An Integrated Location Inventory Model for Designing a Supply Chain Network under Uncertainty

Mir-Bahador Aryanezhad, Seyed Gholamreza Jalali Naini, Armin Jabbarzadeh

[1.] Department of Industrial Engineering, Iran University of Science and Technology, 16846113114, Tehran, Iran
arminj@iust.ac.ir

Abstract: This paper studies a supply chain design problem with an unreliable supplier and random demand. Due to imperfect performance of the supplier, the quantity of the product received from the supplier may be less than the quantity ordered by distribution centers (DCs). In this system, customers have random demands and the supply chain is flexible in determining which customers to serve. The problem is formulated as a nonlinear integer programming model that simultaneously determines which customers are served, where DCs are located and how DCs are assigned to the customers. The objective of the model is to minimize the total costs including location costs, nonlinear inventory costs, transportation costs, and lost sales costs. In order to solve the model, an effective solution method based on genetic algorithm is developed. Finally, computational results for several instances of the problem are presented to demonstrate the effectiveness of the proposed solution approach.
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1. Introduction

In today's increasingly competitive environment, the efficient design of supply chain plays a decisive role in successful performance of companies. Supply chain design models typically treat strategic decisions and tactical decisions separately. Since ignoring the interaction between long term and short term decisions can lead to sub-optimality (Shen and Qi, 2007; Shu et al., 2005; Ozsen, 2004), recently integrated supply chain design models have been developed. These models incorporate the strategic decisions of facility location and tactical decisions of inventory and transportation management. Most of the integrated models in the literature implicitly assume that supplier always performs perfectly. However, in practice a supplier is not always reliable and what is received from the supplier is not equal to what was ordered. In fact, there exist various factors such as random breakdown, raw material shortage, quality rejection, workforce slow down, maintenance duration, transportation damage, and natural disaster leading to unwanted partial yield (Erdem and Ozekici, 2002). Consequently, in many real cases, the amount of yield is random. This highlights the need for supply chain design models that account for uncertain yield.

Also, the majority of the integrated supply chain design models in the literature are based on the assumption that every customer's demands must be fulfilled. In real life world, tough, it may be more beneficial for the company to lose some potential customers, as the cost of maintaining these customers can be inevitably high. In other words, demand choice flexibility can result in cost saving that is significantly important especially for a profit-maximizing business (Shen, 2006). It implies that supply chain design models with demand choice flexibility deserve further attention.

With this background, the present paper discusses supply chain design problem, where yields and demands are uncertain and there is demand choice flexibility. Specifically, a three echelon supply chain comprised of single unreliable supplier, distribution centers (DCs) and customers, is considered. Following the common assumption in the literature, customers are assumed to have independent probabilistic demands with Poisson distribution such that the variance of demand is equal to the mean (Daskin et al., 2002; Shen et al., 2003; Ozsen et al., 2008). The supplier ships one type of product to customers in order to satisfy their demands. It should be noted that there is no requirement for serving all the costumers and the company is flexible in choosing which customers to serve. DCs function as the direct intermediary between the supplier and customers for shipment of the product. That is, DCs combine the orders from different customers and then order to the supplier. A key problem is that the supplier is not always reliable and the quantity of yield received by each DC may be less than what was ordered. In other words, the amount of yield at each DC is not deterministic.

In order to formulate the problem, an integrated supply chain design model is presented. The proposed model simultaneously determines: 1) where DCs are located; 2) which customers are served; 3) which DCs are assigned to which customers; 4) how much and how often to order at each DC. The objective is to minimize total costs including costs of location, inventory (consisting of working and safety inventory at DCs), shipment, and lost sales. In order to solve the model, a solution approach based on genetic algorithm is developed.

The remainder of the paper is organized as follows. Section 2 discusses some relevant models in the literature. In section 3 the integrated supply chain design model for the problem is proposed. Section 4 develops a solution approach to solve the model. In section 5 the related computational results for testing the effectiveness of the developed solution approach are provided. Finally, section 6 concludes the paper along with directions for future research.

2. Literature review

As this paper investigates the design of a supply chain with random yield, first the literature on integrated supply chain design is reviewed briefly. The reader is referred to Shen (2006) for a thorough review of the integrated supply chain design models. The research by Baumol and Wolfe (1985) is among the earliest works that incorporate inventory costs into location models. They discuss that inventory costs should be considered in the location model with a square root term. After Baumol and Wolfe's work, a number of joint location-inventory models have appeared in the literature. However, in most of these models nonlinear inventory costs either are overlooked, or approximated with linear functions (Ozsen et al., 2008).

In the recent years researchers have focused on the integrated models in which location and nonlinear inventory costs are included in the same model (Shen and Qi, 2007). For instance, Erlebacher and Meller (2000) provide a joint location inventory model with complicated nonlinear objective function. They applied a continuous approximation along with some heuristics techniques to solve the model. Daskin et al. (2002), Shen et al. (2003) and Shen (2000) introduce a location model with risk pooling (LMRP) that incorporates inventory decisions into the location model. LMRP minimizes the sum of fixed facility location costs, linear shipment costs and nonlinear inventory costs. Shen et al. (2003) and Shen (2000) use column generation, while Daskin et al. (2002) present Lagrangian relaxation to solve the LMRP. Another efficient approach to solve the LMRP is presented by Shu et al. (2005).

Shen and Daskin (2005) extend the LMRP to include a customer service element and propose useful techniques for evaluation of cost/service trade-offs. Ozsen (2008) develops LMRP in the condition that each DC has limited capacity. Her capacitated model is noticeably harder to solve than LMRP. Shen and Qi (2007) study an integrated supply chain design model that contains location, inventory, and routing decisions; in fact, they add routing decisions to the LMRP framework. Snyder et al. (2007) propose stochastic version of LMRP (called SLMRP) that handles uncertainty by describing discrete scenarios. The goal of SLMRP is to minimize the expected system cost across all scenarios. The authors argue how to use SLMRP to solve multi-commodity and multi-period problems.

Similar integrated supply chain design models are developed by Shen (2006), Sourirajan et al. (2007; 2008). Sourirajan et al. (2007) study the two-stage supply chain with a production facility where the replenishment lead time at the DCs depends on the volume of flow through the DC. They formulate the relationship between the flows in the network, lead times, and safety stock levels and develop a Lagrangian heuristic to obtain near-optimal solutions for the proposed model. Sourirajan et al. (2008) extend the problem to incorporate arbitrary demand variance at the retailers. They suggest genetic algorithm to solve the model and imply that the genetic algorithm outperforms the Lagrangian heuristic developed in the earlier work in some respects. None of these integrated supply chain design models consider random yield at DCs.

Another issue considered in this paper is random yield which has been discussed several times in the literature (Noori and Keller, 1986; Ehrhardt and Taube, 1987; Gerchak et al., 1988; Erdem and Ozekici, 2002; Qi and Shen, 2007; He and Zhang, 2008; Maddah et al., 2009). Most of these paper use newsboy problem to formulate the inventory problem (Noori and Keller, 1986; Ehrhardt and Taube, 1987; Gerchak et al., 1988; Qi and Shen, 2007). For instance, using newsboy problem, Qi and Shen (2007) provide a profit-maximizing model when the price of the product at each retailer is given and the yield is not deterministic. Parlar and Berkin (1991) and Gurler and Parlar (1997) study the problem where supply is presented only during an interval of random length. Henig and Gerchak (1996), also, examine the inventory policies when the amount of yield is stochastic. Yano and Lee (1995) and Tang (2006) survey the inventory models with random yield and provide general reviews.

The present paper differs from the earlier works in some main directions. First, unlike the most of supply chain design models in the literature, the proposed model of this study takes account of random yields at DCs. Moreover, the presented model dismisses the common restrictive assumption in the literature that demands of all customers must be satisfied necessarily. In fact, the model provides a simple but effective technique for determining the profitable customers. Finally, unlike the most of supply chain design models in the literature, the presented model consider limited capacities at DCs.

3. Model formulation

This section formulates a model for the problem explained in section 1. The objective of the model is to minimize the expected total cost including: 1) the fixed cost to locate DCs, 2) the working inventory cost at the located DCs (containing order costs, shipment costs from supplier to DCs, and holding costs), 3) safety stock cost at the located DCs, 4) shipment cost from located DCs to customers, and 5) the lost sales cost of not serving some customers. To develop the proposed model, following notations are used throughout the paper. Additional notations will be given out when required.

I : set of customers indexed by i;
J : set of candidate DC locations indexed by j;
D_i : mean of demand at customer i, for each $i \in I$;
f_j : fixed cost of locating a DC at j, for each $j \in J$;
F_j : fixed cost of placing an order at j, for each $j \in J$;
g_j : fixed cost per shipment from the supplier to DC at j, for each $j \in J$;
A_j : per-unit shipment cost from the supplier to DC at j, for each $j \in J$;
h : inventory holding cost per unit of product;
d_{ij} : per-unit cost to ship from distribution center j to customer i, for each $i \in I$ and for each $j \in J$;
C_j : capacity of DC at j, for each $j \in J$;
α : desired percentage of customers orders satisfied;
β: weight factor associated with the shipment cost;
θ : weight factor associated with the inventory cost;
z_α : standard normal deviate such that $P(z \le z_\alpha) = \alpha$;
L: lead time from supplier to DCs, in days;
P: number of DCs which should be located;
u_i : penalty cost of not serving customer i, per unit of demand (it can be interpreted as lost sales cost, or the cost of serving customer i by purchasing product from a competitor).

3.1. Working inventory cost

This subsection details the inventory policy the DCs follow and calculates the resulting expected working inventory cost. As stated in section 1, due to unreliable performance of the supplier, the quantity of yield received by each DC may be different from what was ordered. Specifically, it is assumed that the supplier has two modes for each DC, and each mode is associated with a constant partial yield. In other words, at each mode the supplier can provide only a fraction of the order placed by each DC. For convenience, assume that the supplier can provide a_j % of the order placed by distribution center j at the first mode, whereas it can satisfy b_j % of the order placed by distribution center j at the second mode. It should be noted that a_j and b_j are known given parameters. Also, durations of the first and second modes of the supplier for distribution center j follow exponential distribution with parameters λ_{1j} and λ_{2j}, respectively. Let Q_j denotes the reorder quantity of distribution center j, $Q_{[a_j]}$ indicates integer value of $Q_j \times a_j\,\%$, $Q_{[b_j]}$ represents integer value of $Q_j \times b_j\,\%$ and μ_j indicates the demand arrival rate to distribution center j in Poisson process. Then, regarding the memoryless property of the exponential distribution, the inventory transition related to distribution center j can be modeled as a birth-death process as Figure 1. In Figure 1, inventory quantities are considered as states of the birth-death process (Ross, 2007). By equating the rate at which the process leaves a state with the rate at which it enters that state, following equations are gained (Wu, 2008):

$$\pi(1) = \pi(2) = \ldots = \pi(Q_{[a_j]}) = \frac{1}{p_{1j} Q_{[a_j]} + p_{2j} Q_{[b_j]}} \quad (1)$$

$$\pi(Q_{[a_j]}+1) = \pi(Q_{[a_j]}+2) = \ldots = \pi(Q_{[b_j]}) = \frac{p_{2j}}{p_{1j} Q_{[a_j]} + p_{2j} Q_{[b_j]}} \quad (2)$$

where $\pi(k)$ denotes the limiting probability of state k (for k = 1 to $Q_{[b_j]}$) in Figure 1. In addition, $p_{1j} = \dfrac{\lambda_{1j}}{\lambda_{1j} + \lambda_{2j}}$ and $p_{2j} = 1 - p_{1j}$. At this stage, the annual working inventory cost at distribution center j can be obtained by:

$$F_j N_j + \beta(g_j N_j + A_j \mu_j) + \theta h \sum_{k=1}^{Q_{[b_j]}} k\,\pi(k) \quad (3)$$

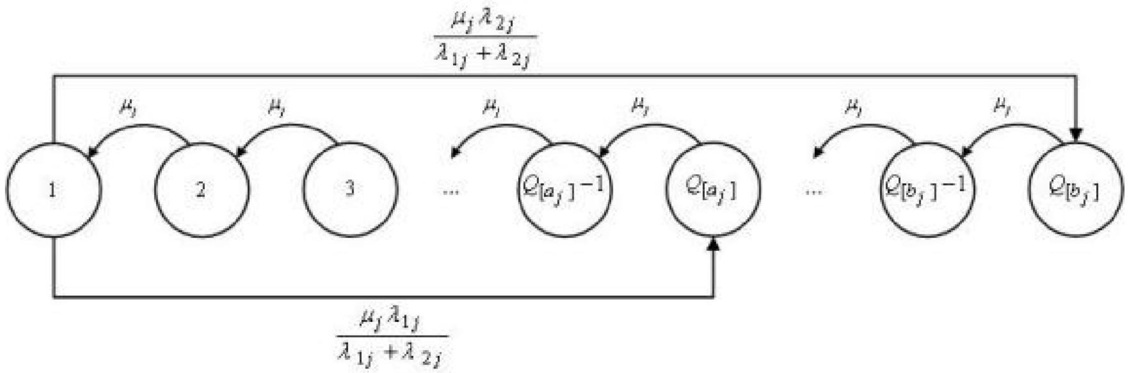

Figure 1. Inventory transition diagram related to distribution center *j*

where N_j denotes the number of orders and is equal to $\dfrac{(\lambda_{1j}+\lambda_{2j})\mu_j}{\lambda_{1j}Q_{[a_j]}+\lambda_{2j}Q_{[b_j]}}$.

The first term of equation (3) is the annual fixed cost of placing orders. The second term indicates the annual cost of shipping orders, assuming the shipment cost from the supplier to distribution center *j* has a fixed cost g_j and volume dependent cost A_j. The last term represents the cost of holding average of $\sum_{k=1}^{Q_{[b_j]}} k\pi(k)$ units of inventory.

Substituting limiting probabilities in (1) and (2) into (3), the annual working inventory cost at distribution center *j* will be:

$$\frac{100(F_j+\beta g_j)\mu_j(\lambda_{1j}+\lambda_{2j})}{a_j\lambda_{1j}Q_j+b_j\lambda_{2j}Q_j}+\beta A_j\mu_j +\frac{\theta h a_j(a_jQ_j+100)}{200(a_jp_{1j}+b_jp_{2j})} +\frac{hp_{2j}(b_jQ_j+a_jQ_j+100)(b_j-a_j)}{200(a_jp_{1j}+b_jp_{2j})} \quad (4)$$

To determine the optimal reorder quantity, we take derivative of (4) respect to Q_j and set the derivative to zero. By this way, the optimal value will be:

$$Q_j=100\sqrt{\frac{2(F_j+\beta g_j)(\lambda_{1j}+\lambda_{2j})}{\theta h(a_j{}^2\lambda_{1j}+b_j{}^2\lambda_{2j})}} \quad (5)$$

Plugging (5) into (4), an annual working inventory cost at distribution center *j* can be calculated as following:

$$\frac{1}{a_j\lambda_{1j}+b_j\lambda_{2j}} \times\sqrt{2(F_j+\beta g_j)\mu_j\theta h(a_j{}^2\lambda_{1j}+b_j{}^2\lambda_{2j})(\lambda_{1j}+\lambda_{2j})} +\beta A_j\mu_j \quad (6)$$

3.2. Shipment cost

Transporting the product from each distribution center *j* to each customer *i* has linear shipment cost. Let S_j be the set of customers assigned to the distribution center *j*. Then, the total demand assigned to distribution center *j* is $\sum_{i\in S_j} D_i$, and the shipment cost from distribution center *j* to the costumers will be:

$$\beta\sum_{i\in S_j} d_{ij}D_i \quad (7)$$

3.3. Lost sales cost

As stated in section 1, in case that the cost of serving a customer is not profitable, the demand of the customer is not provided and the system incurs lost sales cost. This happens when the cost of assigning the customer to any of the DCs is more than u_i. To model the lost sales cost, it is expedient to define a dummy DC with index *u*. Assigning the customer *i* to this dummy DC represents not serving the customer i (Snyder and Daskin, 2005). Regarding distribution center u, we assume that it has the shipment cost $d_{ij} = u_i$ to customer $i \in I$ and there is no other cost. Accordingly, when customer $i \in I$ is assigned to distribution center u it means that customer i is not served and the cost u_i is incurred.

3.4. Safety stock cost

Each DC retains a certain amount of safety stocks to deal with possible stockouts during replenishment lead time. Montgomery et al. (1998) show that a Poisson process with sufficiently large demand values can be approximated by Normal distribution appropriately. Thus, it is assumed that the demands at the customers are normally distributed when calculating the safety stock requirement. Since the customers' demands are assumed to be uncorrelated and normally distributed, the lead time demand variance at distribution center j can be gained by $\sqrt{\sum_{i \in S_j} LV_i^2}$, where V_i^2 denotes the variance of demand at customer i. Therefore, the needed safety stock to guarantee that the stockouts occur with a probability of α or less is $z_\alpha \sqrt{\sum_{i \in S_j} LV_i^2}$. Given the assumption that the variance and mean are equal, the corresponding holding cost for the safety stock at distribution center j is:

$$\theta h z_\alpha \sqrt{\sum_{i \in S_j} LD_i} \tag{8}$$

3.5. Integrated Model

In order to determine the locations of the DCs and assignments of the customers to DCs, two sets of decision variables are defined:

- $X_j = 1$, if j is selected as a DC location, and 0, otherwise, for each $j \in J$;
- $Y_{ij} = 1$, if customer i is assigned to a DC based at j, and 0 otherwise, for each $i \in I$ and $j \in J$.

Now the model can be formulated as follows:

$$\begin{aligned} \text{Min} & \sum_{j \in J} f_j X_j + \left(\beta \sum_{j \in J} \sum_{i \in I} d_{ij} D_i Y_{ij} \right) \\ & + \left(\begin{aligned} & \sum_{j \in J} \sqrt{2(F_j + \beta g_j)\theta h (a_j^2 \lambda_{1j} + b_j^2 \lambda_{2j})(\lambda_{1j} + \lambda_{2j}) \sum_{i \in I} D_i Y_{ij}} \\ & \times \frac{1}{a_j \lambda_{1j} + b_j \lambda_{2j}} + \beta \sum_{j \in J} A_j \sum_{i \in I} D_i Y_{ij} \end{aligned} \right) \\ & + \theta h z_\alpha \sum_{j \in J} \sqrt{\sum_{i \in I} L D_i Y_{ij}} \end{aligned} \tag{9}$$

subject to:

$$\sum_{j \in J} Y_{ij} = 1 \qquad \forall i \in I \tag{10}$$

$$\sum_{i \in I} D_i Y_{ij} \le C_j X_j \qquad \forall j \in J \tag{11}$$

$$X_u = 1 \tag{12}$$

$$\sum_{j \in J} X_j = P + 1 \tag{13}$$

$$X_j \in \{0,1\} \qquad \forall j \in J \tag{14}$$

$$Y_{ij} \in \{0,1\} \quad \forall i \in I,\ \forall j \in J \tag{15}$$

The objective function (9) is composed of four components. The first component represents the fixed cost of locating DCs. Considering equation (7), the second part indicates the expected shipment cost from the DCs to customers. Note that we added dummy distribution center u to the set J to take lost sales cost into account in the model. Also, $\sum_{i \in I} D_i Y_{ij}$ indicates the total annual demand assigned to the distribution center j. With regards to equation (6), it is easy to find that the third component represents the working inventory cost where $\mu_j = \sum_{i \in I} D_i Y_{ij}$. Finally, the fourth part indicates the safety stock cost and can be obtained by considering equation (8).

Constraints (10) stipulate that each customer is assigned to a DC. Recall that assigning a customer to dummy distribution center u is equivalent to not serving the customer. Constraints (11) state that the mean demand flow through a DC should be less than the capacity of that DC. Constraint (12) requires the dummy distribution center u to be located. Constraint (13) assures that the number of located DCs is exactly $P + 1$ (this means that P distribution centers must be located in addition to dummy distribution center u). Constraints (14) and (15) are binary constraints.

Objective function (9) can easily be reorganized as follows:

$$\text{Min} \sum_{j\in J} f_j X_j + \left(\beta \sum_{j\in J}\sum_{i\in I}(d_{ij}+A_j)D_i Y_{ij}\right)$$
$$+\left(\begin{array}{l}\sum_{j\in J}\sqrt{2(F_j+\beta g_j)\theta h(a_j{}^2\lambda_{1j}+b_j{}^2\lambda_{2j})(\lambda_{1j}+\lambda_{2j})\sum_{i\in I} D_i Y_{ij}} \\ \times \dfrac{1}{a_j\lambda_{1j}+b_j\lambda_{2j}}\end{array}\right)$$
$$+\theta h z_{\alpha}\sum_{j\in J}\sqrt{\sum_{i\in I} L D_i Y_{ij}}$$
$$=\sum_{j\in J}\left\{f_j X_j + \sum_{i\in I}\tilde{d}_{ij} Y_{ij} + k_j\sqrt{\sum_{i\in I} D_i Y_{ij}}\right\} \qquad (16)$$

where:

$$\tilde{d}_{ij} = \beta(d_{ij}+A_j)D_i$$
$$k_j = \sqrt{2(F_j+\beta g_j)\theta h(a_j{}^2\lambda_{1j}+b_j{}^2\lambda_{2j})(\lambda_{1j}+\lambda_{2j})} \times \frac{1}{a_j\lambda_{1j}+b_j\lambda_{2j}} + \theta h z_{\alpha}\sqrt{L}$$

4. Solution approach

Meta-heuristic algorithms have been very successful in solving complex mathematical models (Khalilzadeh et al., 2011). In order to solve the model formulated in section 3, a solution approach based on genetic algorithm (GA) is developed. GA is a stochastic search and heuristic optimization technique based on the mechanism of natural genetics which has been successfully applied to various complex problems. It starts with an initial set of random solution called population. Each solution in the population is called chromosome and each component of chromosome is designated by gene. The chromosomes evolve through successive iterations, called generations. During each generation, the chromosomes are evaluated, using some measures of fitness. To create next generation, new chromosomes (called offspring) are formed by crossover or mutation operators. Crossover operator combines two chromosomes from current generation, while mutation operator modifies a chromosome to form offspring. A new generation is created by (a) selecting some of current chromosomes (called parents) and offspring based on the fitness values, (b) rejecting others so as to keep the population size constant. Fitter chromosomes have higher probabilities of being selected. After several generations, the algorithms converge to the best chromosome, which may represent the optimum or suboptimal solution to the problem (Gen and Cheng, 1996). For more details of GA and its application in location problems refer to Sourirajan et al. [(2008), Goldberg (1989) and Jaramillo et al. (2002). In the following subsections, the developed GA for the problem is outlined.

4. 1. Chromosome representation

In this GA-based approach, each chromosome is indicated as a single dimensional array. If m is the number of candidate DCs, each chromosome C can be demonstrated by:

$$C=(X_j, Y_i)=(X_1, X_2 \ldots X_m, X_{m+1}, Y_1, Y_2 \ldots Y_m).$$

Where X_j correspond to the location genes and Y_i correspond to the assignment genes. These genes determine where the DCs are located and how the customers are assigned to the located DCs, respectively. More precisely, if $X_j = 1$, it means that candidate site j is selected as a DC location, while if $X_j = 0$, candidate location j is not chosen as a DC site. The gene X_{m+1} corresponds to dummy distribution center u; thus, it always takes the value 1. Also, $Y_i = j$ represents that customer i is assigned to distribution center j. If customer i is assigned to a dummy distribution center u, the corresponding assignment gene takes the value of m+1; in other words, $Y_i = m+1$.

4.2. Generating the first population

Half the chromosomes of the first population are generated from the feasible region randomly. The next half chromosomes are set identical to the obtained lower bound solution. Note that some customers may be allocated to more than one DC in the lower bound solution. In this case, the chromosome is modified to a feasible one, by assigning such customers to the nearest located DCs.

4.3. Chromosomes fitness

The rank-based evaluation function is defined as the objective function (16) for the chromosomes. In fact, we calculate the objective function (16) for each of the chromosomes. Obviously, the chromosome which results in less value of objective function (16) has the better rank.

4.4. Crossover operator

Crossover operator generates offspring by merging parent chromosomes. In order to determine which of chromosomes C_k, k=1, 2 … *pop-size* are selected as parents for crossover operation, the following procedure is repeated from k=1 to *pop-size*: generating a random number r from the interval [0, 1], the chromosome C_k will be selected as a parent provided that $r < P_C$, where the parameter P_C is the probability of crossover. Then randomly we group the selected parents C_1', C_2', C_3', … to the pairs (C_1', C_2'), (C_3', C_4'),…. Without loss of generality let us explain the crossover operator on each pair by (C_1', C_2').

Crossover operator assigns each customer *i* in offspring chromosome either to the DC which is allocated to customer *i* in parent chromosome C_1', or to the DC which is assigned to customer *i* in parent chromosome C_2'. This occurs randomly and with probability of 0.5. The resulted offspring may be infeasible. If a customer is allocated to an unselected candidate DC site, this infeasibility is removed by locating DC in that candidate location. If the number of located DCs exceeds $P+1$, the number of selected DCs is reduced to $P+1$ by closing some DCs randomly. The customers which are allocated to these closed DCs are allocated randomly to the opened DCs. By this way, the offspring is modified to a feasible chromosome.

4.5. Mutation operator

Mutation operator may modify chromosomes C_k, *k* = 1, 2 … *Pop-size* to form offspring chromosomes. In order to determine which of chromosomes C_k undergo mutation, the following practice is repeated from *k* = 1 to *Pop-size*: generating a random number *r* from the interval [0, 1], the chromosome C_k will be selected as a parent provided that $r < P_M$, where the parameter P_M is the probability of mutation. Each selected chromosome is modified by one of the two following types of mutation several times (each type of mutation is occurred with probability 0.5). The first type of mutation generates offspring by modifying the assignment genes of parent chromosome. Namely, in the first type of mutation two located DCs are selected randomly; let *s* and *t* denote them. Then, if any customer in parent chromosome is assigned to *s*, that customer will be assigned to *t* and if any customer is assigned to *t*, it will be allocated to *s*.

The second type of mutation modifies location genes of parent chromosome to form offspring. Indeed, the second type of mutation randomly selects a location in which no DC is located; let *t* denotes it. Next, a DC is selected randomly from the located DCs and is named *s*. This type of mutation closes distribution center *s* and instead of it locates a DC at *t*. Then, all the customers assigned to distribution center *s*, are allocated to distribution center *t*. Similar to crossover process, if the resulted offspring does not belong to feasible region, it is repaired to become a feasible chromosome.

5. Computational results

This section summarizes the computational experience with the solution approach outlined in the previous section. Also, the performance of the proposed GA is compared with simulated annealing (SA) algorithm developed by Azad and Davoudpour (2010). The solution methods were tested on the 49-node, 88-node, and 150-node data sets described in Daskin (1995). The 49-node data set indicates the capitals of the lower 48 United States plus Washington, DC; the 88-node data set represents the 50 largest cities in the 1990 U.S. census along with the 49-node data set, minus duplicates; and the 150-node data set includes the 150 largest cities in the 1990 U.S. census.

For all three data sets, the mean of demand was obtained by dividing the population data given in Daskin (1995) by 1000. Fixed costs of locating DCs (f_j) were gained by dividing the fixed cost in Daskin (1995) by 10 for the 49-node problem and by 100 for 88-node problem. For the 150-node problem, fixed locating costs were set to 10000 for all the candidate DC locations. We set the per-unit cost to ship from distribution center *j* to customer *i*, d_{ij}, to the great-circle distance between these locations. The fixed ordering F_j and shipping costs g_j were set to 10 and the variable shipping cost A_j was set to 5 for all DCs. The parameters b_j and z_α were set to 100 and 1.96 (corresponding to 97.5% service level), respectively. We set the holding cost *h* and the lead time *L* to 1. For all three data sets, the values of a_j, λ_{1j} and λ_{2j} were set randomly. The other parameters used for the solution method are given in Table 1.

Table 1. Parameters for solution approach

Parameter	Value
Population size of GA	100
Probability of crossover in GA	0.9
Probability of mutation in GA	0.01
The number of generations in GA	500

The developed Solution approach was coded in Visual Basic.Net and executed on Pentium 5 computer with 1.00 GB RAM and 2.00 GHz CPU. Tables 2-4 summarize the results for our computational study on 49-node, 88-node, and 150-node problems with different values for the parameters P, u_i, θ and β, respectively.

Table 2. Computational results for 49-node

	P	*u*	*Θ*	*β*	GA	SA	Percent of Improved Cost
1	5	10	0.01	0.0005	299636	309734	3.37
2	5	10	0.005	0.0001	299525	307133	2.54
3	5	100	0.01	0.0005	299654	308673	3.01
4	5	100	0.005	0.0001	299528	306776	2.42
5	10	10	0.01	0.0005	668011	700543	4.87
6	10	10	0.005	0.0001	667799	695512	4.15
7	10	100	0.01	0.0005	668028	698958	4.63
8	10	100	0.005	0.0001	667801	695315	4.12
9	15	10	0.01	0.0005	1055635	1107889	4.95
10	15	10	0.005	0.0001	1055321	1103760	4.59
11	15	100	0.01	0.0005	1055652	1105373	4.71
12	15	100	0.005	0.0001	667801	696250	4.26
13	20	10	0.01	0.0005	1468258	1545047	5.23
14	20	10	0.005	0.0001	1467843	1541969	5.05
15	20	100	0.01	0.0005	1468275	1544332	5.18
16	20	100	0.005	0.0001	1467846	1539771	4.9

Table 3. Computational results for 88-node

	P	*u*	*Θ*	*β*	GA	SA	Percent of Improved Cost
1	5	10	0.01	0.0005	241015	252632	4.82
2	5	10	0.005	0.0001	240996	250756	4.05
3	5	100	0.01	0.0005	241025	252859	4.91
4	5	100	0.005	0.0001	241000	251267	4.26
5	10	10	0.01	0.0005	547266	581689	6.29
6	10	10	0.005	0.0001	547242	577613	5.55
7	10	100	0.01	0.0005	547276	585749	7.03
8	10	100	0.005	0.0001	547245	572965	4.7
9	15	10	0.01	0.0005	876143	939576	7.24
10	15	10	0.005	0.0001	876113	937528	7.01
11	15	100	0.01	0.0005	876153	932139	6.39
12	15	100	0.005	0.0001	876116	929121	6.05
13	20	10	0.01	0.0005	1223769	1313961	7.37
14	20	10	0.005	0.0001	1223733	1303153	6.49
15	20	100	0.01	0.0005	1223780	1303448	6.51
16	20	100	0.005	0.0001	1223737	1296916	5.98

Table 4. Computational results for 150-node

	P	u	Θ	β	GA	SA	Percent of Improved Cost
1	5	10	0.01	0.0005	625030	661969	5.91
2	5	10	0.005	0.0001	624790	661590	5.89
3	5	100	0.01	0.0005	625065	664632	6.33
4	5	100	0.005	0.0001	624796	663533	6.2
5	10	10	0.01	0.0005	1250039	1346042	7.68
6	10	10	0.005	0.0001	1249579	1344422	7.59
7	10	100	0.01	0.0005	1250074	1351580	8.12
8	10	100	0.005	0.0001	1249584	1349176	7.97
9	20	10	0.01	0.0005	2500055	2768311	10.73
10	20	10	0.005	0.0001	2499159	2760321	10.45
11	20	100	0.01	0.0005	2500084	2772843	10.91
12	20	100	0.005	0.0001	2499164	2762576	10.54
13	30	10	0.01	0.0005	3750068	4241701	13.11
14	30	10	0.005	0.0001	3748739	4235325	12.98
15	30	100	0.01	0.0005	3750099	4269112	13.84
16	30	100	0.005	0.0001	3748744	4238330	13.06

In these tables, the columns marked P , u, θ and β give the parameters P , u_i, θ and β , respectively. The columns labeled GA represent the objective values obtained by GA, whereas the columns marked SA indicate the objective values obtained by SA. The last column in each table indicates the percentage difference between the objective values obtained by GA and SA. In other words, the last column represents the amount of improvement in the objective value when the proposed GA is applied instead of SA, and is obtained by $\frac{(\text{SA-GA})}{\text{GA}} \times 100$.

It follows from Tables 2-4 that the presented solution method based on GA outperforms SA in all the cases. This suggests that the proposed solution approach is effective to solve the model and we can trust it in practice

6. Conclusion

This paper has addressed a stochastic supply chain design problem where a supplier is unreliable. Due to unreliability of the supplier, the yields at DCs are not deterministic. The problem does not assume that all the customers' demands must be satisfied. A nonlinear integer programming model has been presented that minimizes the expected total costs including costs of location, inventory, transportation, and lost sales. The presented model simultaneously determines which customers are served, where DCs are located and how DCs are assigned to the customers. In order to solve the model, a heuristic approach based on genetic algorithm has been proposed. Computational results for different data sets have revealed that the proposed solution approach is quiet effective. In future, it would be interesting to formulate the problem when DCs are unreliable. Furthermore, the model can be extended to consider constraints on the maximum demand that can be provided by a supplier. Finally, incorporating routing decisions in the model makes it more helpful.

Corresponding Author:
Dr. Armin Jabbarzadeh
Department of Industrial Engineering
Iran University of Science and Technology, 16846113114, Tehran, Iran
E-mail: arminj@iust.ac.ir

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