Calculation of Inviscid Compressible Flow past a Symmetric Aerofoil Using Direct Boundary Element Method with Linear Element Approach

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Abstract: In this paper, a steady, irrotational, inviscid compressible flow past a symmetric aerofoil has been calculated using direct boundary element method (DBEM) with linear element approach. The velocity distribution for the flow over the surface of the symmetric aerofoil has been compared with the analytical results.
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Keywords: Direct boundary element method, Inviscid Compressible flow, Symmetric aerofoil.

1. Introduction

In the present period of science and technology, the popularity of boundary element methods (BEMs) rises for solving fluid flow problems. These methods exist under different names such as panel methods, surface singularity method, boundary integral equation method, and boundary integral equation. Previously, finite difference method, finite element method, etc. were being used to solve numerically the problems in computational fluid dynamics. Later on, boundary element method has received much attention from the researchers due to its various advantages over the domain type methods. One of the advantages is that with boundary elements one has to discretize only the surface of the body, whereas with domain methods it is essential to discretize the entire region of the flow field. Moreover, this method is well–suited to problems with an infinite domain. The boundary element method can be classified into two categories i.e. direct and indirect. The direct method takes the form of a statement which provides the values of the unknown variables at any field point in terms of the complete set of all the boundary data. On the other hand, the indirect method utilizes a distribution of singularities over the boundary of the body and computes this distribution as the solution of integral equation (Brebbia, 1978 & 1980). The direct and indirect methods have been used in the past for flow field calculations around bodies (Morino 1975, Hess & Smith, 1967, Kohr, 2000, Luminita, 2008, Muhammad, 2008; Mushtaq, 2008, 2009, 2010, 2011& 2012). Most of the work on fluid flow calculations using boundary element methods has been done in the field of incompressible flow. Very few attempts have been made on flow field calculations using boundary element methods in the field of compressible flow. In this paper, the DBEM has been used for the solution of inviscid compressible flows around a symmetric aerofoil.

2. Mathematical Formulation

We know that equation of motion for two – dimensional, steady, irrotational, and isentropic flow (Mushtaq, 2010, 2011 & 2012, Shah 2011) is

$$(1 - Ma^2)\frac{\partial^2 \Phi}{\partial X^2} + \frac{\partial^2 \Phi}{\partial Y^2} = 0 \qquad (1)$$

where Ma is the Mach number and Φ is the total velocity potential of the flow. Here X and Y are the space coordinates.

Using the dimensionless variables, $x = X$, $y = \beta Y$, where $\beta = \sqrt{1 - Ma^2}$, equation (1) becomes

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0$$

or $\nabla^2 \Phi = 0$ (2)

which is Laplace's equation.

3. Inviscid Compressible Flow Past a Symmetric Aerofoil

Consider the flow past a sy. aerofoil and let the onset flow be the uniform stream with velocity U in the positive direction of the x – axis as shown in figure (1).

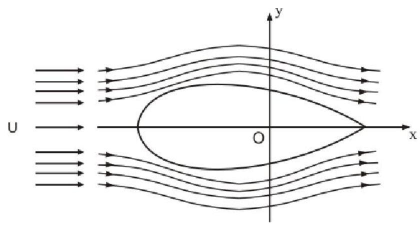


Figure 1: Flow past a symmetric aerofoil.

Exact Velocity

The magnitude of the exact velocity distribution over the surface of a sy. aerofoil is given by Chow [3] & Mushtaq [12, 16, 17,19]

as $$V = U \left| \frac{1 - \left(\frac{r}{z-b}\right)^2}{1 - \left(\frac{a}{z}\right)^2} \right|$$

where r = radius of the circular cylinder,

a = Joukowski transformation constant

and b = a – r = x-coordinates of the centre of the circular cylinder

In Cartesian coordinates, we have

V = U

Error!

$$x \frac{\sqrt{[(x^2+y^2)^2 - a^2(x^2-y^2)]^2 + 4a^4x^2y^2}}{(x^2+y^2)^2 - 2a^2(x^2-y^2) + a^4}$$

Boundary Conditions

Now the condition to be satisfied on the boundary of a symmetric aerofoil is (Mushtaq, 2011)

$$\frac{\partial \phi_{s.a}}{\partial n} = U \frac{(x+b)}{\sqrt{(x+b)^2 + y^2}} \qquad (3)$$

Where the subscript s . a stands for symmetric aerofoil

Equation (3) is the boundary condition which must be satisfied over the boundary of a symmetric aerofoil.

Equation of Direct Boundary Element Method

The equation of DBEM for two–dimensional flow [Mushtaq, 2008, 2009, 2010 & 2011] is :

$$-c_i \phi_i + \frac{1}{2\pi} \int_{\Gamma - i} \phi \frac{\partial}{\partial n} \left[\log \left(\frac{1}{r}\right) \right] d\Gamma + \phi_\infty = \frac{1}{2\pi} \int_{\Gamma} \log\left(\frac{1}{r}\right) \frac{\partial \phi}{\partial n} d\Gamma \qquad (4)$$

where c_i = 0 when i is exterior to Γ

= 1 when i is interior to Γ

= $\frac{1}{2}$ when i lies on Γ and Γ is smooth.

Matrix Formulation with Linear Element Approach

The equation DBEM (4) can be written for this case as

$$-c_i \phi_i + \sum_{j=1}^{m} \int_{\Gamma_j - i} \phi \frac{\partial}{\partial n} \left(\frac{1}{2\pi} \log \frac{1}{r}\right) d\Gamma + \phi_\infty = \sum_{j=1}^{m} \int_{\Gamma_j} \frac{\partial \phi}{\partial n} \left(\frac{1}{2\pi} \log \frac{1}{r}\right) d\Gamma \qquad (5)$$

Since ϕ and $\frac{\partial \phi}{\partial n}$ vary linearly over the element, their values at any point on the element can be defined in terms of their nodal values and the shape functions N_1 and N_2 as

$$\phi = [N_1\, N_2] \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix}$$

$$\frac{\partial \phi}{\partial n} = [N_1\, N_2] \begin{Bmatrix} \frac{\partial \phi_1}{\partial n} \\ \frac{\partial \phi_2}{\partial n} \end{Bmatrix} \qquad (6)$$

The integrals along an element 'j' on the L.H.S. of equation (5) can now be written as

$$\int_{\Gamma_j - i} \phi \frac{\partial}{\partial n} \left(\frac{1}{2\pi} \log \frac{1}{r}\right) d\Gamma = \int_{\Gamma_j - i} [N_1\, N_2] \frac{\partial}{\partial n} \left(\frac{1}{2\pi} \log \frac{1}{r}\right) d\Gamma \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix} = \left[h_{ij}^1 \;\; h_{ij}^2 \right] \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix}$$

where $$h_{ij}^k = \int_{\Gamma_j - i} N_k \frac{\partial}{\partial n} \left(\frac{1}{2\pi} \log \frac{1}{r}\right) d\Gamma, \quad k = 1, 2. \qquad (7)$$

The integrals on the R.H.S. of equation (5) can be written as

$$\int_{\Gamma_j} \frac{\partial \phi}{\partial n} \left(\frac{1}{2\pi} \log \frac{1}{r}\right) d\Gamma = \int_{\Gamma_j} [N_1\, N_2] \left(\frac{1}{2\pi} \log \frac{1}{r}\right) d\Gamma \begin{Bmatrix} \frac{\partial \phi_1}{\partial n} \\ \frac{\partial \phi_2}{\partial n} \end{Bmatrix} = \left[g_{ij}^1 \;\; g_{ij}^2 \right] \begin{Bmatrix} \frac{\partial \phi_1}{\partial n} \\ \frac{\partial \phi_2}{\partial n} \end{Bmatrix}$$

where $$g_{ij}^k = \int_{\Gamma_j} N_k \left(\frac{1}{2\pi} \log \frac{1}{r}\right) d\Gamma, \quad k = 1, 2. \qquad (8)$$

Again the integrals in equations (7) and (8) are calculated numerically as before except for the element on which the fixed point 'i' is lying. For this element the integrals are calculated analytically. The integrals h^1_{ii} and h^2_{ii} are zero because r and $\hat{n}$ are orthogonal to each other over the element. The value of the integrals g^1_{ii} and g^2_{ii} are given by

$$g^{(1)}_{ii} = \frac{\ell}{8\pi}[3 - 2\log\ell]$$

$$\text{and } g^{(2)}_{ii} = \frac{\ell}{8\pi}[1 - 2\log\ell]$$

Also the velocity midway between two nodes on the boundary can then be approximated by using the formula

$$\text{Velocity } \overset{\frown}{V} = \frac{\Phi_{k+1} - \Phi_k}{\text{Length from node } k \text{ to } k+1} \qquad (9)$$

Where the total velocity potential Φ is the sum of the perturbation velocity potential $\phi_{s.a}$ and the velocity potential of the uniform stream $\phi_{u.s}$.

Process of Discretization

Now for the discretization of the boundary of the symmetric aerofoil, the coordinates of the extreme points of the boundary elements can be generated within computer programme using Fortran language as follows:

Divide the boundary of the circular cylinder into m elements in the clockwise direction by using the formula (Mushtaq 2009, 2010, 2011 & 2012).

$$\theta_k = [(m+2) - 2k]\frac{\pi}{m},$$

$$k = 1, 2, \ldots\ldots, m \qquad (10)$$

Then the extreme points of these m elements of circular cylinder are found by

$$\xi_k = -b + r\cos\theta_k$$

$$\eta_k = r\sin\theta_k$$

Now by using Joukowski transformation

$$z = \zeta + \frac{a^2}{\zeta}$$

the extreme points of the sy. aerofoil are

$$x_k = \xi_k\left(1 + \frac{a^2}{\xi_k^2 + \eta_k^2}\right)$$

$$y_k = \eta_k\left(1 - \frac{a^2}{\xi_k^2 + \eta_k^2}\right)$$

where $k = 1, 2, \ldots\ldots, m$.

Therefore the boundary condition (3) in this case takes the form

$$\frac{\partial\phi_{s.a}}{\partial n} = U\frac{(x_k + b)}{\sqrt{(x_k + b)^2 + y_k^2}}$$

$$= \frac{(x_k + b)}{\sqrt{(x_k + b)^2 + y_k^2}} \qquad \text{taking } U = 1 \qquad (11)$$

The following tables show the comparison of computed and analytical velocity distribution over the boundary of a sy. aerofoil for 8, 16, 32, and 64 direct linear boundary elements.

Table (1)

ELEMENT	X	Y	$R = \sqrt{X^2 + Y^2}$	VELOCITY	EXACT VELOCITY
1	-1.94	.39	1.98	.72178E+00	.83769E+00
2	-1.39	.94	1.68	.17456E+01	.20086E+01
3	-.62	.93	1.12	.17562E+01	.20216E+01
4	-.01	.38	.38	.80442E+00	.70748E+00
5	-.01	-.38	.38	.80442E+00	.70748E+00
6	-.62	-.93	1.12	.17562E+01	.20216E+01
7	-1.39	-.94	1.68	.17456E+01	.20086E+01
8	-1.94	-.39	1.98	.72178E+00	.83768E+00

Table (2)

ELEMENT	X	Y	$R=\sqrt{X^2+Y^2}$	VELOCITY	EXACT VELOCITY
1	-2.06	.21	2.07	.38552E+00	.39565E+00
2	-1.90	.60	1.99	.10980E+01	.11264E+01
3	-1.60	.89	1.84	.16440E+01	.16923E+01
4	-1.22	1.05	1.61	.19407E+01	.20055E+01
5	-.79	1.05	1.32	.19438E+01	.20115E+01
6	-.40	.89	.98	.16542E+01	.16985E+01
7	-.10	.58	.59	.11197E+01	.10938E+01
8	.11	.20	.23	.48102E+00	.30764E+00
9	.11	-.20	.23	.48102E+00	.30764E+00
10	-.10	-.58	.59	.11197E+01	.10938E+01
11	-.40	-.89	.98	.16542E+01	.16985E+01
12	-.79	-1.05	1.32	.19438E+01	.20115E+01
13	-1.22	-1.05	1.61	.19407E+01	.20055E+01
14	-1.60	-.89	1.84	.16440E+01	.16923E+01
15	-1.90	-.60	1.99	.10980E+01	.11264E+01
16	-2.06	-.21	2.07	.38551E+00	.39565E+00

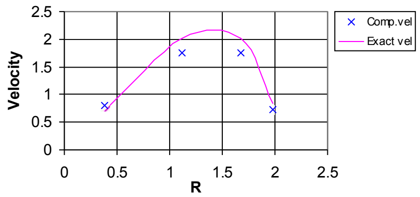
Table (3)

ELEMENT	X	Y	$R=\sqrt{X^2+Y^2}$	VELOCITY	EXACT VELOCITY
1	-2.09	.11	2.10	.19601E+00	.19530E+00
2	-2.05	.32	2.08	.58051E+00	.57871E+00
3	-1.97	.51	2.04	.94275E+00	.94085E+00
4	-1.85	.69	1.98	.12688E+01	.12682E+01
5	-1.70	.84	1.90	.15463E+01	.15485E+01
6	-1.52	.96	1.80	.17645E+01	.17706E+01
7	-1.32	1.04	1.68	.19151E+01	.19256E+01
8	-1.11	1.08	1.55	.19925E+01	.20067E+01
9	-.90	1.08	1.41	.19936E+01	.20097E+01
10	-.69	1.04	1.25	.19187E+01	.19331E+01
11	-.49	.96	1.07	.17707E+01	.17783E+01
12	-.31	.84	.89	.15559E+01	.15493E+01
13	-.16	.68	.70	.12832E+01	.12519E+01
14	-.04	.50	.50	.96547E+00	.89212E+00
15	.06	.29	.29	.63407E+00	.47258E+00
16	.15	.09	.17	.27901E+00	.17793E+00
17	.15	-.09	.17	.27902E+00	.17793E+00
18	.06	-.29	.29	.63407E+00	.47258E+00
19	-.04	-.50	.50	.96547E+00	.89212E+00
20	-.16	-.68	.70	.12832E+01	.12519E+01
21	-.31	-.84	.89	.15559E+01	.15493E+01
22	-.49	-.96	1.07	.17707E+01	.17783E+01
23	-.69	-1.04	1.25	.19187E+01	.19331E+01
24	-.90	-1.08	1.41	.19936E+01	.20097E+01
25	-1.11	-1.08	1.55	.19925E+01	.20067E+01
26	-1.32	-1.04	1.68	.19151E+01	.19256E+01
27	-1.52	-.96	1.80	.17645E+01	.17706E+01
28	-1.70	-.84	1.90	.15463E+01	.15485E+01
29	-1.85	-.69	1.98	.12688E+01	.12682E+01
30	-1.97	-.51	2.04	.94275E+00	.94085E+00
31	-2.05	-.32	2.08	.58050E+00	.57871E+00
32	-2.09	-.11	2.10	.19601E+00	.19529E+00

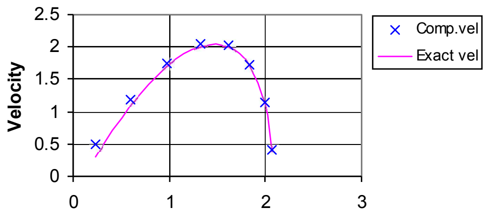
Table (4)

ELEMENT	X	Y	$R=\sqrt{X^2+Y^2}$	VELOCITY	EXACT VELOCITY
1	-2.10	.05	2.10	.98434E-01	.97607E-01
2	-2.09	.16	2.10	.29438E+00	.29139E+00
3	-2.07	.27	2.09	.48748E+00	.48261E+00
4	-2.04	.37	2.07	.67590E+00	.66939E+00
5	-2.00	.47	2.05	.85784E+00	.85001E+00
6	-1.95	.56	2.03	.10315E+01	.10228E+01
7	-1.89	.65	2.00	.11953E+01	.11861E+01
8	-1.82	.74	1.96	.13476E+01	.13383E+01
9	-1.74	.81	1.92	.14870E+01	.14781E+01
10	-1.66	.88	1.88	.16120E+01	.16040E+01
11	-1.57	.94	1.83	.17217E+01	.17148E+01
12	-1.47	.99	1.78	.18147E+01	.18093E+01
13	-1.37	1.03	1.72	.18904E+01	.18867E+01
14	-1.27	1.06	1.66	.19480E+01	.19459E+01
15	-1.17	1.08	1.59	.19868E+01	.19864E+01
16	-1.06	1.09	1.52	.20067E+01	.20075E+01
17	-.95	1.09	1.45	.20073E+01	.20090E+01
18	-.84	1.08	1.37	.19888E+01	.19906E+01
19	-.74	1.06	1.29	.19512E+01	.19524E+01
20	-.63	1.03	1.21	.18950E+01	.18946E+01
21	-.53	.99	1.12	.18208E+01	.18174E+01
22	-.44	.93	1.03	.17294E+01	.17213E+01
23	-.35	.87	.94	.16217E+01	.16072E+01
24	-.27	.80	.85	.14988E+01	.14756E+01
25	-.19	.73	.75	.13620E+01	.13275E+01
26	-.12	.64	.65	.12131E+01	.11637E+01
27	-.06	.55	.55	.10538E+01	.98490E+00
28	-.00	.45	.45	.88675E+00	.79156E+00
29	.04	.34	.35	.71606E+00	.58415E+00
30	.08	.23	.24	.55181E+00	.36688E+00
31	.12	.11	.16	.43243E+00	.18028E+00
32	.17	.03	.17	.12239E+00	.17941E+00
33	.17	-.03	.17	.12238E+00	.17941E+00
34	.12	-.11	.16	.43243E+00	.18028E+00
35	.08	-.23	.24	.55181E+00	.36688E+00
36	.04	-.34	.35	.71605E+00	.58415E+00
37	-.00	-.45	.45	.88676E+00	.79156E+00
38	-.06	-.55	.55	.10538E+01	.98490E+00
39	-.12	-.64	.65	.12131E+01	.11637E+01
40	-.19	-.73	.75	.13620E+01	.13275E+01
41	-.27	-.80	.85	.14988E+01	.14756E+01
42	-.35	-.87	.94	.16217E+01	.16072E+01
43	-.44	-.93	1.03	.17294E+01	.17213E+01
44	-.53	-.99	1.12	.18208E+01	.18174E+01
45	-.63	-1.03	1.21	.18950E+01	.18946E+01
46	-.74	-1.06	1.29	.19512E+01	.19524E+01
47	-.84	-1.08	1.37	.19888E+01	.19906E+01
48	-.95	-1.09	1.45	.20073E+01	.20090E+01
49	-1.06	-1.09	1.52	.20067E+01	.20075E+01
50	-1.17	-1.08	1.59	.19868E+01	.19864E+01
51	-1.27	-1.06	1.66	.19480E+01	.19459E+01
52	-1.37	-1.03	1.72	.18904E+01	.18867E+01
53	-1.47	-.99	1.78	.18147E+01	.18093E+01
54	-1.57	-.94	1.83	.17217E+01	.17148E+01
55	-1.66	-.88	1.88	.16120E+01	.16040E+01
56	-1.74	-.81	1.92	.14870E+01	.14781E+01

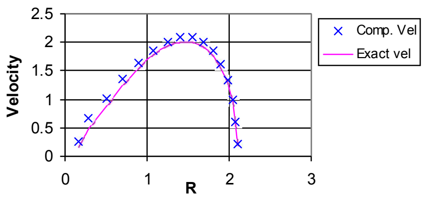
57	-1.82	-.74	1.96	.13476E+01	.13383E+01
58	-1.89	-.65	2.00	.11953E+01	.11861E+01
59	-1.95	-.56	2.03	.10315E+01	.10228E+01
60	-2.00	-.47	2.05	.85783E+00	.85001E+00
61	-2.04	-.37	2.07	.67590E+00	.66939E+00
62	-2.07	-.27	2.09	.48747E+00	.48261E+00
63	-2.09	-.16	2.10	.29438E+00	.29139E+00
64	-2.10	-.05	2.10	.98439E-01	.97606E-01



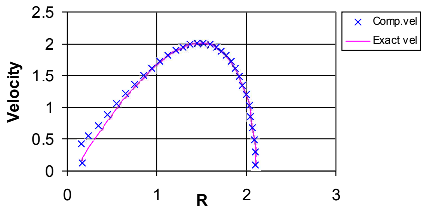
Graph 1: Comparison of computed and analytical velocity distributions over the boundary of a symmetric aerofoil using 8 boundary elements with direct constant element approach for $r = 1.1$, $a = 0.1$ and $Ma = 0.7$.



Graph 2: Comparison of computed and analytical velocity distributions over the boundary of a symmetric aerofoil using 16 boundary elements with direct constant element approach for $r = 1.1$, $a = 0.1$ and $Ma = 0.7$.



Graph 3: Comparison of computed and analytical velocity distributions over the boundary of a symmetric aerofoil using 32 boundary elements with direct constant element approach for $r = 1.1$, $a = 0.1$ and $Ma = 0.7$.



Graph 4: Comparison of computed and analytical velocity distributions over the boundary of a symmetric aerofoil using 64 boundary elements with direct constant element approach for $r = 1.1$, $a = 0.1$ and $Ma = 0.7$.

4. Conclusion

A direct boundary element method has been applied for the calculation of inviscid compressible flow past a symmetric aerofoil with linear element approach. The calculated flow velocities obtained using this method is compared with the analytical solutions for flow over the boundary of a symmetric aerofoil. The tables and graphs, indicate that the computed results obtained by this method are good in agreement with the analytical ones for the body under consideration and the accuracy of the result increases due to increase of number of boundary elements.

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