpha fh+3AF^2(\xi)\alpha gh\\
&+3BF^3(\xi)\alpha gh+3CF^4(\xi)\alpha gh+2AF^3(\xi)\alpha h^2\\
&+2BF^4(\xi)\alpha h^2+2CF^5(\xi)\alpha h^2+f_t+F(\xi)g_t+AB\alpha^2 g_t\\
&+B^2F(\xi)\alpha^2 g_t+2ACF(\xi)\alpha^2 g_t+3BCF^2(\xi)\alpha^2 g_t\\
&+2C^2F^3(\xi)\alpha^2 g_t+F^2(\xi)h_t+2A^2\alpha^2 h_t+6ABF(\xi)\alpha^2 h_t\\
&+4B^2F^2(\xi)\alpha^2 h_t+8ACF^2(\xi)\alpha^2 h_t+10BCF^3(\xi)\alpha^2 h_t\\
&+6C^2F^4(\xi)\alpha^2 h_t+Agq_t+BF(\xi)gq_t+CF^2(\xi)gq_t\\
&+AB^2\alpha^2 gq_t+2A^2C\alpha^2 gq_t+B^3F(\xi)\alpha^2 gq_t\\
&+8ABCF(\xi)\alpha^2 gq_t+7B^2CF^2(\xi)\alpha^2 gq_t+8AC^2F^2(\xi)\alpha^2 gq_t\\
&+12BC^2F^3(\xi)\alpha^2 gq_t+6C^3F^4(\xi)\alpha^2 gq_t+2AF(\xi)hq_t\\
&+2B^2F^2(\xi)hq_t+2C^2F^3(\xi)hq_t+6A^2B\alpha^2 hq_t\\
&+14AB^2F(\xi)\alpha^2 hq_t+16A^2CF(\xi)\alpha^2 hq_t+8B^3F^2(\xi)\alpha^2 hq_t\\
&+52ABCF^2(\xi)\alpha^2 hq_t+38B^2CF^3(\xi)\alpha^2 hq_t\\
&+40AC^2F^3(\xi)\alpha^2 hq_t+54BC^2F^4(\xi)\alpha^2 hq_t\\
&+24C^3F^5(\xi)\alpha^2 hq_t=0
\end{aligned} \qquad (6)$$

From the solutions of the equations, we can found

$$\begin{aligned}
&h=-12\alpha q_t C^2,\ g=-12\alpha q_t BC,\\
&f=-\frac{q_t+\alpha+q_t\alpha^2B^2+8q_t\alpha^2AC}{\alpha},\\
&q_{tt}=0,
\end{aligned} \qquad (7)$$

with the aid of Mathematica. From (7), we can get

$$\begin{aligned}
&q_t=\lambda, q=\lambda t,\\
&h=-12\alpha\lambda C^2, g=-12\alpha\lambda BC,\\
&f=-\frac{\lambda+\alpha+\lambda\alpha^2B^2+8\lambda\alpha^2AC}{\alpha}.
\end{aligned} \qquad (8)$$

where $\lambda=const$.

Substituting (7) or (8) into (5) and using special solutions of Eq. (4), we obtain the following multiple soliton-like and triangular periodic solutions of Eq.(4):

$$u_1=-\frac{\lambda+\alpha+8\lambda\alpha^2}{\alpha}-12\alpha\lambda\tan^2\xi,$$

$$u_2=-\frac{\lambda+\alpha+8\lambda\alpha^2}{\alpha}-12\alpha\lambda\cot^2\xi,$$

$$u_3=-\frac{\lambda+\alpha-8\lambda\alpha^2}{\alpha}-12\alpha\lambda\tanh^2\xi,$$

$$u_4=-\frac{\lambda+\alpha-8\lambda\alpha^2}{\alpha}-12\alpha\lambda\coth^2\xi,$$

$$u_5=-\frac{\lambda+\alpha+2\lambda\alpha^2}{\alpha}-3\alpha\lambda(\tan\xi\pm\sec\xi)^2,$$

$$u_6=-\frac{\lambda+\alpha+2\lambda\alpha^2}{\alpha}-3\alpha\lambda(\csc\xi-\cot\xi)^2,$$
$$u_7=-\frac{\lambda+\alpha+2\lambda\alpha^2}{\alpha}-3\alpha\lambda\frac{\tan^2\xi}{(1\pm\sec\xi)^2},$$
$$u_8=-\frac{\lambda+\alpha+2\lambda\alpha^2}{\alpha}-3\alpha\lambda(\cot\xi\pm\csc\xi)^2,$$
$$u_9=-\frac{\lambda+\alpha+2\lambda\alpha^2}{\alpha}-3\alpha\lambda(\sec\xi-\tan\xi)^2,$$
$$u_{10}=-\frac{\lambda+\alpha+2\lambda\alpha^2}{\alpha}-3\alpha\lambda\frac{\cot^2\xi}{(1\pm\csc\xi)^2}, \qquad (9)$$
$$u_{11}=-\frac{\lambda+\alpha-2\lambda\alpha^2}{\alpha}-3\alpha\lambda(\coth\xi\pm\csc h\xi)^2,$$
$$u_{12}=-\frac{\lambda+\alpha-2\lambda\alpha^2}{\alpha}-3\alpha\lambda(\tanh\xi\pm i\sec h\xi)^2,$$
$$u_{13}=-\frac{\lambda+\alpha-2\lambda\alpha^2}{\alpha}-3\alpha\lambda\left(\frac{\tanh\xi}{1\pm\sec h\xi}\right)^2,$$
$$u_{14}=-\frac{\lambda+\alpha-2\lambda\alpha^2}{\alpha}-3\alpha\lambda\left(\frac{\coth\xi}{1\pm i\csc h\xi}\right)^2,$$
$$u_{15}=-\frac{\lambda+\alpha+20\lambda\alpha^2}{\alpha}+48\alpha\lambda\frac{\tan\xi}{1+\tan\xi}-48\alpha\lambda\left(\frac{\tan\xi}{1+\tan\xi}\right)^2,$$
$$u_{16}=-\frac{\lambda+\alpha+20\lambda\alpha^2}{\alpha}-48\alpha\lambda\frac{\tan\xi}{1-\tan\xi}-48\alpha\lambda\left(\frac{\tan\xi}{1-\tan\xi}\right)^2,$$
$$u_{17}=-\frac{\lambda+\alpha+20\lambda\alpha^2}{\alpha}+48\alpha\lambda\frac{\cot\xi}{1+\cot\xi}-48\alpha\lambda\left(\frac{\cot\xi}{1+\cot\xi}\right)^2,$$
$$u_{18}=-\frac{\lambda+\alpha+20\lambda\alpha^2}{\alpha}-48\alpha\lambda\frac{\cot\xi}{1-\cot\xi}-48\alpha\lambda\left(\frac{\cot\xi}{1-\cot\xi}\right)^2,$$
$$u_{19}=-\frac{\lambda+\alpha}{\alpha}-12\alpha\lambda C^2\left(\frac{-1}{C\xi+c_0}\right)^2.$$

Now, we draw some pictures.

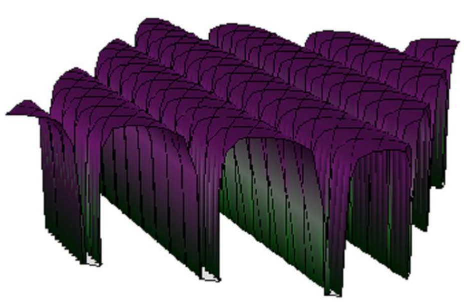

Fig1: u_1; ($\lambda=\alpha=C=1$)

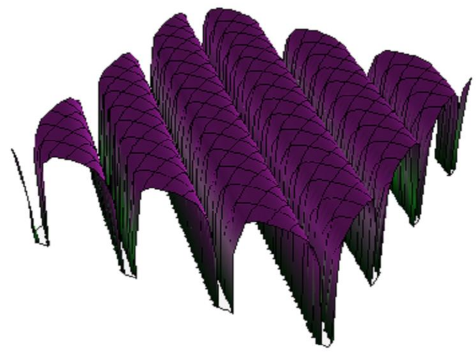

Fig2: u_2; ($\lambda=\alpha=C=1$)

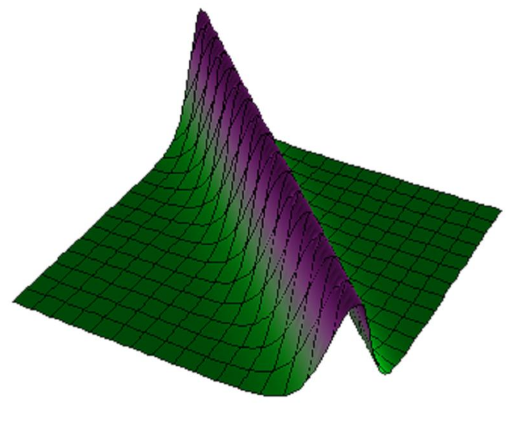

Fig3: u_3; ($\lambda=\alpha=C=1$)

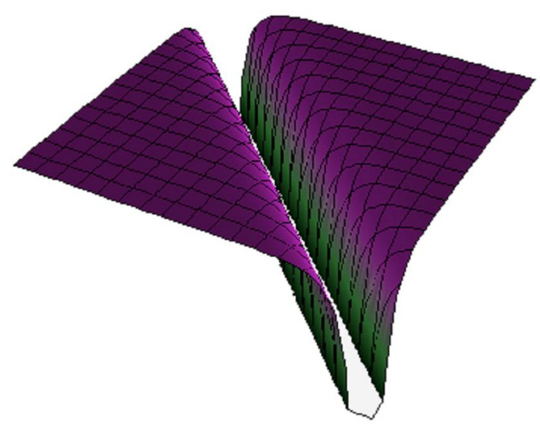

Fig4: u_4; ($\lambda=\alpha=C=1$)

Example 2. In this example we consider the fifth-order KdV equation

$$u_t+au^2u_x+bu_xu_{xx}+cuu_{xxx}+du_{xxxxx}=0, \qquad (10)$$

where a, b, c and d are constants. This equation has been known as the general form of the fifth-order KdV equation. Equation (10) is known as Lax's fifth-order KdV equation with $a = b = 30$, $c = 10$ and $d = 1$.

$$u_t + 30u^2 u_x + 30u_x u_{xx} + 10uu_{xxx} + u_{xxxxx} = 0 \quad (11)$$

When balancing $u^2 u_x$, $u_x u_{xx}$ or uu_{xxx} with u_{xxxxx} then gives $m = 2$. Therefore, we may choose

$$u = f(t) + g(t)F(\xi) + h(t)F^2(\xi) \quad (12)$$

where $\xi = \xi(x,t) = \alpha x + q(t)$.

Substituting (12) into Eq. (11) yields a set of algebraic equations for $f(t), g(t), h(t)$ and $\xi(x,t)$. These equations are finding as

$$\begin{aligned}
&AB^4\alpha^5 g + 22A^2B^2C\alpha^5 g + 16A^3C^2\alpha^5 g + B^5F(\xi)\alpha^5 g \\
&+52AB^3CF(\xi)\alpha^5 g + 136A^2BC^2F(\xi)\alpha^5 g \\
&+31B^4CF^2(\xi)\alpha^5 g + 292AB^2C^2F^2(\xi)\alpha^5 g \\
&+136A^2C^3F^2(\xi)\alpha^5 g + 180B^3C^2F^3(\xi)\alpha^5 g \\
&+480ABC^3F^3(\xi)\alpha^5 g + 390B^2C^3F^4(\xi)\alpha^5 g \\
&+240AC^4F^4(\xi)\alpha^5 g + 360BC^5F^5(\xi)\alpha^5 g \\
&+120C^5F^6(\xi)\alpha^5 g + 10AB^2\alpha^3 fg + 20A^2C\alpha^3 fg \\
&+10B^3F(\xi)\alpha^3 fg + 80ABCF(\xi)\alpha^3 fg \\
&+70B^2CF^2(\xi)\alpha^3 fg + 80AC^2F^2(\xi)\alpha^3 fg \\
&+120BC^2F^3(\xi)\alpha^3 fg + 60C^3F^4(\xi)\alpha^3 fg \\
&+30A\alpha f^2 g + 30BF(\xi)\alpha f^2 g + 30CF^2(\xi)\alpha f^2 g \\
&+30A^2B\alpha^3 g^2 + 70AB^2F(\xi)\alpha^3 g^2 + 80A^2CF(\xi)\alpha^3 g^2 \\
&+40B^3F^2(\xi)\alpha^3 g^2 + 260ABCF^2(\xi)\alpha^3 g^2 \\
&+190B^2CF^3(\xi)\alpha^3 g^2 + 200AC^2F^3(\xi)\alpha^3 g^2 \\
&+270BC^2F^4(\xi)\alpha^3 g^2 + 120C^3F^5(\xi)\alpha^3 g^2 \qquad (13) \\
&+60AF(\xi)\alpha fg^2 + 60BF^2(\xi)\alpha fg^2 + 60CF^3(\xi)\alpha fg^2 \\
&+380B^2CF^3(\xi)\alpha^3 fh + 400AC^2F^3(\xi)\alpha^3 fh \\
&+540BC^2F^4(\xi)\alpha^3 fh + 240C^3F^5(\xi)\alpha^3 fh \\
&+60AF(\xi)\alpha f^2 h + 60BF^2(\xi)\alpha f^2 h + 60CF^3(\xi)\alpha f^2 h \\
&+60A^3\alpha^3 gh + 360A^2BF(\xi)\alpha^3 gh + 570AB^2F^2(\xi)\alpha^3 gh \\
&+600A^2CF^2(\xi)\alpha^3 gh + 270B^3F^3(\xi)\alpha^3 gh \\
&+1680ABCF^3(\xi)\alpha^3 gh + 1110B^2CF^4(\xi)\alpha^3 gh \\
&+1140AC^2F^4(\xi)\alpha^3 gh + 1440BC^2F^5(\xi)\alpha^3 gh \\
&+600C^3F^6(\xi)\alpha^3 gh + 180AF^2(\xi)\alpha fgh \\
&+180BF^3(\xi)\alpha fgh + 180CF^4(\xi)\alpha fgh \\
&+120AF^3(\xi)\alpha g^2 h + 120BF^4(\xi)\alpha g^2 h \\
&+120CF^5(\xi)\alpha g^2 h + 120A^3F(\xi)\alpha^3 h^2 \\
&+540A^2BF^2(\xi)\alpha^3 h^2 + 740AB^2F^3(\xi)\alpha^3 h^2 \\
&+760A^2CF^3(\xi)\alpha^3 h^2 + 320B^3F^4(\xi)\alpha^3 h^2 \\
&+1960ABCF^4(\xi)\alpha^3 h^2 + 1220B^2CF^5(\xi)\alpha^3 h^2 \\
&+1240AC^2F^5(\xi)\alpha^3 h^2 + 1500BC^2F^6(\xi)\alpha^3 h^2 \\
&+600C^3F^7(\xi)\alpha^3 h^2 + 120AF^3(\xi)\alpha fh^2 \\
&+120BF^4(\xi)\alpha fh^2 + 120CF^5(\xi)\alpha fh^2 \\
&+150AF^4(\xi)\alpha gh^2 + 150BF^5(\xi)\alpha gh^2 \\
&+150CF^6(\xi)\alpha gh^2 + 60AF^5(\xi)\alpha h^3 \\
&+60BF^6(\xi)\alpha h^3 + 60CF^7(\xi)\alpha h^3 + f_t \\
&+F(\xi)g_t + F^2(\xi)h_t + Agq_t + BF(\xi)gq_t \\
&+CF^2(\xi)gq_t + 2AF(\xi)hq_t \\
&+2BF^2(\xi)hq_t + 2CF^3(\xi)hq_t = 0
\end{aligned}$$

From the solutions of the equations, we can found

$$\begin{aligned}
&h = \left(-5 \pm \sqrt{13}\right)\alpha^2 C^2, \\
&g = -2(40BC^6\alpha^6\left(-5 \pm \sqrt{13}\right) + 25BC^6\alpha^6\left(-5 \pm \sqrt{13}\right)^2 \\
&\quad + BC^6\alpha^6\left(-5 \pm \sqrt{13}\right)^3)/C^5\alpha^4(4 + 20\left(-5 \pm \sqrt{13}\right) \\
&\quad + 5\left(-5 \pm \sqrt{13}\right)^2), \qquad (14) \\
&f = -(\alpha^2(672(-5B^2 + 2AC) + 32(248B^2 + 451AC) \\
&\quad (-5 \pm \sqrt{13}) + 112(43B^2 + 419AC)(-5 \pm \sqrt{13})^2 \\
&\quad + 16(104B^2 + 2785AC)(-5 \pm \sqrt{13})^3 \\
&\quad + 70(35B^2 + 226AC)(-5 \pm \sqrt{13})^4 \\
&\quad + 2(-73B^2 + 1075AC)(-5 \pm \sqrt{13})^5 + (-51B^2 + 75AC) \\
&\quad (-5 \pm \sqrt{13})^6)) \backslash (6(2 + (-5 \pm \sqrt{13}))(4 + 20(-5 \pm \sqrt{13}) \\
&\quad + 5(-5 \pm \sqrt{13})^2)^2).
\end{aligned}$$

with the aid of Mathematica.

Substituting (14) into (12) and using special solutions of Eq. (11), we obtain the following multiple soliton-like and triangular periodic solutions of Eq.(11):

$$\begin{aligned}
u_1 = &(-1344\alpha^{12} - 14432\alpha^{12}(-5 \pm \sqrt{13}) \\
&- 46928\alpha^{12}(-5 \pm \sqrt{13})^2 - 44560\alpha^{12}(-5 \pm \sqrt{13})^3 \\
&- 15820\alpha^{12}(-5 \pm \sqrt{13})^4 - 2150\alpha^{12}(-5 \pm \sqrt{13})^5 \\
&- 75\alpha^{12}(-5 \pm \sqrt{13})^6)/(6(2\alpha^2 + \alpha^2(-5 \pm \sqrt{13})) \\
&(4\alpha^4 + 20\alpha^4(-5 \pm \sqrt{13}) + 5\alpha^4(-5 \pm \sqrt{13})^2)^2) \\
&+ \alpha^2(-5 \pm \sqrt{13})\tan^2\xi,
\end{aligned}$$

$$u_2=(-1344\alpha^{12}-14432\alpha^{12}(-5\pm\sqrt{13})$$
$$-46928\alpha^{12}(-5\pm\sqrt{13})^2-44560\alpha^{12}(-5\pm\sqrt{13})^3$$
$$-15820\alpha^{12}(-5\pm\sqrt{13})^4-2150\alpha^{12}(-5\pm\sqrt{13})^5$$
$$-75\alpha^{12}(-5\pm\sqrt{13})^6)/(6(2\alpha^2+\alpha^2(-5\pm\sqrt{13}))$$
$$(-4\alpha^4-20\alpha^4(-5\pm\sqrt{13})-5\alpha^4(-5\pm\sqrt{13})^2)^2)$$
$$+\alpha^2(-5\pm\sqrt{13})\cot^2\xi,$$

$$u_3=(1344\alpha^{12}+14432\alpha^{12}(-5\pm\sqrt{13})$$
$$+46928\alpha^{12}(-5\pm\sqrt{13})^2+44560\alpha^{12}(-5\pm\sqrt{13})^3$$
$$+15820\alpha^{12}(-5\pm\sqrt{13})^4+2150\alpha^{12}(-5\pm\sqrt{13})^5$$
$$+75\alpha^{12}(-5\pm\sqrt{13})^6)/(6(2\alpha^2+\alpha^2(-5\pm\sqrt{13}))$$
$$(-4\alpha^4-20\alpha^4(-5\pm\sqrt{13})-5\alpha^4(-5\pm\sqrt{13})^2)^2)$$
$$+\alpha^2(-5\pm\sqrt{13})\tanh^2\xi,$$

$$u_4=(1344\alpha^{12}+14432\alpha^{12}(-5\pm\sqrt{13})$$
$$+46928\alpha^{12}(-5\pm\sqrt{13})^2+44560\alpha^{12}(-5\pm\sqrt{13})^3$$
$$+15820\alpha^{12}(-5\pm\sqrt{13})^4+2150\alpha^{12}(-5\pm\sqrt{13})^5$$
$$+75\alpha^{12}(-5\pm\sqrt{13})^6)/(6(2\alpha^2+\alpha^2(-5\pm\sqrt{13}))$$
$$(-4\alpha^4-20\alpha^4(-5\pm\sqrt{13})-5\alpha^4(-5\pm\sqrt{13})^2)^2)$$
$$+\alpha^2(-5\pm\sqrt{13})\coth^2\xi,$$

$$u_5=(-\frac{21}{256}\alpha^{12}-\frac{451}{512}\alpha^{12}(-5\pm\sqrt{13})$$
$$-\frac{2933}{1024}\alpha^{12}(-5\pm\sqrt{13})^2-\frac{2785}{1024}\alpha^{12}(-5\pm\sqrt{13})^3$$
$$-\frac{3955}{4096}\alpha^{12}(-5\pm\sqrt{13})^4-\frac{1075}{8192}\alpha^{12}(-5\pm\sqrt{13})^5$$
$$-\frac{75}{16384}\alpha^{12}(-5\pm\sqrt{13})^6)/(6(\frac{\alpha^2}{2}+\frac{\alpha^2}{4}(-5\pm\sqrt{13}))$$
$$(\frac{\alpha^4}{8}-\frac{5}{8}\alpha^4(-5\pm\sqrt{13})-\frac{5}{32}\alpha^4(-5\pm\sqrt{13})^2)^2)$$
$$\frac{\alpha^2}{4}(-5\pm\sqrt{13})(\tan\xi\pm\sec\xi)^2,$$

$$u_6=(-\frac{21}{256}\alpha^{12}-\frac{451}{512}\alpha^{12}(-5\pm\sqrt{13})$$
$$-\frac{2933}{1024}\alpha^{12}(-5\pm\sqrt{13})^2-\frac{2785}{1024}\alpha^{12}(-5\pm\sqrt{13})^3$$
$$-\frac{3955}{4096}\alpha^{12}(-5\pm\sqrt{13})^4-\frac{1075}{8192}\alpha^{12}(-5\pm\sqrt{13})^5$$
$$-\frac{75}{16384}\alpha^{12}(-5\pm\sqrt{13})^6)/(6(\frac{\alpha^2}{2}+\frac{\alpha^2}{4}(-5\pm\sqrt{13}))$$
$$(\frac{\alpha^4}{8}-\frac{5}{8}\alpha^4(-5\pm\sqrt{13})-\frac{5}{32}\alpha^4(-5\pm\sqrt{13})^2)^2)$$
$$+\frac{\alpha^2}{4}(-5\pm\sqrt{13})(\csc\xi-\cot\xi)^2,$$

$$u_7=(-\frac{21}{256}\alpha^{12}-\frac{451}{512}\alpha^{12}(-5\pm\sqrt{13})$$
$$-\frac{2933}{1024}\alpha^{12}(-5\pm\sqrt{13})^2-\frac{2785}{1024}\alpha^{12}(-5\pm\sqrt{13})^3$$
$$-\frac{3955}{4096}\alpha^{12}(-5\pm\sqrt{13})^4-\frac{1075}{8192}\alpha^{12}(-5\pm\sqrt{13})^5$$
$$-\frac{75}{16384}\alpha^{12}(-5\pm\sqrt{13})^6)/(6(\frac{\alpha^2}{2}+\frac{\alpha^2}{4}(-5\pm\sqrt{13}))$$
$$(\frac{\alpha^4}{8}-\frac{5}{8}\alpha^4(-5\pm\sqrt{13})-\frac{5}{32}\alpha^4(-5\pm\sqrt{13})^2)^2)$$
$$+\frac{\alpha^2}{4}(-5\pm\sqrt{13})\left(\frac{\tan\xi}{1\pm\sec\xi}\right)^2,$$

$$u_8=(-\frac{21}{256}\alpha^{12}-\frac{451}{512}\alpha^{12}(-5\pm\sqrt{13})$$
$$-\frac{2933}{1024}\alpha^{12}(-5\pm\sqrt{13})^2-\frac{2785}{1024}\alpha^{12}(-5\pm\sqrt{13})^3$$
$$-\frac{3955}{4096}\alpha^{12}(-5\pm\sqrt{13})^4-\frac{1075}{8192}\alpha^{12}(-5\pm\sqrt{13})^5$$
$$-\frac{75}{16384}\alpha^{12}(-5\pm\sqrt{13})^6)/(6(\frac{\alpha^2}{2}+\frac{\alpha^2}{4}(-5\pm\sqrt{13}))$$
$$(-\frac{\alpha^4}{8}-\frac{5}{8}\alpha^4(-5\pm\sqrt{13})-\frac{5}{32}\alpha^4(-5\pm\sqrt{13})^2)^2)$$
$$+\frac{\alpha^2}{4}(-5\pm\sqrt{13})(\cot\xi\pm\csc\xi)^2,$$

$$u_9=(-\frac{21}{256}\alpha^{12}-\frac{451}{512}\alpha^{12}(-5\pm\sqrt{13})$$
$$-\frac{2933}{1024}\alpha^{12}(-5\pm\sqrt{13})^2-\frac{2785}{1024}\alpha^{12}(-5\pm\sqrt{13})^3$$
$$-\frac{3955}{4096}\alpha^{12}(-5\pm\sqrt{13})^4-\frac{1075}{8192}\alpha^{12}(-5\pm\sqrt{13})^5$$
$$-\frac{75}{16384}\alpha^{12}(-5\pm\sqrt{13})^6)/(6(\frac{\alpha^2}{2}+\frac{\alpha^2}{4}(-5\pm\sqrt{13}))$$
$$(-\frac{\alpha^4}{8}-\frac{5}{8}\alpha^4(-5\pm\sqrt{13})-\frac{5}{32}\alpha^4(-5\pm\sqrt{13})^2)^2)$$
$$+\frac{\alpha^2}{4}(-5\pm\sqrt{13})(\sec\xi-\tan\xi)^2,$$

$$u_{10}=(-\frac{21}{256}\alpha^{12}-\frac{451}{512}\alpha^{12}(-5\pm\sqrt{13}) \qquad (14)$$
$$-\frac{2933}{1024}\alpha^{12}(-5\pm\sqrt{13})^2-\frac{2785}{1024}\alpha^{12}(-5\pm\sqrt{13})^3$$
$$-\frac{3955}{4096}\alpha^{12}(-5\pm\sqrt{13})^4-\frac{1075}{8192}\alpha^{12}(-5\pm\sqrt{13})^5$$
$$-\frac{75}{16384}\alpha^{12}(-5\pm\sqrt{13})^6)/(6(\frac{\alpha^2}{2}+\frac{\alpha^2}{4}(-5\pm\sqrt{13}))$$
$$(-\frac{\alpha^4}{8}-\frac{5}{8}\alpha^4(-5\pm\sqrt{13})-\frac{5}{32}\alpha^4(-5\pm\sqrt{13})^2)^2)$$
$$+\frac{\alpha^2}{4}(-5\pm\sqrt{13})\left(\frac{\cot\xi}{1\pm\csc\xi}\right)^2,$$

$$u_{11} = (\frac{21}{256}\alpha^{12} + \frac{451}{512}\alpha^{12}(-5\pm\sqrt{13}) + \frac{2933}{1024}\alpha^{12}(-5\pm\sqrt{13})^2 + \frac{2785}{1024}\alpha^{12}(-5\pm\sqrt{13})^3 + \frac{3955}{4096}\alpha^{12}(-5\pm\sqrt{13})^4 + \frac{1075}{8192}\alpha^{12}(-5\pm\sqrt{13})^5 + \frac{75}{16384}\alpha^{12}(-5\pm\sqrt{13})^6)/(6(\frac{\alpha^2}{2} + \frac{\alpha^2}{4}(-5\pm\sqrt{13})) (-\frac{\alpha^4}{8} - \frac{5}{8}\alpha^4(-5\pm\sqrt{13}) - \frac{5}{32}\alpha^4(-5\pm\sqrt{13})^2)^2) + \frac{\alpha^2}{4}(-5\pm\sqrt{13})(\coth\xi \pm \csc h\xi)^2,$$

$$u_{12} = (\frac{21}{256}\alpha^{12} + \frac{451}{512}\alpha^{12}(-5\pm\sqrt{13}) + \frac{2933}{1024}\alpha^{12}(-5\pm\sqrt{13})^2 + \frac{2785}{1024}\alpha^{12}(-5\pm\sqrt{13})^3 + \frac{3955}{4096}\alpha^{12}(-5\pm\sqrt{13})^4 + \frac{1075}{8192}\alpha^{12}(-5\pm\sqrt{13})^5 + \frac{75}{16384}\alpha^{12}(-5\pm\sqrt{13})^6)/(6(\frac{\alpha^2}{2} + \frac{\alpha^2}{4}(-5\pm\sqrt{13})) (-\frac{\alpha^4}{8} - \frac{5}{8}\alpha^4(-5\pm\sqrt{13}) - \frac{5}{32}\alpha^4(-5\pm\sqrt{13})^2)^2) + \frac{\alpha^2}{4}(-5\pm\sqrt{13})(\tanh\xi \pm i\sec h\xi)^2,$$

$$u_{13} = (\frac{21}{256}\alpha^{12} + \frac{451}{512}\alpha^{12}(-5\pm\sqrt{13}) + \frac{2933}{1024}\alpha^{12}(-5\pm\sqrt{13})^2 + \frac{2785}{1024}\alpha^{12}(-5\pm\sqrt{13})^3 + \frac{3955}{4096}\alpha^{12}(-5\pm\sqrt{13})^4 + \frac{1075}{8192}\alpha^{12}(-5\pm\sqrt{13})^5 + \frac{75}{16384}\alpha^{12}(-5\pm\sqrt{13})^6)/(6(\frac{\alpha^2}{2} + \frac{\alpha^2}{4}(-5\pm\sqrt{13})) (-\frac{\alpha^4}{8} - \frac{5}{8}\alpha^4(-5\pm\sqrt{13}) - \frac{5}{32}\alpha^4(-5\pm\sqrt{13})^2)^2) + \frac{\alpha^2}{4}(-5\pm\sqrt{13})\left(\frac{\tanh\xi}{1\pm\sec h\xi}\right)^2,$$

$$u_{14} = (\frac{21}{256}\alpha^{12} + \frac{451}{512}\alpha^{12}(-5\pm\sqrt{13}) + \frac{2933}{1024}\alpha^{12}(-5\pm\sqrt{13})^2 + \frac{2785}{1024}\alpha^{12}(-5\pm\sqrt{13})^3 + \frac{3955}{4096}\alpha^{12}(-5\pm\sqrt{13})^4 + \frac{1075}{8192}\alpha^{12}(-5\pm\sqrt{13})^5 + \frac{75}{16384}\alpha^{12}(-5\pm\sqrt{13})^6)/(6(\frac{\alpha^2}{2} + \frac{\alpha^2}{4}(-5\pm\sqrt{13})) (-\frac{\alpha^4}{8} - \frac{5}{8}\alpha^4(-5\pm\sqrt{13}) - \frac{5}{32}\alpha^4(-5\pm\sqrt{13})^2)^2) + \frac{\alpha^2}{4}(-5\pm\sqrt{13})\left(\frac{\coth\xi}{1\pm i\csc h\xi}\right)^2,$$

$$u_{15} = (44040192\alpha^{12} - 62062592\alpha^{12}(-20\pm4\sqrt{13}) - 28958720\alpha^{12}(-20\pm4\sqrt{13})^2 - 6129664\alpha^{12}(-20\pm4\sqrt{13})^3 - 663040\alpha^{12}(-20\pm4\sqrt{13})^4 - 14864\alpha^{12}(-20\pm4\sqrt{13})^5 + 54\alpha^{12}(-20\pm4\sqrt{13})^6)/(6(8\alpha^2 + \alpha^2(-20\pm4\sqrt{13})) (128\alpha^4 + 160\alpha^4(-20\pm4\sqrt{13}) + 10\alpha^4(-20\pm4\sqrt{13})^2)^2) + \alpha^2(-5\pm\sqrt{13})\left(\frac{\tan\xi}{1+\tan\xi}\right)^2 - ((-1280\alpha^6(-20\pm4\sqrt{13}) - 200\alpha^6(-20\pm4\sqrt{13})^2 - 2\alpha^6(-20\pm4\sqrt{13})^3)\tan\xi)/ ((64\alpha^4 + 80\alpha^4(-20\pm4\sqrt{13}) + 5\alpha^4(-20\pm4\sqrt{13})^2)(1+\tan\xi)),$$

$$u_{16} = (44040192\alpha^{12} - 62062592\alpha^{12}(-20\pm4\sqrt{13}) - 28958720\alpha^{12}(-20\pm4\sqrt{13})^2 - 6129664\alpha^{12}(-20\pm4\sqrt{13})^3 - 663040\alpha^{12}(-20\pm4\sqrt{13})^4 - 14864\alpha^{12}(-20\pm4\sqrt{13})^5 + 54\alpha^{12}(-20\pm4\sqrt{13})^6)/(6(8\alpha^2 + \alpha^2(-20\pm4\sqrt{13})) (128\alpha^4 + 160\alpha^4(-20\pm4\sqrt{13}) + 10\alpha^4(-20\pm4\sqrt{13})^2)^2) - ((1280\alpha^6(-20\pm4\sqrt{13}) + 200\alpha^6(-20\pm4\sqrt{13})^2 + 2\alpha^6(-20\pm4\sqrt{13})^3)\tan\xi)/((64\alpha^4 + 80\alpha^4(-20\pm4\sqrt{13}) + 5\alpha^4(-20\pm4\sqrt{13})^2)(1-\tan\xi)) + \alpha^2(-20\pm4\sqrt{13})\left(\frac{\tan\xi}{1-\tan\xi}\right)^2,$$

$$u_{17} = (44040192\alpha^{12} - 62062592\alpha^{12}(-20\pm4\sqrt{13}) - 28958720\alpha^{12}(-20\pm4\sqrt{13})^2 - 6129664\alpha^{12}(-20\pm4\sqrt{13})^3 - 663040\alpha^{12}(-20\pm4\sqrt{13})^4 - 14864\alpha^{12}(-20\pm4\sqrt{13})^5 + 54\alpha^{12}(-20\pm4\sqrt{13})^6)/(6(8\alpha^2 + \alpha^2(-20\pm4\sqrt{13})) (-128\alpha^4 - 160\alpha^4(-20\pm4\sqrt{13}) - 10\alpha^4(-20\pm4\sqrt{13})^2)^2) + ((1280\alpha^6(-20\pm4\sqrt{13}) + 200\alpha^6(-20\pm4\sqrt{13})^2 + 2\alpha^6(-20\pm4\sqrt{13})^3)\cot\xi)/((64\alpha^4 + 80\alpha^4(-20\pm4\sqrt{13}) + 5\alpha^4(-20\pm4\sqrt{13})^2)(1+\cot\xi)) + \alpha^2(-20\pm4\sqrt{13})\left(\frac{\cot\xi}{1+\cot\xi}\right)^2,$$

$$u_{18} = (44040192\alpha^{12} - 62062592\alpha^{12}(-20\pm4\sqrt{13}) - 28958720\alpha^{12}(-20\pm4\sqrt{13})^2 - 6129664\alpha^{12}(-20\pm4\sqrt{13})^3 - 663040\alpha^{12}(-20\pm4\sqrt{13})^4 - 14864\alpha^{12}(-20\pm4\sqrt{13})^5 + 54\alpha^{12}(-20\pm4\sqrt{13})^6)/(6(8\alpha^2 + \alpha^2(-20\pm4\sqrt{13})) (-128\alpha^4 - 160\alpha^4(-20\pm4\sqrt{13}) - 10\alpha^4(-20\pm4\sqrt{13})^2)^2) + ((-1280\alpha^6(-20\pm4\sqrt{13}) - 200\alpha^6(-20\pm4\sqrt{13})^2 - 2\alpha^6(-20\pm4\sqrt{13})^3)\cot\xi)/((64\alpha^4 + 80\alpha^4(-20\pm4\sqrt{13}) + 5\alpha^4(-20\pm4\sqrt{13})^2)(1-\cot\xi)) + \alpha^2(-20\pm4\sqrt{13})\left(\frac{\cot\xi}{1-\cot\xi}\right)^2,$$

$$u_{19} = \frac{\alpha^2(-20\pm4\sqrt{13})}{(c_0 - 2\xi)^2}.$$

Example 3. In this example we consider the Drinfeld-Sokolov-Wilson equation system

$$u_t - 3ww_x = 0, \qquad (15)$$
$$w_t - 2w_{xxx} - 2uw_x - wu_x = 0.$$

When balancing u_t with $3ww_x$ and w_{xxx} with uw_x then gives $m_1 = 2$ and $m_2 = 1$. Therefore, we may choose

$$u = f(t) + g(t)F(\xi) + h(t)F^2(\xi),$$
$$w = f_1(t) + g_1(t)F(\xi), \qquad (16)$$

where $\xi = \xi(x,t) = \alpha x + q(t)$.

Substituting (16) into Equation (15) yields a set of algebraic equations for $f(t), g(t), h(t), f_1(t), g_1(t)$ and $\xi(x,t)$. These equations are finding as

$$-3A\alpha f_1 g_1 - 3BF(\xi)\alpha f_1 g_1 - 3CF^2(\xi)\alpha f_1 g_1$$
$$-3AF(\xi)\alpha g_1^2 - 3BF^2(\xi)\alpha g_1^2 - 3CF^3(\xi)\alpha g_1^2$$
$$+ f_t + F(\xi)g_t F^2(\xi)h_t + Agq_t + BF(\xi)gq_t + CF^2(\xi)gq_t$$
$$+2AF(\xi)hq_t + 2BF^2(\xi)hq_t + 2CF^3(\xi)hq_t = 0$$

$$-A\alpha g f_{1t} - BF(\xi)\alpha g f_1 - CF^2(\xi)\alpha g f_1$$
$$-2AF(\xi)\alpha h f_1 - 2BF^2(\xi)\alpha h f_1 - 2CF^3(\xi)\alpha h f_1$$
$$-2AB^2\alpha^3 g_1 - 4A^2C\alpha^3 g_1 - 2B^3F(\xi)\alpha^3 g_1 \qquad (17)$$
$$-16ABCF(\xi)\alpha^3 g_1 - 14B^2CF^2(\xi)\alpha^3 g_1 - 16AC^2F^2(\xi)\alpha^3 g_1$$
$$-24BC^2F^3(\xi)\alpha^3 g_1 - 12C^3F^4(\xi)\alpha^3 g_1 - 2A\alpha f g_1 - 2BF(\xi)\alpha f g_1$$
$$-2CF^2(\xi)\alpha f g_1 - 3AF(\xi)\alpha g g_1 - 3BF^2(\xi)\alpha g g_1$$
$$-3CF^3(\xi)\alpha g g_1 - 4AF^2(\xi)\alpha h g_1 - 4BF^3(\xi)\alpha h g_1 - 4CF^4(\xi)\alpha h g_1$$
$$+Ag_1 q_t + BF(\xi)g_1 q_t + CF^2(\xi)g_1 q_t + f_{1t} + F(\xi)g_{1t} = 0$$

From the solutions of the equations, we can found;

a) for $g_1 = iC\sqrt{2\alpha q_t}$,

$$h = -3\alpha^2C^2, g = -3BC\alpha^2, f = \frac{-\alpha^3B^2 - 8AC\alpha^3 + 2q_t}{4\alpha}, \quad (18)$$
$$f_1 = \frac{iB\sqrt{\alpha q_t}}{\sqrt{2}}, q_{tt} = 0,$$

b) for $g_1 = -iC\sqrt{2\alpha q_t}$,

$$h = -3\alpha^2C^2, g = -3BC\alpha^2, f = \frac{-\alpha^3B^2 - 8AC\alpha^3 + 2q_t}{4\alpha}, \quad (19)$$
$$f_1 = -\frac{iB\sqrt{\alpha q_t}}{\sqrt{2}}, q_{tt} = 0,$$

with the aid of Mathematica. From (18) and (19), we can get

$$q_t = \lambda, q = \lambda t,$$
$$h = -3\alpha^2C^2, g = -3BC\alpha^2, f = \frac{-\alpha^3B^2 - 8AC\alpha^3 + 2\lambda}{4\alpha}, \quad (20)$$
$$f_1 = \pm\frac{iB\sqrt{\alpha\lambda}}{\sqrt{2}}, g_1 = \pm iC\sqrt{2\alpha\lambda}.$$

where $\lambda, \theta =$ const.

Substituting (19) or (20) into (16) and using special solutions of Eq. (15), we obtain the following multiple soliton-like and triangular periodic solutions of Eq.(15):

$$u_1 = \frac{-8\alpha^3 + 2\lambda}{4\alpha} - 3\alpha^2\tan^2\xi,$$
$$w_1 = \pm i\sqrt{2\alpha\lambda}\tan\xi,$$
$$u_2 = \frac{-8\alpha^3 + 2\lambda}{4\alpha} - 3\alpha^2\cot^2\xi,$$
$$w_2 = \pm i\sqrt{2\alpha\lambda}\cot\xi,$$
$$u_3 = \frac{8\alpha^3 + 2\lambda}{4\alpha} - 3\alpha^2\tanh^2\xi,$$
$$w_3 = \pm i\sqrt{2\alpha\lambda}\tanh\xi,$$
$$u_4 = \frac{8\alpha^3 + 2\lambda}{4\alpha} - 3\alpha^2\coth^2\xi,$$
$$w_4 = \pm i\sqrt{2\alpha\lambda}\coth\xi,$$
$$u_5 = \frac{-2\alpha^3 + 2\lambda}{4\alpha} - \frac{3}{4}\alpha^2(\tan\xi \pm \sec\xi)^2,$$
$$w_5 = \frac{\pm i\sqrt{\alpha\lambda}}{\sqrt{2}}(\tan\xi \pm \sec\xi),$$
$$u_6 = \frac{-2\alpha^3 + 2\lambda}{4\alpha} - \frac{3}{4}\alpha^2(\csc\xi - \cot\xi)^2,$$
$$w_6 = \frac{\pm i\sqrt{\alpha\lambda}}{\sqrt{2}}(\csc\xi - \cot\xi),$$
$$u_7 = \frac{-2\alpha^3 + 2\lambda}{4\alpha} - \frac{3}{4}\alpha^2\left(\frac{\tan\xi}{1 \pm \sec\xi}\right)^2,$$
$$w_7 = \frac{\pm i\sqrt{\alpha\lambda}}{\sqrt{2}}\left(\frac{\tan\xi}{1 \pm \sec\xi}\right),$$
$$u_8 = \frac{-2\alpha^3 + 2\lambda}{4\alpha} - \frac{3}{4}\alpha^2(\cot\xi \pm \csc\xi)^2,$$
$$w_8 = \frac{\pm i\sqrt{\alpha\lambda}}{\sqrt{2}}(\cot\xi \pm \csc\xi),$$
$$u_9 = \frac{-2\alpha^3 + 2\lambda}{4\alpha} - \frac{3}{4}\alpha^2(\sec\xi - \tan\xi)^2,$$
$$w_9 = \frac{\pm i\sqrt{\alpha\lambda}}{\sqrt{2}}(\sec\xi - \tan\xi), \qquad (21)$$
$$u_{10} = \frac{-2\alpha^3 + 2\lambda}{4\alpha} - \frac{3}{4}\alpha^2\left(\frac{\cot\xi}{1 \pm \csc\xi}\right)^2,$$
$$w_{10} = \frac{\pm i\sqrt{\alpha\lambda}}{\sqrt{2}}\left(\frac{\cot\xi}{1 \pm \csc\xi}\right),$$

$$u_{11} = \frac{2\alpha^3 + 2\lambda}{4\alpha} - \frac{3}{4}\alpha^2(\coth\xi \pm \csc h\xi)^2,$$
$$w_{11} = \frac{\pm i\sqrt{\alpha\lambda}}{\sqrt{2}}(\coth\xi \pm \csc h\xi),$$
$$u_{12} = \frac{2\alpha^3 + 2\lambda}{4\alpha} - \frac{3}{4}\alpha^2(\tanh\xi \pm i\sec h\xi)^2,$$
$$w_{12} = \frac{\pm i\sqrt{\alpha\lambda}}{\sqrt{2}}(\tanh\xi \pm i\sec h\xi),$$

$$u_{13} = \frac{2\alpha^3 + 2\lambda}{4\alpha} - \frac{3}{4}\alpha^2\left(\frac{\tanh\xi}{1 \pm \sec h\xi}\right)^2,$$
$$w_{13} = \frac{\pm i\sqrt{\alpha\lambda}}{\sqrt{2}}\left(\frac{\tanh\xi}{1 \pm \sec h\xi}\right),$$
$$u_{14} = \frac{2\alpha^3 + 2\lambda}{4\alpha} - \frac{3}{4}\alpha^2\left(\frac{\coth\xi}{1 \pm i\csc h\xi}\right)^2,$$
$$w_{14} = \frac{\pm i\sqrt{\alpha\lambda}}{\sqrt{2}}\left(\frac{\coth\xi}{1 \pm i\csc h\xi}\right),$$
$$u_{15} = \frac{-20\alpha^3 + 2\lambda}{4\alpha} + 12\alpha^2\frac{\tan\xi}{1 + \tan\xi} - 12\alpha^2\left(\frac{\tan\xi}{1 + \tan\xi}\right)^2,$$
$$w_{15} = \pm i\sqrt{2\alpha\lambda} \mp 2i\sqrt{2\alpha\lambda}\left(\frac{\tan\xi}{1 + \tan\xi}\right),$$
$$u_{16} = \frac{-20\alpha^3 + 2\lambda}{4\alpha} - 12\alpha^2\frac{\tan\xi}{1 - \tan\xi} - 12\alpha^2\left(\frac{\tan\xi}{1 - \tan\xi}\right)^2,$$
$$w_{16} = \mp i\sqrt{2\alpha\lambda} \mp 2i\sqrt{2\alpha\lambda}\left(\frac{\tan\xi}{1 - \tan\xi}\right),$$
$$u_{17} = \frac{-20\alpha^3 + 2\lambda}{4\alpha} + 12\alpha^2\left(\frac{\cot\xi}{1 + \cot\xi}\right) - 12\alpha^2\left(\frac{\cot\xi}{1 + \cot\xi}\right)^2,$$
$$w_{17} = \mp i\sqrt{2\alpha\lambda} \pm 2i\sqrt{2\alpha\lambda}\left(\frac{\cot\xi}{1 + \cot\xi}\right),$$
$$u_{18} = \frac{-20\alpha^3 + 2\lambda}{4\alpha} - 12\alpha^2\left(\frac{\cot\xi}{1 - \cot\xi}\right) - 12\alpha^2\left(\frac{\cot\xi}{1 - \cot\xi}\right)^2,$$
$$w_{18} = \pm i\sqrt{2\alpha\lambda} \pm 2i\sqrt{2\alpha\lambda}\left(\frac{\cot\xi}{1 - \cot\xi}\right),$$
$$u_{19} = \frac{\lambda}{2\alpha} - \frac{12\alpha^2}{(c_0 - 2\xi)^2},$$
$$w_{19} = \frac{\mp 2i\sqrt{2\alpha\lambda}}{c_0 - C\xi}.$$

4. Discussions

In this paper, the generalized tanh method has been successfully applied with aid of Mathematica, implement it in a computer algebraic system. The generalized tanh method is used to find new exact traveling wave solutions of the Benjamin-Bona-Mahony equation, Lax's fifth-order KdV equation and Drinfeld-Sokolov-Wilson equation system. We also obtain some new and more general solutions at same time. The method can be used to many other nonlinear equations or coupled ones. In addition, this method is also computerizable, which allows us to perform complicated and tedious algebraic calculation on a computer.

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References

1. W. Malfliet, Solitary wave solutions of nonlinear wave equations, Amer. J. Phys. 60, 650 (1992).
2. L. Debtnath, Nonlinear Partial Differential Equations for Scientist and Engineers, Birkhauser, Boston, MA, 1997.
3. A.M. Wazwaz, Partial Differential Equations: Methods and Applications, Balkema, Rotterdam, 2002.
4. W. Hereman, P.P. Banerjee, A. Korpel, G. Assanto, A. van Immerzeele, A. Meerpoel, Exact solitary wave solutions of nonlinear evolution and wave equations using a direct algebraic method, J. Phys. A: Math. Gen. 19 (1986) 607.
5. E. J. Parkes, Exact solutions to the two-dimensional Korteweg-de Vries-Burgers equation, J. Phys. A 27, L497 (1994).
6. E. J. Parkes and B. R. Duffy, An automated tanh-function method for finding solitary wave solutions to nonlinear evolution equations, Computer Phys. Commun. 98, 288 (1996).
7. H. Chen, H. Zhang, New multiple soliton-like solutions to the generalized (2 + 1)- dimensional KP equation, Appl. Math. Comput. 157 (2004) 765-773.
8. E. G. Fan, Y. C. Hon, Generalized tanh Method Extended to Special Types of Nonlinear Equations, Z. Naturforsch. 56a, 312 (2001).
9. E. G. Fan, Soliton solutions for a generalized HirotaCSatsuma coupled KdV equation and a coupled MKdV equation, Phys. Lett. A 282, 18 (2001).
10. E. G. Fan, J. Zhang, and Y. C. Hon, A new complex line soliton for the two-dimensional KdV-Burgers equation, Phys. Lett. A 291, 376
11. A.H. Khater, M.A. Helal, O.H. El-Kalaawy, Bäcklund transformations: exact solutions for the KdV and the Calogero--Degasperis--Fokas mKdV equations, Math. Methods Appl. Sci. 21 (1998) 719.

12. A.M. Wazwaz, A study of nonlinear dispersive equations with solitary-wave solutions having compact support, Math. Comput. Simul. 56 (2001) 269-276.
13. C.H. Gu,H. S.Hu, andZ.X.Zhou, DarbouxTransformations in Solition Theory and its Geometric Applications, Shanghai Sci. Tech. Publ. 1999.
14. S.A. Elwakil, S.K. El-Labany, M.A. Zahran, R. Sabry, Modified extended tanh-function method for solving nonlinear partial differential equations,Phys. Lett. A 299 (2002) 179.
15. Y. Lei, Z. Fajiang, W. Yinghai, The homogeneous balance method, Lax pair, Hirota transformation and a general fifth-order KdV equation, Chaos Solitons Fractals 13 (2002) 337.
16. T. Yoshinaga, M.Wakamiya, and T. Kakutani, Recurrence and chaotic behavior resulting from nonlinear interaction between long and short waves Phys. Fluids, A 3, 83 (1991).
17. M. Airault, H. Mckean, and J. Moser, Rational and Elliptic Solutions of the Korteweg-DeVries Equation and a Related Many-Body Problem. Commun. Pure. Appl. Math. 30, 95 (1977).
18. M. Adler and J. Moser, Some Finite Dimensional Integrable Systems and Their Behavior, Commun. Math. Phys. 19, 1 (1978).
19. J.F. Zhang, New exact solitary wave solutions of the KS equation, Int. J. Theor. Phys. 38 (1999) 1829.
20. M.L. Wang, Exact solutions for a compound KdV-Burgers equation, Phys. Lett. A 213 (1996) 279.
21. M.L. Wang, Y.B. Zhou, Z.B. Li, Application of a homogeneous balance method to exact solutions of nonlinear equations in mathematical physics, Phys. Lett. A 216(1996),67-75.
22. Z.S. Körpınar, Solitions of some nonlinear partial differential equations by using analytical and semi-analytical methods, Ph. D. Thesis, Fırat University, (in preparation)
23. H.I. Abdel-Gawad, M. Osman, N. S. Elazab, Exact Solutions of Space-Time Dependent Korteweg-de Vries Equation by The Extended Unified Method, *Life Sci J* 2013;10(2):2598-2604.
24. M. Alghamdi, E. M. Elsayed and M. M. Eldessoky, On the Solutions of Some Systems of Second Order Rational Difference Equations, *Life Sci J* 2013;10(3):344-351.

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