Linear Interaction of Inhomogeneity in Electron Beam-Plasma System

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Abstract: The linear interaction of cold inhomogeneous electron beam (EB) with inhomogeneous magnetized warm plasma is investigated. The unperturbed density of the EB is considered to vary linearly over the length of the system such that: $N_{0b} = n_{0b}(1+\frac{x}{L})$, where L is the length scale of the variation ($L >> x$). The dispersion relation of the system is derived. In addition, the growth rate of the instability is calculated. The presence of external static magnetic field, warmness plasma electrons and inhomogeneous EB leads to a decrease of the growth rate in comparison with the case of magnetized cold plasma and homogeneous EB.

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1. Introduction

A great number of publications (Fainberg, 1961, Nezlin, 1977, Bogdankevich, 1971, Bobylev, 2004, El-Shorbagy, 2012, El-Shorbagy, 2009 and El-Sharif, 2009) and refs. Cited there are devoted to the theoretical study of beam-plasma interaction.

Treatments of the beam-plasma instability usually study the behaviour of the growth rate as a function of the parameters of the problem for one or two oscillation modes, which have the largest growth rate

(Ivanov, 1984, Popkov, 1984, Kondratenko, 1989, Karbushev, 1989 and Kachalov, 1989). This approach has permitted considerable insight into the important role of transverse transport of the oscillation energy which the group velocity out of the beam region in determining the behaviour of the beam-plasma interaction (Bilienkm, 1993 and El-Shorbagy, 2014).

In the present work, we study the linear interaction between the inhomogeneous EB and inhomogeneous warm plasma under the effect of the external static magnetic field directed along z-direction, ($\vec{H}_{ext.} = H_0\vec{e}_z$). The field equation that describes the system is differential equations of third order. To solve this equation to obtain the dispersion relation we consider the density and velocity of inhomogeneous EB is in such form:

$$N_{0b} = n_{0b}(1+\frac{x}{L}),\quad V_{0b} = v_{0b}(1+\frac{x}{L})$$

Where L is the length scale of the variation ($L >> x$). The growth rate of the instability is calculated. Waves are excited more strongly in this case compared for homogeneous electron beam and magnetized cold plasma.

2. Fundamental Waves

The initial linearized set of equations (the equation of motion and the continuity equation) describing the oscillations in 1-D, for a relativistic electron beam, which travels along the magnetic field, are:

$$\frac{\partial \vec{V}_b}{\partial t} + (\vec{V}_b\cdot\vec{\nabla})\vec{V}_b) = -\frac{e}{m}\vec{E};\quad \vec{V}_b = \vec{V}_{0b} + \vec{V}_{1b},$$
$$\vec{V}_{0b} = V_{0b}\,\vec{e}_z \qquad (1)$$

$$\frac{\partial N_b}{\partial t} + \vec{\nabla}\cdot(N_b\vec{V}_b) = 0;\quad N_b = n_{0b} + n_{1b} \qquad (2)$$

The initial linearized set of equations (the equation of motion and the continuity equation) describing the oscillations in 1-D, for inhomogeneous plasma electrons in the oscillating electric field and a static magnetic field $\vec{H}_{ext.}$ perpendicular to the plasma density gradient are given by: -

$$\frac{\partial \vec{V}_P}{\partial t} + (\vec{V}_P \cdot \vec{\nabla})\vec{V}_P = -\frac{e}{m}[\vec{E} + \frac{1}{C}(\vec{V}_P \times \vec{H}_{ext.})] - \frac{1}{n_P m}\nabla P \quad (3)$$

$$\frac{\partial N_P}{\partial t} + \vec{\nabla} \cdot (N_P \vec{V}_P) = 0;\ N_P = n_{0P} + n_{1P} \quad (4)$$

In equations (1-4), V_{0b}, n_{0b} are the unperturbed velocity and density of the beam while n_{0P}, n_{1P} are the unperturbed and perturbed density of the plasma, respectively. P is the plasma pressure. All other terms have their usual meaning. From (1)-(4), we can derive the following expressions for the perturbed densities: -

$$n_{1b} = \left. \begin{array}{l} -\exp(\frac{i\omega}{V_{0b}}x).\int \frac{e\gamma^2 n_{0b}}{mV_{0b}^2} E_x \\ \exp(\frac{-i\omega}{V_{0b}}x)dx - \exp(\frac{i\omega}{V_{0b}}x) \\ \int \frac{i\omega e\gamma^2 n_{0b}}{mV_{0b}^3} \int \exp(\frac{-i\omega}{V_{0b}}x)E_x dx \\ -\exp(\frac{i\omega}{V_{0b}}x)\int \frac{e\gamma^2}{mV_{0b}^2}\frac{dn_{0b}}{dx} \\ \int \exp(\frac{-i\omega}{V_{0b}}x)E_x dx \end{array} \right\} \quad (5)$$

$$n_{1P} = -\frac{\partial}{\partial x}\left[\frac{e}{m\omega\tilde{\omega}} n_{0P}(x)\, E(x)\right] - \frac{V_{T_e}^2}{\omega\tilde{\omega}} \frac{\partial^2}{\partial x^2}\left(n_{0P}(x)\right) \quad (6)$$

Where, $\tilde{\omega} = (\omega^2 - \omega_c^2)^{1/2}$, $\omega_c = \frac{eH_0}{mc}$ is the electron cyclotron frequency and $V_{T_e} = \sqrt{\frac{kT_e}{m}}$ is the electron thermal velocity.

Using Poisson's equation

$$\frac{dE}{dx} = -4\pi e(n_{1P} + n_{1b}) \quad (7)$$

Moreover, by substituting from (5) and (6) into (7) we have:

$$\left. \begin{array}{l} \frac{\partial^3}{\partial x^3}(\hat{\varepsilon}E_x) - \frac{2i\omega}{V_{0b}}\frac{\partial^2}{\partial x^2}(\hat{\varepsilon}E_x) \\ -\frac{\omega^2}{V_{0b}^2}\frac{\partial^2}{\partial x^2}(\hat{\varepsilon}E_x) = -\frac{\partial}{\partial x}(\frac{\omega_b^2}{V_{0b}^2}E_x) \\ -\frac{1}{V_{0b}^2}\frac{\partial}{\partial x}(\omega_b^2 E_x) \ -\exp(\frac{i\omega}{V_{0b}}x) \\ \frac{1}{V_{0b}^2}\frac{\partial^2}{\partial x^2}(\omega_b^2)\int \exp(\frac{i\omega}{V_{0b}}x)E_x dx \end{array} \right\} \quad (8)$$

Where; $\hat{\varepsilon} = \varepsilon + \frac{V_{T_e}^2}{\omega\tilde{\omega}}\frac{\partial^2}{\partial x^2}\left(\frac{\omega_P^2(x)}{\omega\tilde{\omega}}\right)$, $\varepsilon = 1 - \frac{\omega_P^2(x)}{\omega\tilde{\omega}}$, is the dielectric permeability of the plasma, $\omega_P^2(x) = \frac{4\pi e^2 n_{0P}(x)}{m}$ is the plasma Langmuir frequency and $\omega_b^2(x) = \frac{4\pi e^2 n_{0b}(x)}{m}$ is the frequency of the inhomogeneous EB.

When $N_b = const.$, $V_{0b} = const.$, and $V_{T_e} = 0$, i.e., the case of a homogeneous electron beam with magnetized cold plasma, equation (7) becomes:

$$\left(-i\omega + V_{0b}\frac{\partial}{\partial x}\right)^2 (\varepsilon E_x) + \omega_b^2 E_x = 0 \quad (9)$$

Equation (9) is in agreement with the work of (Amein, 1975 and Akiezer, 1975).

From (2) into (8) we have

$$\frac{\partial}{\partial x}\left(\begin{array}{l}\frac{\partial^2}{\partial x^2}(\hat{\varepsilon}_P^{(0)}+\delta\varepsilon_P)E_x-2i\chi_0(1-\frac{x}{L})\\ \frac{\partial}{\partial x}\left\{(\hat{\varepsilon}_P^{(0)}+\delta\varepsilon_P)E_x\right\}\\ +\left[\begin{array}{l}\chi_b^2(1-\frac{x}{L})-(\hat{\varepsilon}_P^{(0)}+\delta\varepsilon_P)\\ \chi_0^2(1-\frac{2x}{L})\end{array}\right]E_x\end{array}\right)$$

$$=-E_x\frac{\partial}{\partial x}\left\{\chi_b^2(1-\frac{2x}{L})\right\} \qquad (10)$$

where;

$$\hat{\varepsilon}_P^{(0)}=\varepsilon_P^{(0)}+\frac{V_{T_e}^2}{\omega\tilde{\omega}}\left[\frac{\omega_P^2(0)}{\omega\tilde{\omega}}\right],\ \varepsilon_P^{(0)}=1-\frac{\omega_P^2(0)}{\omega\tilde{\omega}},$$

$$\delta\varepsilon_P=-\frac{\omega_P^2}{\omega\tilde{\omega}}\frac{x}{L},\ \chi_0=\frac{\omega}{V_{0b}},$$

and $\chi_b^2=\frac{\omega_b^2(0)}{V_{0b}^2}(1-\frac{x}{L})$.

$\omega_P(0),\ \omega_b(0)$ is the first approximation for the plasma and beam respectively, and $\varepsilon_P^{(0)}$ is the first approximation of plasma electric permittivity.

If we neglect small terms $\frac{x}{L}$ or $\frac{x^2}{L^2}$ and takes $E_x\sim\exp(ikx)$, we have from (10), that:

$$1-\frac{\omega_P^2}{\omega\tilde{\omega}}+\frac{V_{T_e}^2}{\omega\tilde{\omega}}\left[\frac{\omega_P^2(0)}{\omega\tilde{\omega}}\right]-\frac{\omega_{Rb}^2}{(\omega-kV_{0p})^2}\left[1+\frac{2ik^2V_{0P}^2\omega_P^2}{kL\omega_{Rb}^2\omega\tilde{\omega}}\left\{1-\frac{\omega\tilde{\omega}\frac{n_b}{n_P}}{2k^2V_{0b}^2}\right\}\right]=0 \qquad (11)$$

Since $\omega=\omega_P+\gamma$ then from dispersion relation (11), we obtain the growth rate as

$$\gamma=\left\{\frac{\omega_{Rb}^2\omega_P}{2\left(1+\frac{\omega_c}{\omega_P}\right)}\left[\begin{array}{c}1+2i\frac{n_p}{n_b}\\ \frac{1}{kL}\left(1+\frac{ikLV_{T_e}^2}{2}\right)\end{array}\right]\right\}^{1/3} \qquad (12)$$

where;

$$kL<<1,\ \frac{n_b}{n_P}=10^{-2}-10^{-4}.$$

At $N_b=const.,\ V_{0b}=const.,$ and $V_{T_e}=0$, equation (12) is agreement with the equation in case of homogeneous beam-magnetized cold plasma.

In the case of inhomogeneous electron beam interaction with inhomogeneous magnetized cold plasma,

$$\gamma_{cold}=\left\{\frac{\omega_b^2\omega_p}{2(1+\frac{\omega_c}{\omega_p})}\left[1+2i\frac{n_p}{n_b}\frac{1}{kL}\right]\right\}^{1/3} \qquad (13)$$

By comparing relation (12) with (13), we can see that the effect of inhomogeneous warm plasma is appear in term $\frac{\omega_{Rb}^2\omega_p}{2(1+\frac{\omega_c}{\omega_p})}\cdot\left[2i\frac{n_p}{n_b}\frac{1}{kL}\left(1+\frac{ikLV_{T_e}^2}{2}\right)\right]$ i.e., the instability is decreasing in the case of inhomogeneous magnetized warm plasma as comparing with inhomogeneous magnetized cold plasma.

We conclude that the growth rate of the instability decreased more in inhomogeneous EB-magnetized warm plasma than in inhomogeneous electron beam-magnetized cold plasma.

3. Conclusions

We study the linear interaction between the inhomogeneous EB and warm plasma under the effect of the external static magnetic field. The field equation which describes the system is differential equations of third order. The growth rate of the instability is calculated. Waves are excited more strongly in this case compared for homogeneous electron beam and magnetized cold plasma.

The differential equation which we obtained is in agreement with the case of homogeneous beam-magnetized cold plasma instability if we put in our case $N_b = const.$, $V_{0b} = const.$, and $V_{T_e} = 0$. The linear dispersion relation describes the system is obtained (relation (11)).

Relation (12) shows that the growth rate of beam-plasma instability decreased. This means that, the interaction of beam with plasma waves are weaker due to the local character of the interaction i.e., the Chernkov resonance condition in this case (inhomogeneous density of beam and plasma) could satisfied only locally because of the dependence of ω_p and ω_b on the variable x.

In conclusion, it is shown that the growth rate of the instability in inhomogeneous EB- magnetized warm plasma has been reduced as compared to inhomogeneous electron beam- magnetized cold plasma.

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