Impact of Using Thinking Maps in Teaching Algebra on Cognitive Achievement among Second Year Preparatory Students

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Abstract: This study aimed to investigate the impact of using thinking maps in teaching "real numbers" unit on the cognitive achievement for second year preparatory students. The study sample comprised 110 students from the second year in Saqulta Preparatory School for Girls in Saqulta, divided into two groups; an experimental group that studied "real numbers" unit using thinking maps, while the other studied the same unit in the ordinary way. The study concluded that there are statistically significant differences between the means of marks of the experimental group students who studied "real numbers" unit using the thinking maps and those of the control group who studied the same unit in the ordinary way in the post-test of cognitive achievement at the levels of knowledge, comprehension and application in favor of the experimental group. In addition, using thinking maps in teaching "real numbers" unit to the experimental group students had a significant impact on the total cognitive achievement and on the levels of knowledge, comprehension and application, where the impact value reached 0.964.
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1. Introduction

The nature and content of mathematics and the way itdeals with problems make it hold a distinct position. The nature of deductive mathematics helps understand the world and train the mind to think Ainley (2012, p. 85). Algebra helps learner represent the world in which the person lives, analyze mathematical situations using algebraic symbols, developthe way of thinking by developing the ability to deduce, provide the learner with the techniques required to solve problems and apply them in other fields of mathematics (Blanton & Kaput, 2011, p. 8).

Furthermore, algebra is one of the important branches of mathematics as it covers aspects of learning essential to understand and interpret cognitive learning aspects included in other branches. It is an important area for developing the abilities of learners related to different thinking skills, such as problems that stimulate the learners' minds, challenge their mental abilities, provide them the opportunity to analyze and solve problems, and understand terminology, symbols and abstractions (Mason, 2005, p. 24).

Mathematics contains symbols and theoretical rules. Thus, it is difficult for many learners, even if the teacher utilizes logical teaching methods, to simplify the content. Accordingly, the problem should be linked with illustrations and visual tools that enable the learner to reach the conclusion(Alabd, 2012, p. 3).

Thinking maps are one of the third-generation visual tools used to organize information; generate and classify ideas, words, and elements related to a theme (Abed, 2014, p. 4).

Several educational studies as(Alper, Williams, & Hyerle, 2012; Lopez, 2011; Mapeala & Sopiah, 2016; Mashal & Kasirer, 2011; Sunseri, 2011; Woodford, 2015) have used thinking maps in teaching some subjects. They concluded that thinking maps:

1. Link the previous and new knowledge, link concepts and activities, and organize the content of the lesson which enables learners to analyze and categorize ideas and evaluate their learning outcomes.
2. Help increase the ability to imagine and to develop a system that processes and organizes information in an appropriate thinking map. As a result, learner's persistence and thinking are developed.
3. Enable to develop a work plan to accomplish activities. Such a plan is kept in mind, reflected on and assessed on completion. Utilizing visual language to form a mental image help reduce verbalization in content.
4. Develop some thinking skills, e.g. asking questions, taking notes, organizing data, and defining the relationship between cause and effect. They help

the learner deduce and formulate data and observations from these maps, and ask questions to understand the contents of the map and the interrelated relations.

Literature above shows the significance of thinking maps and their impact on some educational outcomes process in different subjects. Cognitive achievement is an important learning outcome which reveals the knowledge, information and skills that students acquired in the learning process.

Problem of the study:

Examining and analyzing marks of the students in the first semester test Hamidiya Preparatory School and Saqulta Preparatory School for Girls in 2013-2014, showed low level of cognitive achievement of second year preparatory students in algebra. The means of students' marks in the two schools were: 8.39/25 and 10.84/25. Hence, the problem of the study has been defined in the low level of cognitive achievement of second year preparatory students in algebra.

Objectives of the study:

The current study aimed to examine the impact of using thinking maps in teaching "real numbers" unit on the cognitive achievement among second year preparatory students.

Question of the study

What is the impact of using thinking maps in teaching "real numbers" unit on the cognitive achievement among second year preparatory students?

To answer this question, the following hypothesis was tested:

There are no statistically significant differences at the level of 0.05among the means of marks of the second year preparatory students who studied "real numbers" unit using thinking maps and those who studied the same unit in the ordinary way in the post-test of cognitive achievement as a whole on knowledge, comprehension and application levels.

Significance of the study:

1- This study provides a model of some lessons in algebra course prepared using thinking maps to the teachers of mathematics.

2- This is a significant study benefits planners and developers of mathematics courses of the preparatory stage in designing other modules of algebra courses using thinking maps.

Limitations of the study:

The study has been limited to:

1. A sample of second year students from Saqulta Preparatory School for Girls in Sohag, Egypt. The school was chosen from the schools that randomly distribute students to the classes and there are no distinct classes.

2. "Real numbers" unit of algebra course in the first semester of 2016/2017, after being reformulated using thinking maps. It was chosen according to the opinions of some supervisors of mathematics.

3. Cognitive achievement on knowledge, comprehension and application levels.

4. Six forms of thinking maps: circle map, bubble map, flow map, bridge map, tree map, analysis map because they are appropriate to the content of the unit in question.

Variables of the study:

Independent variable: Teaching algebra using thinking maps.

Dependent variable: Cognitive achievement among second year preparatory students.

2. Materials and tools of the study:

Materials of the study:

(1) The student's book after reformulating the content of "real numbers" unit using thinking maps.

(2) Teacher Guide for teaching "real numbers" unit using thinking maps.

Tool of the Study: Cognitive achievement test.

Definition of terms

Thinking maps:

They are visual forms associated with basic thinking skills used by second year preparatory students as a set of visual tools in studying algebra to help them organize information, concepts and experiences; and to create relationships and links among them.

Cognitive Achievement:

It is procedurally defined as what the second-preparatory students acquired of the cognitive learning aspects included in "real numbers" unit of algebra course. It is measured by the marks obtained by the students of the sample when applying the test of achievement prepared for this purpose.

Thinking maps and teaching mathematics

Thinking maps are typical visual tools for the integration of previous and subsequent lessons. They are a new method to organize information so that the learner can retrieve, interpret and analyze it. The main purpose of using thinking maps is to simplify information and help the learner remember, organize, process and apply it to new situations (Hyerle, 2004, p. 2).

Schlesinger (2007) described thinking maps as effective and highly efficient thinking tools that represent visual content, and creative models of content information which help achieve a learner's deep understanding and acceptance of it.

According to Alabd (2012), thinking maps comprise eight visual tools that reflect eight basic thinking processes to enhance learner's abilities, generate and evaluate ideas, collect and arrange information, and help solve problems. They are flexible forms that allow the learner to choose and expand the appropriate map to allow complete the mission and reach the conclusion.

I. Forms of thinking maps

Thinking maps comprise eight forms of visual schematic maps, namely the maps of the circle, bubble, double bubble, tree, analysis, flow chart, multiple flow, and bridge (Holzman, 2004, pp. 1- 4; Hyerle, 2000, pp. 104- 106). They are as follows:

A- Circle map:

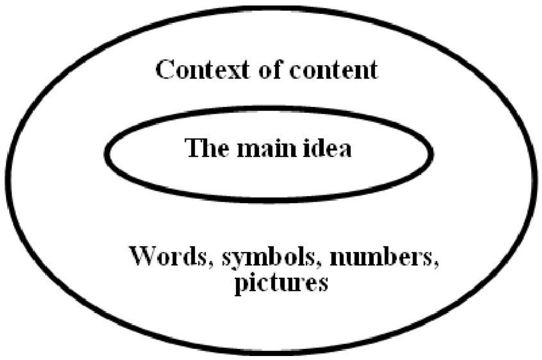

Figure 1. Circle map

The circle map consists of two circles with the same center and a different diameter. In the center of the first circle, there are the ideas, names, numbers, pictures, words, symbols and any idea that is meant to be defined. In the outer circle, any item relevant to the main subject, whether in the form of writing or drawing.

Hyerle (2000) explained the schematic diagram of the circle map as in figure1.

Jabir (2013) argued that the circle map helps clarify abstract concepts, without limiting the number of elements, encouraging learners to brainstorm.

B- Bubble map

It consists of a central circle surrounded by a number of external circles. While the central circle includes the concept, word, or thing whose properties are to be determined, the external circles contains the most important characteristics and qualities of this object.

Hyerle (2000, p. 104) illustrated the diagram of the bubble map in figure 2.

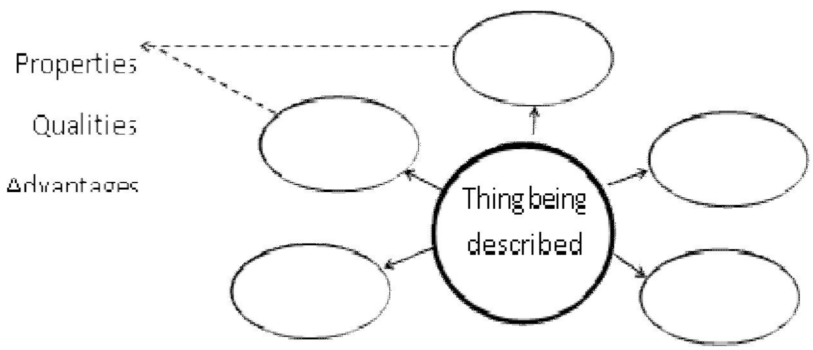

Figure 2. Bubble map

Jabir (2013, p. 73) indicated that the bubble map helps comprehend abstract concepts by recognizing the concept and its characteristics without being limited to a certain number of characteristics.

C- Double bubble map

It consists of two adjacent central circles. Each circle comprises an aspect of the comparison. In between, a number of circles contain the similar characteristics of the two, and on both sides of the two central circles their differences are written.

Hyerle (2000, p. 104) depicted the schematic diagram of the double bubble map in figure 3.

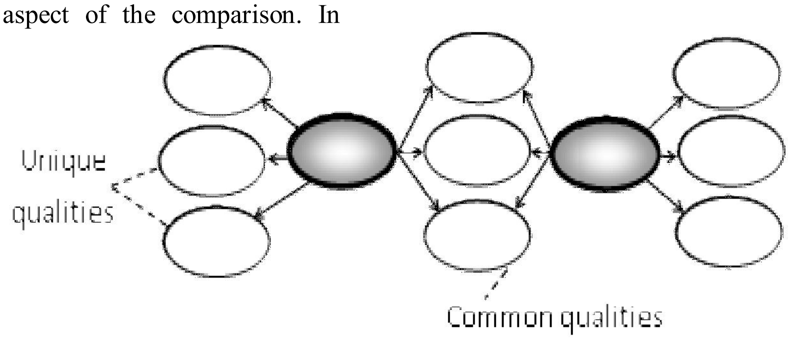

Figure 3. Double- bubble map

He states that the double bubble map is used to organize the comparison process, to clarify the differences between two topics, and to identify the most important information in the comparison process.

D- Tree map

It is a main branch in which the main idea is written and the sub-ideas are written below with specific details of each branch. Further branches may be drawn.

Hyerle (2000, p. 105) depicted the schematic diagram of the double bubble map in figure 4.

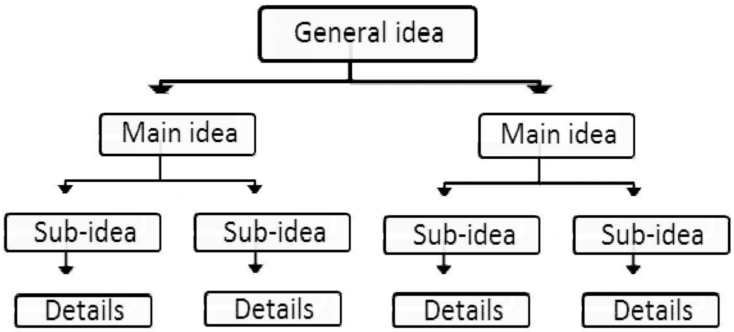

Figure 4. Tree map

Musa (2011, p. 43) indicated that the tree map shows the correlation of the aspects of mathematical knowledge provided to learners and its full comprehension.

E- Brace map

It consists of two parts. While the object or subject to be studied is written on the right, the main parts are written on the left. Sub-links representing the minor components of these parts are drawn. This continues until the analysis of this object or subject is completed. This map serves as an anatomy of objects or subjects on paper.

Hyerle (2000, p. 105) has drawn the schematic of the analysis / brace map in figure 5.

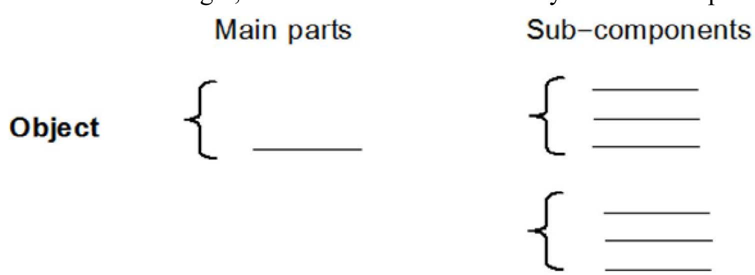

Figure 5. Brace map

Hyerle (2004, p. 6) argues that this map helps the learner understand the relationship between the whole and the part and analyze the subjects into elements.

F- Flow map:

It is a set of consecutive rectangles. In the first rectangle, the subject or event is written. Then, the successive events are logically and systematically shown in the others. They all sequentially express the event from beginning to end. Results, numbers or symbols are written in minor and smaller rectangles.

Hyerle (2000, p. 105) shown the schematic diagram of the flow chart in figure 6.

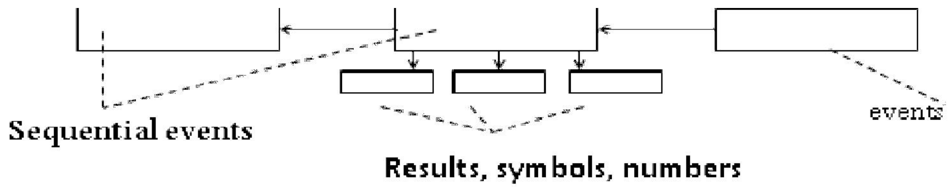

Figure 6. Flow Map

Hyerle (2004, p. 6) argues that the flow chart is used to:

- Determine the precedence of events and steps, and regularly retrieve them.
- Show sequence and schedules.
- Arrange the required tasks according to importance.
- Solve mathematical problems.

G- Multi- Flow map

It consists of a main rectangular in the middle in which in the event or subject is written and it is surrounded by a number of rectangles on both sides left and right. While the causes of the event are placed on the right, its effects are placed on the left.

Hyerle (2000, p. 106) showed the schematic diagram of the multiple flow chart in figure 7.

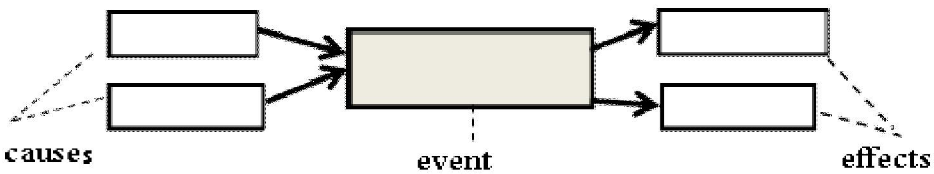

Figure 7. Multi- Flow Map

Jabir (2013, p. 73) noted that this map contributes to data analysis and understanding the relationship between data and what is required.

H- Bridge map

It consists of two sides separated by a bridge. While the objects or information to be learned are placed on the left, the former knowledge of the learner is placed on the right. It should be noted that the same relation is provided on both sides. Additionally, the bridge may extend using more relevant factors.

Hyerle (2000, p. 106) showed the schematic diagram of the bridge map in figure 8.

Figure 8. Bridge map

Hyerle (2004, p. 6) states that the bridge map is used to illustrate the relationship between abstract and concrete item and to understand symmetries and similarities.

II. The importance of thinking maps in mathematics

Holzman (2004, p. 2); Long and Carlson (2011, p. 5); Spiegel (2003, p. 49) agree that thinking maps can be used in any course content and at all levels. Spiegel (2003, p. 50) argues that thinking maps are not only limited to organizing information, but they are a way of solving problems. They give both the teacher and learner an opportunity to interact positively and think about what they do.

Some previous studies including (Abu Al – Qasim, 2010; Alabd, 2012; Hickie, 2006; Holzman, 2004; Jabir, 2013; Musa, 2011; Shankland, 2010; Shehata, 2012) are concerned with the use of thinking maps in the teaching of mathematics.

They concluded that thinking maps:

1. Contribute to the formulation and organization of topics simply and smoothly, allowing the flow of information, retrieve the previous experiences of learners, and foresight of mathematical relations.
2. Help master mathematical concepts, deeply understand the content, and solve mathematical problems.
3. Help the learner have flexible thinking, discover relationships, predict new mathematical ideas, and reorganize the structure of the learner's mathematical knowledge and display it in another form that suits the learner's style.
4. Contribute to the organization and arrangement of mathematical knowledge to help students recall former experiences and recognize the relationship between them and the new knowledge.
5. Help translate the verbal formulations of the problem, and organize the steps of the solution in a logical and sequential manner.
6. Contribute to the analysis of data and understanding the relationship between data and what is required, and help the learner solve problems and exercises in different directions.

To conclude, thinking maps have eight basic visual forms, each which has its own simple scheme, as shown in table 1.

Table1. Thinking maps and their use and importance map in mathematics

ng	Use	Importance in mathematics
Circle map	Used to define. It is an effective tool to demonstrate the flexibility of learners' thinking and their ability to present ideas	- Definition of mathematical concepts - Evaluation of students' learning - Writing examples of a mathematical concept - Identification of prior knowledge - Presentation of new knowledge
Bubble map	Used to identify features or characteristics. It helps identify the concept and its characteristics.	- Defining the properties of mathematical concepts - Defining the conditions to be met - Defining the characteristics of a mathematical concept
Double bubble map	Used to compare. It helps highlight the similarities and differences between two topics or concepts.	- Defining the characteristics of each concept - Identifying similarities and differences - Defining the advantages of each concept - Comparing two mathematical concepts - Identifying the features and common characteristics between two concepts - Identify the similarities between two mathematical concepts - Defining the common conditions to be met
Tree map	Used for classification. It helps classify things and define the relationship between the main ideas and sub-details.	- Formation of an integrated vision of mathematical knowledge - Classifying the components of mathematical knowledge into key ideas - Organization of information - Classifying key ideas into sub-ideas - Categorizing sub-ideas into details
Brace map	Used to analyze the subject to be studied into elements and it helps to clarify the relationship between the whole and the part in the subjects.	- Analysis of the mathematical problem to data and required - Analysis of the number to its primary factors - Understanding the relationship between the whole and the part
Flow map	Used to show sequence of events and sequence steps. It helps arrange tasks to be easily remembered.	- Organizing steps to solve the problem - Arranging the stages of solving the problem - Arranging the required tasks - Determining the precedence of the steps and retrieving them systematically
Multi Flow map	Used to predict results in the light of causes or data. It helps identify the causes and effects of an event.	- Understanding the mathematical problem - Identification of causes or data - Results prediction - Interpretation of mathematical ideas
Bridge map	Used to translate ideas into other similar ones. It helps transform concepts and ideas from one form to another and understand their similarities.	- Translating verbal data into geometric shapes - Translating abstract concepts into concrete ones. - Translating a mathematical image into another equal one.

III. Qualities of thinking maps

Hyerle (2009, p. 123) showed the characteristics of thinking maps in figure 9.

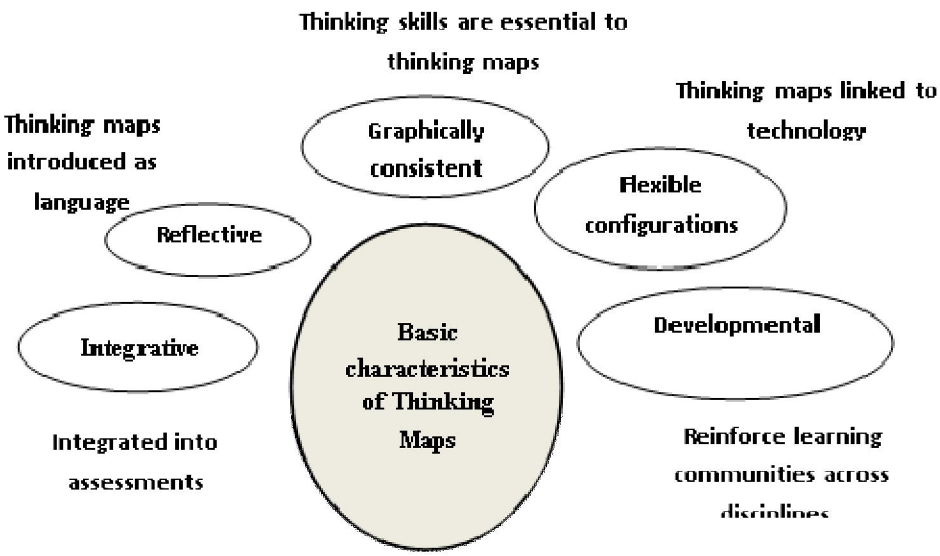

Figure 9. A bubble map of the characteristics of thinking maps

Hyerl and Curtis (2004, p. 106) report that the characteristics of thinking maps help the learner learn more effectively and efficiently. The objectives set can be achieved in less time while retaining the material in a larger way and they help provide learners with new ways to practice higher thinking levels.

Additionally, thinking maps have five characteristics that make them important in teaching and learning. Such characteristics make thinking maps integrative visual tools that can be used in different areas and they are consistent so that more than one map can be used to accomplish a task. They are developmental so they can be used in all educational stages because they can evolve according to the nature of each stage. They are also flexible to be expanded to achieve the desired goal. They are reflective to allow the learner to see thinking on paper.

III. The stages of using thinking maps

Hyerle (2009, p. 120) states that the learner passes through four stages when using thinking maps, as shown in figure 10.

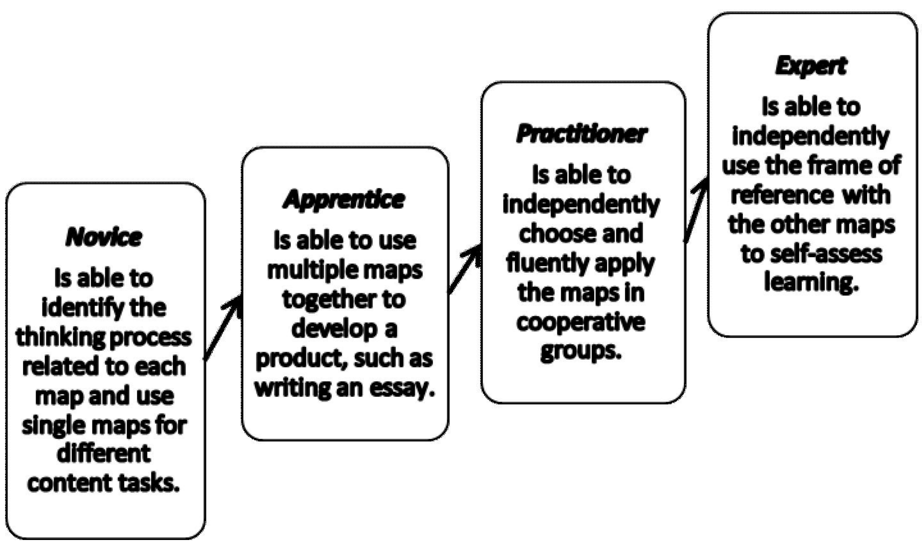

Figure 10. The stages of using thinking maps

Accordingly, the mapping of thinking can be defined as follows:

1. Identify main ideas.
2. Identify sub-ideas.
3. Determining the involved thinking skill.
4. Choose the appropriate thinking map based on the identified skill.
5. Start drawing the thinking map.
6. Add sub-ideas to the map.
7. Evaluate thinking map in terms of the appropriateness of the type of map for the defined skill and the content of the map.
8. Reorder ideas if needed.

Procedures of the study:

To achieve the objectives of the study, answer the questions and test the validity of its hypothesis, the following procedures have been followed:

1. Reviewing literature related to thinking maps in order to benefit from them in preparing the materials and the tools of the study.
2. Analyzing the content of “real numbers” unit of algebra course in the second preparatory year to define the concepts, generalizations and skills and estimate the validity and reliability of the analysis included in the unit.
3. Preparing the study materials, including:
   - Student's book after reformulating the content of the unit according to thinking maps.
   - Teacher guide of the same unit.
4. preparing cognitive achievement test in “real numbers” unit on the levels of knowledge, comprehension and application.
5. Submitting the study materials and tool to a group of reviewers to judge their validity and make the suggested modifications.
6. Pilot application of the materials to a sample of the preparatory stage students.
7. Pilot application of the tool to a sample of the preparatory stage students.
8. Controlling and statistically processing the tool to be applied to the main study group, calculating the coefficients of validity and reliability, and calculating the time required for the application of each test.
9. Selecting a sample of the study of the second year in Saqulta Preparatory School for Girls in Saqulta.
10. Dividing the sample into two groups; control and experimental.
11. Obtaining and statistically processing the marks of students in the final exam of algebra for the scholastic year 2015/2016 in order to know the equivalence of the two groups in cognitive achievement.
12. Teaching “real numbers” of algebra course using thinking maps to the experimental group and teaching the same unit to the control one in the ordinary way.
13. Post-test of the tool to the study groups.
14. Monitoring, statistically processing, analyzing, and interpreting results; answering the questions of the study; testing the validity of this hypothesis.

Experimental design of the study

To achieve the objective of the study, the researchers utilized the semi-experimental method based on two equal groups of students. While the experimental group studied "real numbers" unit using the independent variable, i.e. thinking maps, the other studied the same unit in the ordinary way.

Implementation

The experimental group was taught according to the teacher’s guide at the beginning of the first semester of 2016/2017. It took six weeks, two periods 4 lessonsper week for both groups. After excludingnine students for frequent absence during the study of the unit and pre-and post-test of the tool, the sample of the study comprised two groups; experimental of fifty five students and a control one offifty five students.

Their marks were obtained in the final test of 2015/2016 in algebra to ascertain the equivalence of the two groups and to statistically analyze them. Results indicated that the value of "T" calculated is less than that of "T" table at a significance level of 0.05, indicating that the difference between the means of students’ marks in the two groups is not statistically significant. Accordingly, the two groups are equivalent in achievement according to the test the previous year in algebra before applying the experiment.

The cognitive achievement test was applied periodically to the experimental and control groups, and the results were statistically treated.

3. Results

To answer the study question and test the hypothesis validity, “T” value was calculated for two independent samples using SPSS, as shown in table 2.

Table 2 shows that the value of T calculated is greater than the value of t-table at the level of 0.05 and a freedom degree of 108 at all sub-levels and the test as a whole. This indicates that there are statistically significant differences between the means of the experimental and control groups in the post-test of cognitive achievement at the levels of knowledge, comprehension and application favoring the experimental group.

Table 2 shows that the effect size of using thinking maps on the cognitive achievement of the experimental group was 0.964 indicating that thinking maps significantly the cognitive achievement of the female students because it is greater than 0.80 marginal value of the great effect size (Abdel - Hamid, 2012, p. 283).

Accordingly, the hypothesis was rejected and the alternative one is accepted that:

There were statistically significant differences between the mean marks of the experimental group that studied "real numbers" unit using thinking maps and the control group that studied the same unit in the ordinary way in the post-test of cognitive achievement in the test as a whole and the levels of knowledge, comprehension and application favoring the experimental group.

Table2. Significance of the differences between the means of the two groups in the post- test of cognitive achievement as a whole and at all sub-levels

Level	Group	Number	Means	SD	T Calculated	η2	ES
Remember	Experimental	55	7.054	2.921	4.280	0.06	0.840 Great
	Control	55	4.545	3.219			
Understand	Experimental	55	13.127	5.725	4.416	0.09	0.850 Great
	Control	55	8.018	6.390			
Apply	Experimental	55	10.927	5.521	5.477	0.22	1.062 Great
	Control	55	5.400	5.054			
The whole test	Experimental	55	31.109	13.395	5.010	0.19	0.964 Great
	Control	55	17.963	14.115			

4. Discussion

Figure11 compares the marks of the control and experimental groups in the post-test of cognitive achievement.

Figure 11 shows the increase in the number of students in the control group compared to their number in the experimental one at: 4.5, 13.5 and 22.5 grades. The number of students in the experimental group increased compared to the control one at: 31.5, 40.5 and 49.5 grades, indicating the obvious impact of using thinking maps on the cognitive achievement of the experimental group.

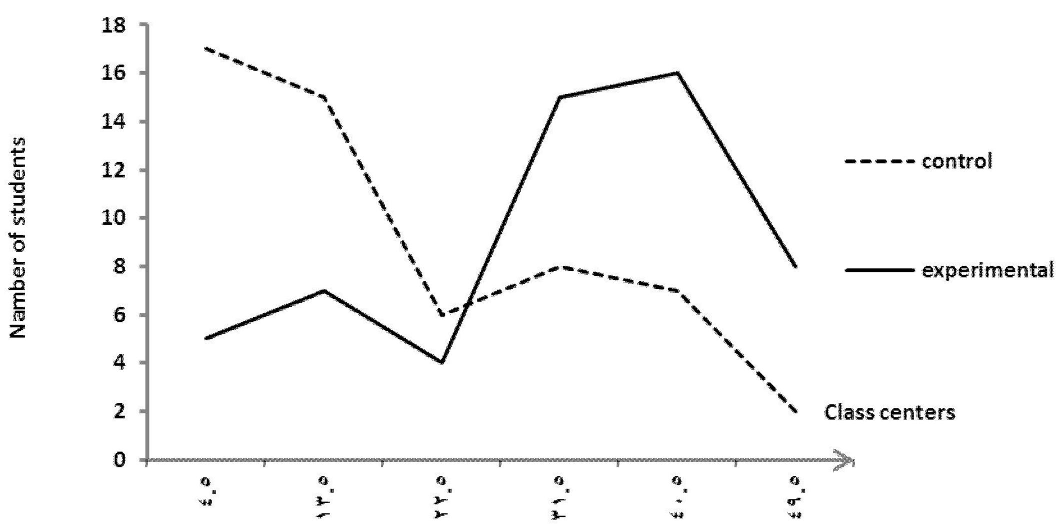

Figure 11. Comparison of students' marks in the control and experimental groups in the post-test of cognitive achievement

This agrees with the results of studies that concluded that thinking maps are effective in the development of cognitive achievement in teaching mathematics such as: Abu Al – Qasim (2010); Musa (2011); Shehata (2012). It also agrees with other studies that utilized thinking maps to develop achievement in other subjects such as: Hindman (2000); Holzman (2004); Hudson (2013); Kessler, Zuercher, and Wong (2013); Kim and Park (2016); Long and Carlson (2011).

These results may be concluded that:

1. Thinking maps, as visual aids, help form a mental image of mathematical concepts and information through the design and drawing of maps, causing the proper organization of ideas and the recall of concepts and generalizations appropriate to the educational situation.
2. The diversity of thinking maps utilized. The study adopted six types of thinking maps, which helped the students accurately determine the mathematical knowledge.
3. The positive role of female students in carrying out the activities in question has a significant impact on the consolidation of knowledge. Accordingly, their cognitive achievement increases, making learning meaningful and increasing understanding.
4. Thinking maps help students logically organize the steps of the solution and retrieve information with analysis and tabulation in order to be used in the solution.
5. The positive, effective and active role of female students in the construction and design of thinking maps increase their ability to transfer information into knowledge to be applied to new situations.
6. Evaluation at the end of each lesson provided an opportunity for female students to deal with questions of various answers which greatly help develop their level of application.

After using thinking maps, some students of the experimental group thought that they:

- Make study easier and simpler and they help better understand the rules.
- Help clarify the steps of the solution in an orderly and sequential manner, providing them with simple and clear ways to solve the problems.
- Help answer these questions while solving mathematical problems: What did I do? What will I do? Is what I did right?
- Help them write the solution in a more structured way and help think about solving mathematical problems logically and sequentially.
- Motivate them to favor algebra because they provide them with clear and simplified visual forms to solve the problems.

Some students reported that the flow map greatly help recall the necessary steps of solving the mathematical problems. The bubble map facilitated the collection of data about a concept or idea. The bridge map helped explain the equal images of mathematical concepts and ideas.

Some students expressed their desire to use thinking maps in other subjects because they helped organize information and improved the understanding of the content.

In view of the results, we have to say that:

A. Using thinking maps can support the teaching and learning of mathematics in general and algebra in particular.

B. Interest in rephrasing the content of mathematics curricula using thinking maps according to the nature of the content not only the textual presentation of information.

C. Using thinking maps in teaching other branches of mathematics in different educational stages.

D. Preparing training programs for mathematics in - service teachers to train them on the construction and use of thinking maps.

E. Providing a learning environment for students that is characterized with freedom, participation and cooperation as it has an effective role in increasing learners' achievement.

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