Plane Wave Spectral Integral Representation of the Dyadic Greens Function of Layered Media

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Abstract: The construction of the solution to the canonical problem of the electromagnetic (EM) radiation by current point source in the presence of horizontally stratified anisotropic medium. This solution is developed in terms of the well known vertical (or z-propagation) plane wave spectrum integral representation for the EM fields. The fields can be expressed in a compact form in terms of the dyadic Green's function for this problem. A rigorous analysis is performed using a dyadic Green's function formulation where the mixed boundary value problem is reduced to a set of coupled vector integral equations using the Fourier transform.
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1. Introduction

The plane wave integral representation of the dyadic Green's function for the canonical problem of electric current sources in the presence of the layered media may be constructed in several ways. One of the most common approaches is to express the Green's function in terms of a magnetic vector potential (Somerfeld, 1949), where as another less used approach is to construct the Green's function from a set of appropriate electric and magnetic vector potentials (Kong, 1986). In the former case, the magnetic vector potential has two components; one parallel and one normal to the interface. In the other approach, the magnetic and electric vector potentials are normal to the interface. If the electric current moment is chosen entirely normal to the interface, then the two approaches become identical since only a single normally directed magnetic vector potential suffices in this case. This is related to the fact that the normally oriented current moment excites only the TM waves (with respect to the normal $\hat{z}$ to the interface), whereas the current moment parallel to the interface excites both TM and TE waves. Therefore, the total electromagnetic field must be constructed either with magnetic vector potential which can produce both TM and TE waves (in this case the magnetic vector potential must have components normal and parallel to the interface in order to satisfy the appropriate boundary conditions), or with the magnetic and electric vector potentials which are both normal to the interface (since a normally directed magnetic vector potential produces TM waves and a normally directed electric vector potential produces TE waves). One of the main advantages of the later formulation is that boundary conditions associated with the differential operators for the two different types of vector potentials can be uncoupled (Ali*et al.,*1982). In the case of a choice of a single type of magnetic vector potential containing both a vertical (normal $\hat{z}$ to the interface) and a horizontal (transverse to $\hat{z}$ or parallel to the interface) components, the transverse component of the magnetic vector potential will contribute to both TE and TM waves, therefore, the boundary conditions for normal and transverse components will be coupled. This disadvantage will be pronounced if one deals with a stratified or multilayer dielectric media, for which the number of coupled boundary conditions increase, and no simple approach exists.

In this study, a unified general approach to the problem of radiation of arbitrary sources in a stratified medium is presented. The model of the medium is considered to consists of N horizontally stratified layers and an upper half-space. First, all layers are assumed to be isotropic, then the more general case of a uniaxial medium is considered where all layers posses both tensor permittivities and tensor permeabilities which in general can be complex. All axes of anisotropy are considered perpendicular to the boundaries separating the different layers.

Starting by defining two types of dyadic Green's functions which are dual to each other, namely an electric type dyadic Green's function $\overline{\overline{G}}$ and a magnetic type dyadic Green's function $\overline{\overline{\Gamma}}$ (Stratton, 1941). These Green's functions are resolved into their TE and TM parts. The resulting integrals are expressed in cartesian coordinates in terms of Weyl-type integral. A simple procedure to obtain the fields in any arbitrary layer is described. Tractable forms are shown to be easily deduced from the physical picture of the waves radiated from the primary sources and the multiple reflections from the stratified medium. The dyadic Green's function in the field region is properly represented by extracting the delta function singularity.

Recursion relations for appropriately defined reflection and transmission coefficients are presented.

2. Material and Methods

2-2Dyadic Green's Functions for Layered Isotropic Media

2-2-1Formulation

Consider the layered medium shown in (Figure. 2.1) with impressed sources located in an arbitrary layer

(j), $j = 0,1,...,t$. The layers are assumed to be isotropic with parameters (ε_j, μ_j) . For an impressed electric current source $\overline{J}_j$ located in layer (j) and varies harmonically with time as $e^{-i\omega t}$, the wave equation for $\overline{E}_i$ in the layer (i) is given by

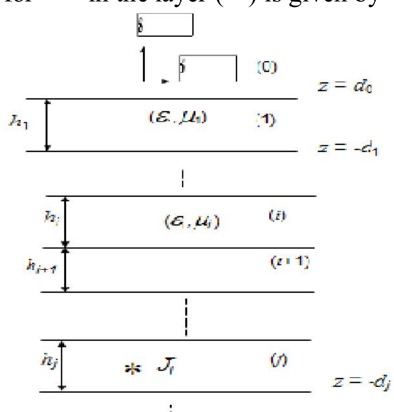

Figure 2.1, Geometric configuration of layered medium

$$\left(\nabla \times \nabla \times \overline{\overline{I}} - k_i^2 \overline{\overline{I}} \right) \cdot \overline{E}_i(\overline{r}) = i\omega\mu_j \overline{J}_j(\overline{r})\delta_{ij} \tag{2.1}$$

where $k_i^2 = \omega^2 \mu_i \varepsilon_i$, $\overline{\overline{I}}$ is the unit dyadic or idem factor, and δ_{ij} is the kronecker delta ($\delta_{ij} = 1$ for $i = j$ and $\delta_{ij} = 0$ for $i \neq j$).

To integrate Eq. 2.1 an electric type dyadic Green's function $\overline{\overline{G}}_{ij}(\overline{r},\overline{r}')$ for the layered medium may be introduced. Using first subscript (i) to denote the layer containing the observation point and the second subscript (j) to indicate that the source is in layer

(j).

The dyadic Green's function $\overline{\overline{G}}_{ij}(\overline{r},\overline{r}')$ satisfies the following relation

$$\left(\nabla \times \nabla \times \overline{\overline{I}} - k_i^2 \overline{\overline{I}} \right) \cdot \overline{\overline{G}}_{ij}(\overline{r},\overline{r}') = \begin{cases} 0 \quad , & i \neq j \\ \overline{\overline{I}}\,\delta(\overline{r},\overline{r}') , & i = j \end{cases} \tag{2.2}$$

It can be easily shown that

$$\overline{E}_i(\overline{r}) = i\omega\mu_j \iiint_{V_j} \overline{\overline{G}}_{ij}(\overline{r},\overline{r}') \cdot \overline{J}_j(\overline{r}')\, dV' \tag{2.3}$$

where V_j is the volume included by the sources in the layer (j).

Thus, if the electric current density $\overline{J}(\overline{r}')$ is taken to be arbitrary oriented point source of strength $\overline{p}_e$ at $\overline{r} = \overline{r}'$, then

$$\overline{J}(\overline{r}') = \overline{p}_e \delta(\overline{r} - \overline{r}')$$

The electric field $\overline{E}$ may now be viewed as

$$\overline{E} = i\omega\mu_o \overline{\overline{G}}(\overline{r} - \overline{r}') \cdot \overline{p}_e$$

where the dielectric layers are assumed to be nonmagnetic with $\mu_j = \mu_0$. Thus, it is important to observe that the components of the dyadic Green's function can be inferred from the equation of the electric field by inspection if $\overline{E}$ is excited by the arbitrary oriented current point source $\overline{p}_e$ is known. It is noted that the units of $\overline{p}_e$ are A-m (and

therefore constitutes a point current moment)

where as $\delta(\overline{r},\overline{r}')$ has the units of $\frac{1}{m^3}$, thus $\overline{J}(\overline{r}') = \overline{p}_e \delta(\overline{r} - \overline{r}')$ consistently has the units of $\frac{A}{m^3}$.

Therefore the dyadic Green's function $\overline{\overline{G}}(\overline{r},\overline{r}')$ can be obtained from the electric field $\overline{E}$ in matrix form as:

$$\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = i\omega\mu_o \begin{bmatrix} G_{xx} G_{xy} G_{xz} \\ G_{yx} G_{yy} G_{yz} \\ G_{zx} G_{zy} G_{zz} \end{bmatrix} \begin{bmatrix} p_{ex} \\ p_{ey} \\ p_{ez} \end{bmatrix}$$

with $\overline{p}_e$ given by

$$\overline{p}_e = \overline{p}_{ex}\hat{x} + \overline{p}_{ey}\hat{y} + \overline{p}_{ez}\hat{z}$$

This explains the physical meaning of the different components of the dyadic Green's function $\bar{\bar{G}}(\bar{r},\bar{r}')$

The dyadic Green's function $\bar{\bar{G}}_{ij}$ satisfies the following boundary conditions:

$$\hat{z}\times\bar{\bar{G}}_{ij} = \hat{z}\times\bar{\bar{G}}_{(i+1)j} \qquad (2.4a)$$

$$\frac{1}{\mu_i}\hat{z}\times\nabla\times\bar{\bar{G}}_{ij} = \frac{1}{\mu_{(i+1)}}\hat{z}\times\nabla\times\bar{\bar{G}}_{(i+1)j} \qquad (2.4b)$$

at the surface $z_i = -d\ i\ , \quad (i = 0,\ldots,n)$.

When the impressed sources are magnetic, a magnetic type dyadic Green's function $\bar{\bar{\Gamma}}_{ij}(\bar{r},\bar{r}')$ which satisfy dual expressions to Eq. 2.3 and Eq. 2.4 may be introduced.

Hence, if $\bar{M}_j$ represents a distribution of magnetic currents in the layer (j), one gets

$$\bar{H}_i(\bar{r}) = i\omega\,\varepsilon_j \iiint_{V_j} \bar{\bar{\Gamma}}_{ij}(\bar{r},\bar{r}')\cdot\bar{M}_j(\bar{r}')\,dV' \qquad (2.5)$$

Since $\bar{\bar{G}}$ and $\bar{\bar{\Gamma}}$ are dual to each other, $\bar{\bar{G}}$ and by duality the results will apply directly to $\bar{\bar{\Gamma}}$ shall deal with. The dyadic Green's function $\bar{\bar{G}}_{jj}$ in the layer (j) containing the sources, can be expressed as a superposition of the unbounded dyadic Green's function $\bar{\bar{G}}^{(p)}$ due to the primary excitation and a scattered dyadic Green's function $\bar{\bar{G}}^{(s)}$.

Hence for any layer (i)

$$\bar{\bar{G}}_{ij} = \bar{\bar{G}}^{(p)}\,\delta_{ij} + \bar{\bar{G}}_{ij}^{(s)} \qquad (2.6)$$

where the scattered dyadic Green's function satisfies the homogeneous wave equation

$$\left(\nabla\times\nabla\times\bar{\bar{I}} - k_i^2\bar{\bar{I}}\right)\cdot\bar{\bar{G}}_{ij}^{(s)}(\bar{r},\bar{r}') = 0 \qquad (2.7)$$

2-2-2 The Unbounded Dyadic Green's Function

In an arbitrary region (j) the unbounded dyadic Green's function satisfy the following differential equation

$$\nabla\times\nabla\times\bar{\bar{G}}^{(p)}(\bar{r},\bar{r}') - k_j^2\,\bar{\bar{G}}^{(p)}(\bar{r},\bar{r}') = \bar{\bar{I}}\,\delta(\bar{r}-\bar{r}') \qquad (2.8)$$

where by applying the Fourier transform, than

$$\delta(\bar{r}-\bar{r}') = \frac{1}{(2\pi)^3}\iint\int_{-\infty}^{\infty} d\bar{k}\ e^{i\bar{k}\cdot(\bar{r}-\bar{r}')} \qquad (2.9)$$

$$\bar{\bar{G}}^{(p)}(\bar{r},\bar{r}') = \frac{1}{(2\pi)^3}\iint\int_{-\infty}^{\infty} d\bar{k}\ \tilde{\bar{\bar{G}}}^{(p)}(\bar{k},\bar{r}')\ e^{i\bar{k}\cdot(\bar{r}-\bar{r}')} \qquad (2.10)$$

where

$$\bar{k} = \bar{k}_s + \hat{z}k_z$$

$$\bar{k}_s = \hat{x}k_x + \hat{y}k_y \qquad d\bar{k} = dk_x\,dk_y\,dk_z$$

Thus, Fourier transforming Eq. 2.8 in the $\bar{r}$ variable, the result will be

$$-\bar{k}\times\bar{k}\times\tilde{\bar{\bar{G}}}^{(p)}(\bar{k},\bar{r}') - k_j^2\tilde{\bar{\bar{G}}}^{(p)}(\bar{k},\bar{r}') = \bar{\bar{I}}e^{-i\bar{k}\cdot\bar{r}'} \qquad (2.11)$$

That is

$$-[\bar{k}\bar{k}\cdot\tilde{\bar{\bar{G}}}^{(p)}(\bar{k},\bar{r}') - k^2\tilde{\bar{\bar{G}}}^{(p)}(\bar{k},\bar{r}')] - k_j^2\tilde{\bar{\bar{G}}}^{(p)}(\bar{k},\bar{r}') = \bar{\bar{I}}e^{-i\bar{k}\cdot\bar{r}'} \qquad (2.12)$$

Dot multiplying Eq. 2.11a by $\bar{k}$, get

$$-k_j^2(\bar{k}\cdot\tilde{\bar{\bar{G}}}^{(p)}(\bar{k},\bar{r}')) = \bar{k}\,e^{-i\bar{k}\cdot\bar{r}'} \qquad (2.13)$$

Multiplying Eq. 2.12 by k_j^2 and using Eq. 2.13, than

$$\tilde{\bar{\bar{G}}}^{(p)}(\bar{k},\bar{r}') = \frac{\bar{\bar{I}}k_j^2 - \bar{k}\bar{k}}{k_j^2(k^2-k_j^2)}e^{-i\bar{k}\cdot\bar{r}'} \qquad (2.14)$$

where

$$k^2 = k_x^2 + k_y^2 + k_z^2 = k_s^2 + k_z^2$$

$$k_s^2 = k_x^2 + k_y^2$$

Thus,

$$\bar{\bar{G}}^{p}(\bar{r},\bar{r}') = \frac{1}{(2\pi)^3}\iint\int_{-\infty}^{\infty} d\bar{k}\ \frac{\bar{\bar{I}}k_j^2 - \bar{k}\bar{k}}{k_j^2\ (k^2-k_j^2)}e^{i\bar{k}\cdot(\bar{r}-\bar{r}')} \qquad (2.15)$$

This Fourier transform representation does not converge. The integrand tends to a constant as $\left|\bar{k}\right|\to\infty$ while k_x and k_y remain finite, than (Chew, 1989):

$$\frac{\bar{\bar{I}}k_j^2 - \bar{k}\bar{k}}{k_j^2(k^2-k_j^2)} = \frac{\bar{\bar{I}}k_j^2 - (\bar{k}_s + \hat{z}k_z)(\bar{k}_s + \hat{z}k_z)}{k_j^2(k^2-k_j^2)}$$

$$=\frac{1}{k_j^2(k^2-k_j^2)}[\overline{\overline{I}}k_j^2-\bar{k}_s\bar{k}_s-\bar{k}_s\hat{z}k_z-\hat{z}\bar{k}_s k_z-\hat{z}\hat{z}k_z^2]$$

$$-\frac{\hat{z}\hat{z}}{k_j^2},k_z\to\infty$$

$\sim$

Thus,

$$\overline{\overline{G}}^{(p)}(\bar{r},\bar{r}')=\frac{1}{(2\pi)^3}\iint\int_{-\infty}^{\infty}d\bar{k}e^{i\bar{k}\cdot(\bar{r}-\bar{r}')}[\frac{\overline{\overline{I}}k_j^2-\bar{k}\bar{k}}{k_j^2(k^2-k_j^2)}+\frac{\hat{z}\hat{z}}{k_j^2}]$$

$$-\frac{\hat{z}\hat{z}}{(2\pi)^3k_j^2}\iint\int_{-\infty}^{\infty}d\bar{k}e^{i\bar{k}\cdot(\bar{r}-\bar{r}')} \qquad (2.16)$$

The 1st integrand vanishes as $\frac{1}{k_z^2}$ when $k_z\to\infty$, while k_x and k_y are held finite.

By virtue of Jordan's Lemma and Cauch'stheorem, the dk_z integral can be evaluated first using residue calculus for $|z-z'|>0$, by picking up the residue contributions at the pole locations given by

$k_z=\pm\sqrt{k_j^2-k_s^2}\equiv\pm k_{jz}$. The second integral in Eq. 2.16 is just the Fourier representation of delta function $\delta(\bar{r}-\bar{r}')$.

Consequently,

$$\overline{\overline{G}}^{(p)}(\bar{r},\bar{r}')=\frac{1}{(2\pi)^3}\int\int_{-\infty}^{\infty}d\bar{k}_s\int_{-\infty}^{\infty}dk_z e^{i\bar{k}_s\cdot(\bar{r}_s-\bar{r}_s')}e^{ik_z(z-z')}[\frac{\overline{\overline{I}}k_j^2-\bar{k}\bar{k}}{k_j^2(k_z-k_{jz})(k_z+k_{jz})}$$

$$+\frac{\hat{z}\hat{z}}{k_j^2}]-\frac{\hat{z}\hat{z}}{k_j^2}\delta(\bar{r}-\bar{r}') \qquad (2.17)$$

where

$$\bar{r}_s=\hat{x}x+\hat{y}y$$

For $z>z'$, we deform the integration path in the complex k_z-plane upwards (see Figure. 2.2), to pick up the contribution of the pole at $k_z=+k_{jz}$.

For $z<z'$, we deform downwards, to pick up the contribution of the pole at $k_z=-k_{jz}$.

The integration path is chosen such that Im(k_{jz}) > 0 in order to satisfy the radiation condition.

Thus.

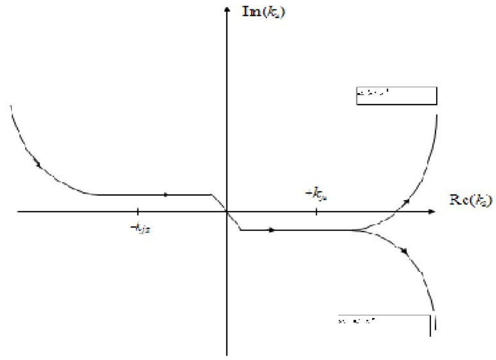

Figure 2.2, Integration Path in the Complex k_z plane

$$\overline{\overline{G}}^{(p)}(\bar{r},\bar{r}')=-\frac{1}{k_j^2}\hat{z}\hat{z}\delta(r-r')$$

$$+\begin{cases}\frac{i}{8\pi^2}\int\int_{-\infty}^{\infty}d\bar{k}_s e^{i\bar{k}_s\cdot(\bar{r}_s-\bar{r}_s')}\frac{1}{k_{jz}}\left[\overline{\overline{I}}-\frac{\bar{k}_j\bar{k}_j}{k_j^2}\right]e^{ik_{jz}(z-z')}, & z>z'\\ \frac{i}{8\pi^2}\int\int_{-\infty}^{\infty}d\bar{k}_s e^{i\bar{k}_s\cdot(\bar{r}_s-\bar{r}_s')}\frac{1}{k_{jz}}\left[\overline{\overline{I}}-\frac{\bar{K}_j\bar{K}_j}{k_j^2}\right]e^{-ik_{jz}(z-z')}, & z<z'\end{cases}$$

(2.18a)

where

$\bar{k}_j=\bar{k}_s+\hat{z}k_{jz}$ (2.18b) $\bar{K}_j=\bar{k}_s-\hat{z}k_{jz}$ (2.18c)

Recognizing that $\hat{k}_j=\frac{\bar{k}_j}{k_j}$, forming an orthogonal system consisting of unit vectors $\hat{k}_j,\hat{v}(k_{jz})$ and $\hat{h}(k_{jz})$ as follows (Kong, 1986)

$$\hat{h}(k_{jz})=\frac{\bar{k}_j\times\hat{z}}{|\bar{k}_j\times\hat{z}|}=\frac{1}{k_s}(\hat{x}k_y-\hat{y}k_x) \qquad (2.19)$$

$$\hat{v}(k_{jz})=\hat{h}(k_{jz})\times\hat{k}_j=\frac{-k_{jz}(\hat{x}k_x+\hat{y}k_y)}{k_jk_s}+\hat{z}\frac{k_s}{k_j}$$

(2.20)

where

$$k_s=|\bar{k}_s|$$

Also, $\hat{K}_j=\frac{\bar{K}_j}{K_j}$, $\hat{v}(-k_{jz})$, and $\hat{h}(-k_{jz})$ form another orthonormal set of unit vectors as follows :

$$\hat{h}(-k_{jz})=\hat{h}(k_{jz}) \qquad (2.21)$$

$\hat{v}(-k_{jz}) = \hat{h}(-k_{jz}) \times \hat{K}_j = \frac{k_{jz}}{k_j k_s}(\hat{x}k_x + \hat{y}k_y) + \hat{z}\frac{k_s}{k_j}$ (2.22)

Since $\bar{\bar{I}}$ can be written as

$\bar{\bar{I}} = \hat{k}_j \hat{k}_j + \hat{h}(k_{jz})\,\hat{h}(k_{jz}) + \hat{v}(k_{jz})\,\hat{v}(k_{jz})$ (2.23a)

or

$\bar{\bar{I}} = \hat{K}_j \hat{K}_j + \hat{h}(-k_{jz})\,\hat{h}(-k_{jz}) + \hat{v}(-k_{jz})\,\hat{v}(-k_{jz})$ (2.23b)

so,

$\bar{\bar{I}} - \hat{k}_j \hat{k}_j = \hat{h}(k_{jz})\,\hat{h}(k_{jz}) + \hat{v}(k_{jz})\,\hat{v}(k_{jz})$ (2.24a)

$\bar{\bar{I}} - \hat{K}_j \hat{K}_j = \hat{h}(-k_{jz})\,\hat{h}(-k_{jz}) + \hat{v}(-k_{jz})\,\hat{v}(-k_{jz})$ (2.24b)

The dyadic Green's function $\bar{\bar{G}}^{(p)}(\bar{r},\bar{r}')$ in a region (j), can be represented in the following form

$$\bar{\bar{G}}^{(p)}(\bar{r},\bar{r}') = \frac{-\hat{z}\hat{z}}{k_j^2}\delta(\bar{r}-\bar{r}') + \frac{i}{8\pi^2}\int\int_{-\infty}^{\infty} d\bar{k}_s\, e^{i\bar{k}_s\cdot(\bar{r}-\bar{r}')}\frac{1}{k_{jz}} \begin{cases} \left[\hat{h}(k_{jz})\hat{h}(k_{jz}) + \hat{v}(k_{jz})\hat{v}(k_{jz})\right]e^{ik_{jz}(z-z')} & , z > z' \\ \left[\hat{h}(-k_{jz})\hat{h}(-k_{jz}) + \hat{v}(-k_{jz})\hat{v}(-k_{jz})\right]e^{-ik_{jz}(z-z')} & , z < z' \end{cases} \quad (2.25)$$

Notice that $\hat{h}(\pm k_{jz})$ is a unit vector in the direction of the electric field for a horizontally polarized TE wave and $\hat{v}(\pm k_{jz})$ is a unit vector in the direction of the electric field for a vertically polarized TM wave.

In the above for $z > z'$, the TE dyad $\hat{h}(k_{jz})\,\hat{h}(k_{jz})e^{ik_{jz}(z-z')}$ is formed by two unit vectors, the anterior one is related to the field point where the wave vector has a positive

z-component (wave is propagating upward). The posterior unit vector is related to the source point where the wave vector has also a positive z-component.

The propagator $e^{ik_{jz}(z-z')}$ accounts for the phase difference as the wave propagates from the source point at z' to the field point at z. For $z > z'$, $\hat{h}(-k_{jz})\,\hat{h}(-k_{jz})e^{ik_{jz}(z-z')}$. Similar discussion holds for the dyad of the TM waves.

2-3Source and Observation Points in the Same Layer

2-3-1The Dyadic Green's Function $\bar{\bar{G}}_{00}(\bar{r},\bar{r}')$

Consider electric sources be located in the upper half-space of the layered isotropic medium of Figure. 2.1, and it is required to find the field in the upper half-space also.

In this case $\bar{\bar{G}}_{00}$ is the superposition of the unbounded dyadic Green's function $\bar{\bar{G}}^{(p)}$ given by Eq. 2.25 and a scattered dyadic Green's function $\bar{\bar{G}}_{00}^{(s)}$ which is the contribution caused by layered medium.

(i) For $z < z'$:

It is possible to construct the dyadic Green's function $\bar{\bar{G}}_{00}(\bar{r},\bar{r}')$ from the physical picture of the waves radiated from the source and reaching the field point. From Figure. 2.3, it is clear that adding the unbounded dyadic Green's function (the direct waves from the source point to the field point) and the waves reflected from the layered stratified medium with generalized reflection coefficient $R^s_{\cap 0}$ at the lower boundary of layer (0) together. The superscript S stands for TE or TM waves.

Thus,

$$\bar{\bar{G}}_{00}(\bar{r},\bar{r}') = -\frac{\hat{z}\hat{z}}{k_0^2}\delta(\bar{r}-\bar{r}') + \frac{i}{8\pi^2}\int\int_{-\infty}^{\infty} d\bar{k}_s\, e^{i\bar{k}_s\cdot(\bar{r}_s-\bar{r}_s')}\frac{1}{k_{0z}} \left\{ \begin{bmatrix} \hat{h}(k_{0z})\hat{h}(k_{0z})e^{ik_{0z}(z_0-z_0')} + \\ R^{TE}_{\cap 0}\,\hat{h}(k_{0z})\hat{h}(-k_{0z})e^{ik_{0z}(z_0+z_0')} \end{bmatrix} + \begin{bmatrix} \hat{v}(k_{0z})\hat{v}(k_{0z})e^{ik_{0z}(z_0-z_0')} + \\ R^{TM}_{\cap 0}\,\hat{v}(k_{0z})\hat{v}(-k_{0z})e^{ik_{0z}(z_0+z_0')} \end{bmatrix} \right\} \quad (2.26)$$

2-3-2The Generalized Reflection Coefficient $R^s_{\cap l}$

The generalized reflection coefficient $R^s_{\cap l}$ with $l = 0,1,...(n-1)$, can be obtained by using the picture of the multiple reflections shown in Figure. 2.4.

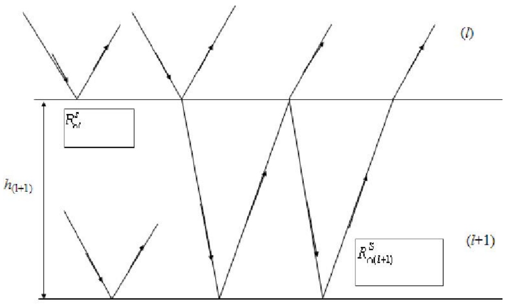

Figure 2.4, Determination of $R^s_{\cap l}$

Note that $R^s_{\cap l}$ is the reflection occurring at the lower boundary of layer (l) and contains the contribution of all the underlying layers.

$$R^s_{\cap l} = R^s_{l(l+1)} + T^s_{l(l+1)} e^{ik_{(l+1)z} h_{(l+1)}} R^s_{\cap(l+1)} e^{ik_{(l+1)z} h_{(l+1)}} T^s_{(l+1)l} + T^s_{l(l+1)} e^{ik_{(l+1)z} h_{(l+1)}} R^s_{\cap(l+1)} e^{ik_{(l+1)z} h_{(l+1)}} R^s_{(l+1)l} e^{ik_{(l+1)z} h_{(l+1)}} R^s_{\cap(l+1)} e^{ik_{(l+1)z} h_{(l+1)}} T^s_{(l+1)l} + \ldots$$

$$= R^s_{l(l+1)} + T^s_{l(l+1)} R^s_{\cap(l+1)} T^s_{(l+1)l} e^{2ik_{(l+1)z} h_{(l+1)}} \left[1 + m + m^2 \ldots\right] \quad (2.27)$$

where

$$m = R^s_{\cap(l+1)} R^s_{(l+1)l} e^{2ik_{(l+1)z} h_{(l+1)}}$$

The recurrence relation Eq. 2.27 can be put in the form

$$R^s_{\cap l} = R^s_{l(l+1)} + \frac{T^s_{l(l+1)} R^s_{\cap(l+1)} T^s_{(l+1)l} e^{2ik_{(l+1)z} h_{(l+1)}}}{1 - R^s_{\cap(l+1)} R^s_{(l+1)l} e^{2ik_{(l+1)z} h_{(l+1)}}} \quad (2.28)$$

where $l = 0, \ldots, (n-1)$ and $R^s_{\cap n} = R^s_{nt}$.

$R^s_{ij} = -R^s_{ji}$ and $T^s_{ij} = 1 + R^s_{ij}$ are the Fresnel's reflection and transmission coefficients, respectively.

$$R^{TE}_{ij} = \frac{\mu_j k_{iz} - \mu_i k_{jz}}{\mu_j k_{iz} + \mu_i k_{jz}} \quad (2.29)$$

$$R^{TM}_{ij} = \frac{\varepsilon_j k_{iz} - \varepsilon_i k_{jz}}{\varepsilon_j k_{iz} + \varepsilon_i k_{jz}} \quad (2.30)$$

Relation Eq. 2.28 can be put in the other form given by

$$R^s_{\cap l} = \frac{R^s_{l(l+1)} + R^s_{\cap(l+1)} e^{2ik_{(l+1)z} h_{(l+1)}}}{1 + R^s_{\cap(l+1)} R^s_{l(l+1)} e^{2ik_{(l+1)z} h_{(l+1)}}} \quad (2.31)$$

where $l = 0, \ldots, (n-1)$ and $R^s_{\cap n} = R^s_{nt}$.

So, to get $R^s_{\cap 0}$ we start with $R^s_{\cap n} = R^s_{nt}$ and proceed upward using either of the recurrence relations Eq. 2.28 or Eq. 2.31

(ii) For $z < z'$:

Using Fig. 2.5, than $\overline{\overline{G}}_{00}(\bar{r}, \bar{r}')$ the following expression

$$\overline{\overline{G}}_{00}(\bar{r}, \bar{r}') = -\frac{\hat{z}\hat{z}}{k_0^2}\delta(\bar{r} - \bar{r}') + \frac{i}{8\pi^2}\iint_{-\infty}^{\infty} d\bar{k}_s\, e^{i\bar{k}_s \cdot (\bar{r}_s - \bar{r}_s')} \frac{1}{k_{0z}} \Big\{ \left[\hat{h}(-k_{0z})\hat{h}(-k_{0z})e^{-ik_{0z}(z_0 - z_0')} + R^{TE}_{\cap 0}\hat{h}(k_{0z})\hat{h}(-k_{0z})e^{ik_{0z}(z_0 + z_0')}\right] + \left[\hat{v}(-k_{0z})\hat{v}(-k_{0z})e^{-ik_{0z}(z_0 - z_0')} + R^{TM}_{\cap 0}\hat{v}(k_{0z})\hat{v}(-k_{0z})e^{ik_{0z}(z_0 + z_0')}\right] \Big\} \quad (2.32)$$

where

$$\hat{h}(k_{0z}) = \hat{h}(-k_{0z}) \quad (2.33)$$

2-3-3The Dyadic Green's Function $\overline{\overline{G}}_{tt}(\bar{r}, \bar{r}')$

If the source and observation points are located in the lower half-space of the stratified medium, $\overline{\overline{G}}_{tt}(\bar{r}, \bar{r}')$ can be obtained as a superposition of the unbounded dyadic Green's function plus the contribution of the reflected waves from the layered medium. Using the local coordinates shown in Fig. 2.6, the result will be

(i) For $z > z'$:

$$\overline{\overline{G}}_{tt}(\bar{r}, \bar{r}') = -\frac{\hat{z}\hat{z}}{k_t^2}\delta(\bar{r} - \bar{r}') + \frac{i}{8\pi^2}\iint_{-\infty}^{\infty} d\bar{k}_s\, e^{i\bar{k}_s \cdot (\bar{r}_s - \bar{r}_s')} \frac{1}{k_{tz}} \quad (2.33)$$

$$\left\{ \left[\hat{h}(k_{tz})\hat{h}(k_{tz})e^{ik_{tz}(z_t - z_t')} + R^{TE}_{\cup t}\,\hat{h}(-k_{tz})\hat{h}(k_{tz})e^{-ik_{tz}(z_t + z_t')} \right] + \left[\hat{v}(k_{tz})\hat{v}(k_{tz})e^{ik_{tz}(z_t - z_t')} + R^{TM}_{\cup t}\,\hat{v}(-k_{tz})\hat{v}(k_{tz})e^{-ik_{tz}(z_t + z_t')} \right] \right\} \quad (2.34a)$$

where $R^S_{\cup t}$ (S = TE and TM) is the generalized reflection coefficient at the upper boundary of layer (t).

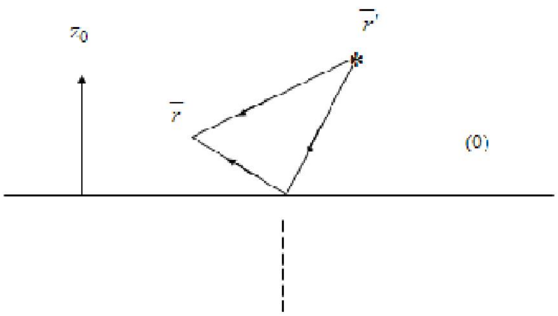

Figure 2.5, Source and Observation points in the upper half-space ($z < z'$)

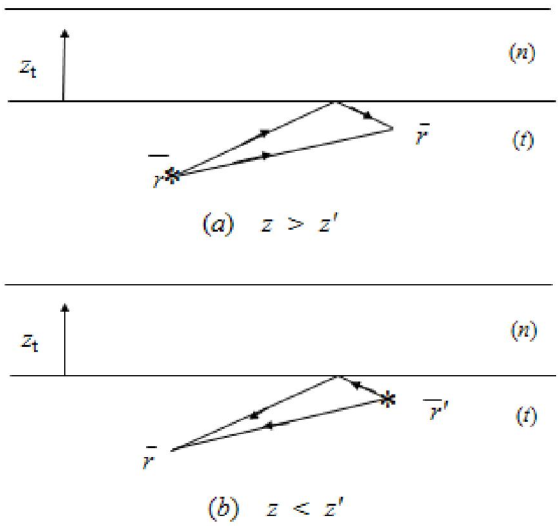

Figure 2.6, Source and Observation points in the lower half-space

(ii) For $z < z'$

$$\overline{\overline{G}}_{tt}(\bar r, \bar r') = -\frac{\hat z\hat z}{k_t^2}\delta(\bar r - \bar r') + \frac{i}{8\pi^2}\iint_{-\infty}^{\infty} d\bar k_s\, e^{i\bar k_s\cdot(\bar r_s - \bar r_s')}\frac{1}{k_{tz}}$$
$$\left\{\left[\hat h(-k_{tz})\hat h(-k_{tz})e^{-ik_{tz}(z_t - z_t')} + R^{TE}_{\cup t}\hat h(-k_{tz})\hat h(k_{tz})e^{-ik_{tz}(z_t + z_t')}\right] + \left[\hat v(-k_{tz})\hat v(-k_{tz})e^{-ik_{tz}(z_t - z_t')} + R^{TM}_{\cup t}\hat v(-k_{tz})\hat v(k_{tz})e^{-ik_{tz}(z_t + z_t')}\right]\right\} \quad (2.34b)$$

2-3-4 The Generalized Reflection Coefficient $R^s_{\cup l}$

The generalized reflection coefficient $R^s_{\cup l}$ at the upper boundary of layer (l), can be obtained in a similar way to $R^s_{\cap l}$. With the aid of Fig. 2.7, the recurrence relation for $R^s_{\cup l}$ can be obtained in a similar way as

$$R^s_{\cup l} = R^s_{l(l-1)} + \frac{T^s_{l(l-1)}\, R^s_{\cup(l-1)}\, T^s_{(l-1)l}\, e^{2ik_{(l-1)z}h_{(l-1)}}}{1 - R^s_{\cup(l-1)}\, R^s_{(l-1)l}\, e^{2ik_{(l-1)z}h_{(l-1)}}} \quad (2.35)$$

where $l = 2,3,\ldots, t$ and $R^s_{\cup 1} = R^s_{10}$.

Relation Eq. 2.35 can be put in the other form

$$R^s_{\cup l} = \frac{R^s_{l(l-1)} + R^s_{\cup(l-1)}\, e^{2ik_{(l-1)z}h_{(l-1)}}}{1 + R^s_{\cup(l-1)}\, R^s_{l(l-1)}\, e^{2ik_{(l-1)z}h_{(l-1)}}} \quad (2.36)$$

So, to get $R^s_{\cup t}$ starting with $R^s_{\cup 1} = R^s_{10}$ and proceed downward using either of the recurrence relations

Eq. 2.35 or Eq. 2.36.

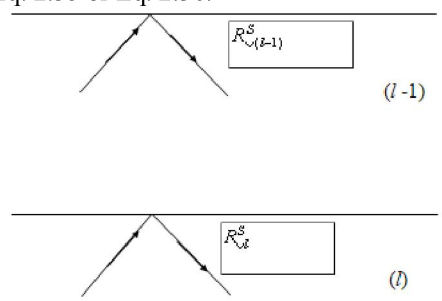

Figure 2.7, Determination of $R^s_{\cup l}$

2-3-5The Dyadic Green's Function $\overline{\overline{G}}_{ll}(\bar r, \bar r')$

When the source and observation points are in an arbitrary layer (l), $l \neq 0$ or t, the waves reaching the observation point from the source are those shown in Figure 2.8 plus those due to multiple reflections. If the local coordinates are at the upper boundary of layer (l), the dyadic Green's function takes the form.

(i) For $z > z'$:

Using Fig. 2.8a, can be written as the following expression

$$\overline{\overline{G}}_{ll}(\bar r, \bar r') = -\frac{\hat z\hat z}{k_l^2}\delta(\bar r - \bar r') + \frac{i}{8\pi^2}\iint_{-\infty}^{\infty} d\bar k_s\, e^{i\bar k_s\cdot(\bar r_s - \bar r_s')}\frac{1}{k_{lz}}$$

$$\left\{ r_l^{TE} \left[\hat{h}(k_{lz})\hat{h}(k_{lz})e^{ik_{lz}(z_l - z_l')} + R_{\cup l}^{TE}\ \hat{h}(-k_{lz})\hat{h}(k_{lz})e^{-ik_{lz}(z_l + z_l')} \right. \right.$$

$$+ R_{\cap l}^{TE}\ \hat{h}(k_{lz})\hat{h}(-k_{lz})e^{2ik_{lz}h_l}\, e^{ik_{lz}(z_l + z_l')}$$

$$+$$

$$\left. \left[R_{\cup l}^{TE} R_{\cap l}^{TE}\ \hat{h}(-k_{lz})\hat{h}(-k_{lz})e^{2ik_{lz}h_l}\, e^{-ik_{lz}(z_l - z_l')} \right. + TM\ terms \right\}$$

which can be written in the more compact form :

$$\overline{\overline{G}}_{ll}(\bar{r},\bar{r}') =$$

$$-\frac{\hat{z}\hat{z}}{k_l^2}\delta(\bar{r}-\bar{r}') + \frac{i}{8\pi^2}\iint_{-\infty}^{\infty} d\bar{k}_s\, e^{i\bar{k}_s\cdot(\bar{r}_s - \bar{r}_s')}\, \frac{1}{k_{lz}}$$

$$\left\{ r_l^{TE}\left[\hat{h}(k_{lz})e^{ik_{lz}z_l} + R_{\cup l}^{TE}\ \hat{h}(-k_{lz})e^{-ik_{lz}z_l}\right]\left[\hat{h}(k_{lz})e^{-ik_{lz}z_l'} + R_{\cap l}^{TE}\ e^{2ik_{lz}h_l}\ \hat{h}(-k_{lz})e^{ik_{lz}z_l'}\right] \right.$$

$$\left. + r_l^{TM}\left[\hat{v}(k_{lz})e^{ik_{lz}z_l} + R_{\cup l}^{TM}\ \hat{v}(-k_{lz})e^{-ik_{lz}z_l}\right]\left[\hat{v}(k_{lz})e^{-ik_{lz}z_l'} + R_{\cap l}^{TM}\ e^{2ik_{lz}h_l}\ \hat{v}(-k_{lz})e^{ik_{lz}z_l'}\right] \right\}$$

(2.37)

where

$$r^{TE} = \frac{1}{1 - R_{\cap l}^{TE} R_{\cup l}^{TE} e^{2ik_{lz}h_l}} \quad (2.38a)$$

$$r_l^{TM} = \frac{1}{1 - R_{\cap l}^{TM}\ R_{\cup l}^{TM}\ e^{2ik_{lz}h_l}} \quad (2.38b)$$

r_l^{TE} and r_l^{TM} account, respectively, for the multiple reflections of the TE and TM waves inside layer (l).

(ii) For $z < z'$:

Using Figure 2.8b, can be written as the following expression for $\overline{\overline{G}}_{ll}(\bar{r},\bar{r}')$:

$$\overline{\overline{G}}_{ll}(\bar{r},\bar{r}') =$$

$$-\frac{\hat{z}\hat{z}}{k_l^2}\delta(\bar{r}-\bar{r}') + \frac{i}{8\pi^2}\iint_{-\infty}^{\infty} d\bar{k}_s\, e^{i\bar{k}_s\cdot(\bar{r}_s - \bar{r}_s')}\, \frac{1}{k_{lz}}$$

$$\left\{ r_l^{TE}\left[\hat{h}(-k_{lz})e^{-ik_{lz}z_l} + R_{\cap l}^{TE}\, e^{2ik_{lz}h_l}\ \hat{h}(k_{lz})e^{ik_{lz}z_l}\right]\left[\hat{h}(-k_{lz})e^{ik_{lz}z_l'} + R_{\cup l}^{TE}\ \hat{h}(k_{lz})e^{-ik_{lz}z_l'}\right] \right.$$

$$\left. + r_l^{TM}\left[\hat{v}(-k_{lz})e^{-ik_{lz}z_l} + R_{\cap l}^{TM}\, e^{2ik_{lz}h_l}\, \hat{v}(k_{lz})e^{ik_{lz}z_l}\right]\left[\hat{v}(-k_{lz})e^{ik_{lz}z_l'} + R_{\cup l}^{TM}\ \hat{v}(k_{lz})e^{-ik_{lz}z_l'}\right] \right\}$$

(2.39)

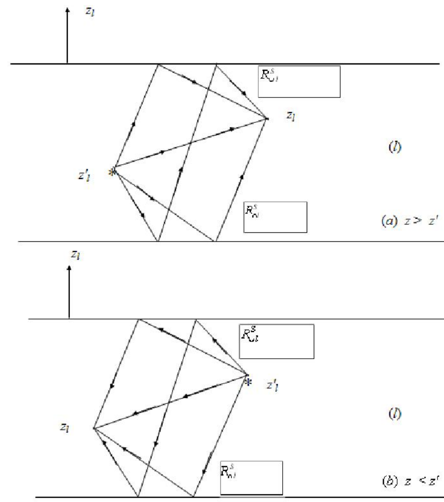

Figure 2.8, Source and Observation points in a general layer (l) ($z_l = z + d_{(l-1)}$)

Choosing the local coordinates of layer (l) as shown in Figure 2.9, than the following expression for the dyadic Green's function $\overline{\overline{G}}_{ll}(\bar{r},\bar{r}')$:

(i) For $z > z'$:

Using Figure 2.9a, than

$$\overline{\overline{G}}_{ll}(\bar{r},\bar{r}') =$$

$$-\frac{\hat{z}\hat{z}}{k_0^2}\delta(\bar{r}-\bar{r}') + \frac{i}{8\pi^2}\iint_{-\infty}^{\infty} d\bar{k}_s\, e^{i\bar{k}_s\cdot(\bar{r}_s - \bar{r}_s')}\, \frac{1}{k_{lz}}$$

$$\left\{ r_l^{TE}\left[\hat{h}(k_{lz})e^{ik_{lz}z_l} + R_{\cup l}^{TE}\, e^{2ik_{lz}h_l}\ \hat{h}(-k_{lz})e^{-ik_{lz}z_l}\right]\left[\hat{h}(k_{lz})e^{-ik_{lz}z'_l} + R_{\cap l}^{TE}\ \hat{h}(-k_{lz})e^{ik_{lz}z'_l}\right] \right.$$

$$\left. + r_l^{TM}\left[\hat{v}(k_{lz})e^{ik_{lz}z_l} + R_{\cup l}^{TM}\, e^{2ik_{lz}h_l}\, \hat{v}(-k_{lz})e^{-ik_{lz}z_l}\right]\left[\hat{v}(k_{lz})e^{-ik_{lz}z'_l} + R_{\cap l}^{TM}\ \hat{v}(-k_{lz})e^{ik_{lz}z'_l}\right] \right\}$$

(2.40)

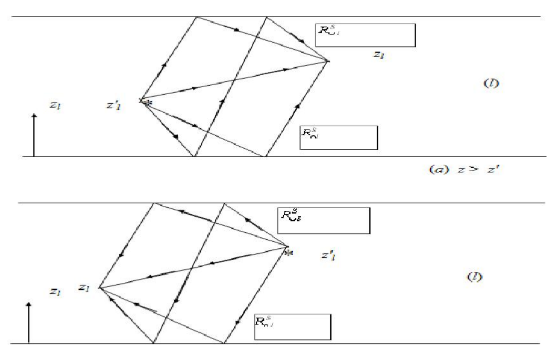

Figure 2.9, Source and Observation points in a general layer (l) ($z_l = z + d_l$)

(ii) For $z<z'$:
Using Figure 2.9a, than

$$\overline{\overline{G}}_{ll}(\bar r,\bar r') = -\frac{\hat z\hat z}{k_l^2}\delta(\bar r-\bar r') + \frac{i}{8\pi^2}\iint_{-\infty}^{\infty} d\bar k_s\, e^{i\bar k_s\cdot(\bar r_s-\bar r_s')}\frac{1}{k_{lz}}$$
$$\Big\{ r_l^{TE}\left[\hat h(-k_{lz})e^{-ik_{lz}z_l} + R_{\cap l}^{TE}\,\hat h(k_{lz})e^{ik_{lz}z_l}\right]$$
$$\left[\hat h(-k_{lz})e^{-ik_{lz}z_l'} + R_{\cup l}^{TE}\,e^{2ik_{lz}h_l}\hat h(k_{lz})e^{-ik_{lz}z_l'}\right]$$
$$+r_l^{TM}\left[\hat v(-k_{lz})e^{-ik_{lz}z_l} + R_{\cap l}^{TM}\,\hat v(k_{lz})e^{ik_{lz}z_l}\right]$$
$$\left[\hat v(-k_{lz})e^{ik_{lz}z_l'} + R_{\cup l}^{TM}\,e^{2ik_{lz}h_l}\,\hat v(k_{lz})e^{-ik_{lz}z_l'}\right]\Big\}$$
(2.41)

2.4 Source and Observation Points in Different Layers

2-4-1The Dyadic Green's Function $\overline{\overline{G}}_{lm}(\bar r,\bar r')$

The geometry of the problem is shown in Figure 2.10, where the sources are in layer (m) and the observation point is in the layer (l) below layer (m).

In this case choosing the local coordinates of both layers to be at the boundaries most near to each other.

With all multiple reflections, assuming that the waves transmitted from the source in the layer (m) propagate downwards and enter layer (l) with a transmission coefficient $X^S_{\cap l,m}$ (S = TE or TM) which shall call downward transmission coefficient. Thus,

$$\overline{\overline{G}}_{lm}(\bar r,\bar r') = \frac{i}{8\pi^2}\iint_{-\infty}^{\infty} d\bar k_s\, e^{i\bar k_s\cdot(\bar r_s-\bar r_s')}\frac{1}{k_{mz}}$$
$$\Big\{ r_m^{TE}X^{TE}_{\cap l,m}\left[\hat h(-k_{lz})e^{-ik_{lz}z_l} + R_{\cap l}^{TE}e^{2ik_{lz}h_l}\,\hat h(k_{lz})e^{ik_{lz}z_l}\right]$$
$$\left[\hat h(-k_{mz})e^{ik_{mz}z_m'} + R_{\cup m}^{TE}e^{2ik_{mz}h_m}\,\hat h(k_{mz})e^{-ik_{mz}z_m'}\right]$$
$$+r_m^{TM}X^{TM}_{\cap l,m}\left[\hat v(-k_{lz})e^{-ik_{lz}z_l} + R_{\cap l}^{TM}e^{2ik_{lz}h_l}\,\hat v(k_{lz})e^{ik_{lz}z_l}\right]$$
$$\left[\hat v(-k_{mz})e^{ik_{mz}z_m'} + R_{\cup m}^{TM}e^{2ik_{mz}h_m}\,\hat v(k_{mz})e^{-ik_{mz}z_m'}\right]\Big\}$$
(2.42)

2-4-2 The Downward Transmission Coefficient $X^S_{\cap l,m}$

The downward transmission coefficient $X^S_{\cap l,m}$ from layer (m) to layer (l) is obtained by writing $\overline{\overline{G}}_{(l-1)m}$ and applying the boundary condition at $z_l = 0$.

Thus,

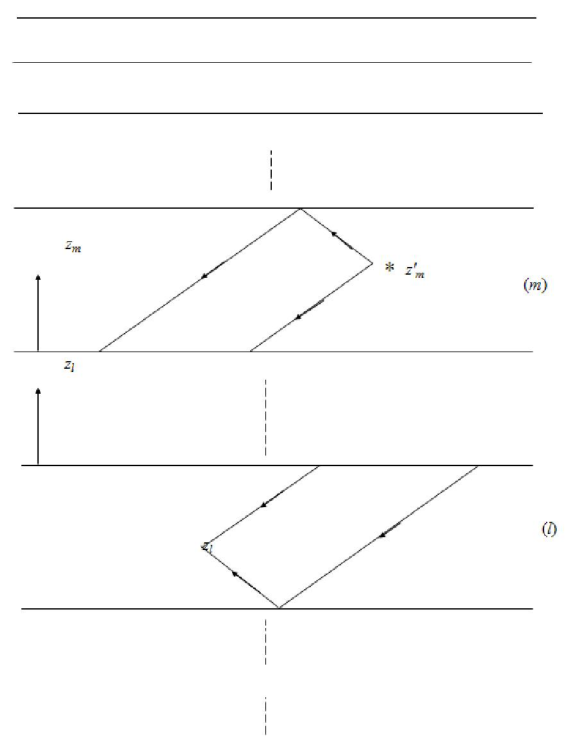

Figure 2.10, Source and Observation points in two different layers ($z<z'$) ($z_l = z+d_{(l-1)}, z_m = z+d_m$)

$$\overline{\overline{G}}_{(l-1)m}(\bar r,\bar r') = \frac{i}{8\pi^2}\iint_{-\infty}^{\infty} d\bar k_s\, e^{i\bar k_s\cdot(\bar r_s-\bar r_s')}\frac{1}{k_{mz}}$$
$$\Big\{ r_m^{TE}X^{TE}_{\cap(l-1),m}\left[\hat h(-k_{(l-1)z})e^{-ik_{(l-1)z}z_{(l-1)}} + R^{TE}_{\cap(l-1)}e^{2ik_{(l-1)z}h_{(l-1)}}\,\hat h(k_{(l-1)z})e^{ik_{(l-1)z}z_{(l-1)}}\right]$$
$$\left[\begin{matrix}\hat h(-k_{mz})e^{ik_{mz}z_m'} \\ +R^{TE}_{\cup m}e^{2ik_{mz}h_m}\,\hat h(k_{mz})e^{-ik_{mz}z_m'}\end{matrix}\right]$$
$$+r_m^{TM}X^{TM}_{\cap(l-1),m}\left[\hat v(-k_{(l-1)z})e^{-ik_{(l-1)z}z_{(l-1)}} + R^{TM}_{\cap(l-1)}e^{2ik_{(l-1)z}h_{(l-1)}}\,\hat v(k_{(l-1)z})e^{ik_{(l-1)z}h_{(l-1)}}\right]$$
$$\left[\hat v(-k_{mz})e^{ik_{mz}z_m'} + R^{TM}_{\cup m}e^{2ik_{mz}h_m}\,\hat v(k_{mz})e^{-ik_{mz}z_m'}\right]\Big\} \quad (2.43)$$

At the boundary $z_l = 0,\ \ z_{(l-1)} = h_{(l-1)}$, having

$$\hat z\times\overline{\overline{G}}_{lm} = \hat z\times\overline{\overline{G}}_{(l-1)m} \quad (2.44)$$

So, equality of the TE part gives

$$X^{TE}_{\cap l,m}\left[1+R^{TE}_{\cap l}\,e^{2ik_{lz}h_l}\right] = X^{TE}_{\cap(l-1),m}\left[e^{ik_{(l-1)z}h_{(l-1)}}+R^{TE}_{\cap(l-1)}e^{ik_{(l-1)z}h_{(l-1)}}\right] \quad (2.45)$$

which gives

$$X^{\mathrm{TE}}_{\cap l,m} = X^{TE}_{\cap(l-1),m}\,e^{ik_{(l-1)z}h_{(l-1)}}\frac{\left(1+R^{TE}_{\cap(l-1)}\right)}{\left(1+R^{TE}_{\cap l}\,e^{2ik_{lz}h_l}\right)} \quad (2.46a)$$

where $l = m+2,\ m+3,\ldots,t$.

For $l = m+1$, it can be easily shown that

$$X^{TE}_{\cap(m+1),m} = \frac{\left(1+R^{TE}_{\cap m}\right)}{\left(1+R^{TE}_{\cap(m+1)}e^{2ik_{(m+1)z}h_{(m+1)}}\right)} \quad (2.46b)$$

Similarly, for $X^{TM}_{\cap l,m}$

$$\frac{k_{lz}}{k_l}X^{TM}_{\cap l,m}\left(1-R^{TM}_{\cap l}\,e^{2ik_{lz}h_l}\right) = \frac{k_{(l-1)z}}{k_{(l-1)}}X^{TM}_{\cap(l-1),m}\,e^{ik_{(l-1)z}h_{(l-1)}}\left(1-R^{TM}_{\cap(l-1)}\right) \quad (2.47)$$

which gives

$$X^{TM}_{\cap l,m} = \frac{k_l}{k_{lz}}\frac{k_{(l-1)z}}{k_{(l-1)}}X^{TM}_{\cap(l-1),m}\,e^{ik_{(l-1)z}h_{(l-1)}}\frac{\left(1-R^{TM}_{\cap(l-1)}\right)}{\left(1-R^{TM}_{\cap l}\,e^{2ik_{lz}h_l}\right)}$$

(2.48a)

where $l = m+2,\ m+3,\ldots,\ t$

For $l = m+1$, we have

$$X^{TM}_{\cap(m+1),m} = \frac{k_{(m+1)}}{k_{(m+1)z}}\frac{k_{mz}}{k_m}\frac{\left(1-R^{TM}_{\cap m}\right)}{\left(1-R^{TM}_{\cap(m+1)}e^{ik_{(m+1)z}h_{(m+1)}}\right)}$$

(2.48b)

So, the transmission coefficient $X^{S}_{\cap l,m}$ is calculated recursively starting from the source layer with

$X^{S}_{\cap m,m} = 1$ and proceeding downwards.

It is clear that $\overline{\overline{G}}_{tm}(\bar{r},\bar{r}')$ can be obtained from Eq. 2.43 where $l = m$ and $R^{\mathrm{S}}_{\cap t} = 0$.

Also, $\overline{\overline{G}}_{t0}(\bar{r},\bar{r}')$ can be obtained from Eq2.43 where $m = 0$ and $R^{\mathrm{S}}_{\cup 0} = 0$.

2-4-3The Upward Transmission Coefficient $X^{\mathrm{S}}_{\cup l,m}$

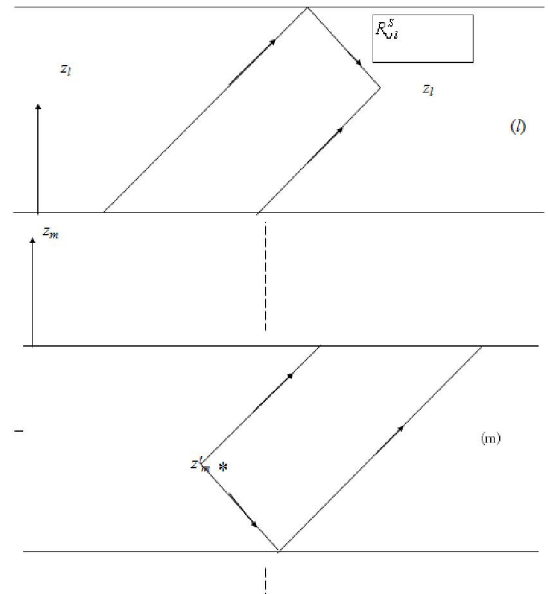

Figure 2.11, Source and Observation points in two different layers ($z>z'$) $z_l = z+d_l, z_m = z+d_{(m-1)}$

The geometry of the problem is shown in Figure 2.11 where sources exist in a layer below that of the observation point. In this case, the local coordinates have been chosen at the boundaries most near to each other. Assuming that waves transmitted from the sources in layer (m) together with multiple reflections propagate upward and enter layer (l) with transmission coefficient $X^{S}_{\cup l,m}$ which will be called upward transmission coefficient.

Using Figure 2.11 than

$$\overline{\overline{G}}_{lm}(\bar{r},\bar{r}') = \frac{i}{8\pi^2}\int\int_{-\infty}^{\infty} d\bar{k}_s\; e^{i\bar{k}_s\cdot(\bar{r}_s-\bar{r}_s')}\frac{1}{k_{mz}}$$

$$\Big\{ r^{TE}_m X^{TE}_{\cup l,m}\left[\hat{h}(k_{lz})e^{ik_{lz}z_l}+R^{TE}_{\cup l}\,e^{2ik_{lz}h_l}\;\hat{h}(-k_{lz})e^{-ik_{lz}z_l}\right]$$

$$\left[\hat{h}(k_{mz})e^{-ik_{mz}z'_m}+R^{TE}_{\cap m}\,e^{2ik_{mz}h_m}\;\hat{h}(-k_{mz})e^{ik_{mz}z'_m}\right]$$

$$+$$

$$r^{TM}_m X^{TM}_{\cup l,m}\left[\hat{v}(k_{lz})e^{ik_{lz}z_l}+R^{TM}_{\cup l}\,e^{2ik_{lz}h_l}\;\hat{v}(-k_{lz})e^{-ik_{lz}z_l}\right]$$

$$\left[\hat{v}(k_{mz})e^{-ik_{mz}z'_m}+R^{TM}_{\cap m}\,e^{2ik_{mz}h_m}\;\hat{v}(-k_{mz})e^{ik_{mz}z'_m}\right]\Big\}$$

(2.49)

To find the recurrence relations for $X^{S}_{\cup l,m}$, the expression can be written for $\overline{\overline{G}}_{(l+1)m}$ and apply the boundary conditions at $z_l = 0$. In this case, getting

$$X_{\cup l,m}^{TE} = X_{\cup(l+1),m}^{TE}\; e^{ik_{(l+1)z}h_{(l+1)}}\; \frac{\left(1+R_{\cup(l+1)}^{TE}\right)}{\left(1+R_{\cup l}^{TE}\, e^{2ik_{lz}h_l}\right)} \qquad (2.50a)$$

where $l = (m-2), (m-3),..., 0$.

For $l = m-1$,

$$X_{\cup(m-1),m}^{TE} = \frac{\left(1+R_{\cup m}^{TE}\right)}{\left(1+R_{\cup(m-1)}^{TE} e^{2ik_{(m-1)z}h_{(m-1)}}\right)} \qquad (2.50b)$$

Similarly, for $X_{\cup l,m}^{TM}$

$$X_{\cup l,m}^{TM} = X_{\cup(l+1),m}^{TM}\; \frac{k_l}{k_{lz}}\, \frac{k_{(l+1)z}}{k_{(l+1)}}\, e^{ik_{(l+1)z}h_{(l+1)}}\; \frac{\left(1-R_{\cup(l+1)}^{TM}\right)}{\left(1-R_{\cup l}^{TM}\, e^{2ik_{lz}h_l}\right)}$$

(2.51a)

where $l = (m-2), (m-3),..., 0$. For $l = m-1$, we have

$$X_{\cup(m-1),m}^{TM} = \frac{k_{(m-1)}}{k_{(m-1)z}}\, \frac{k_{mz}}{k_m}\, \frac{\left(1-R_{\cup m}^{TM}\right)}{\left(1-R_{\cup(m-1)}^{TM}\, e^{2ik_{(m-1)z}h_{(m-1)}}\right)}$$

(2.51b)

It is clear that the dyadic Green's functions $\overline{\overline{G}}_{0m}$, $\overline{\overline{G}}_{0t}$ and $\overline{\overline{G}}_{lt}$ can be obtained as special cases of Eq. 2.49.

3. Conclusion

A solution of the canonical problem of the electromagnetic radiation by current point in the presence of horizontally stratified anisotropic medium has been constructed.

This solution is developed in terms of the well known vertical (or z-propagation) plane wave spectrum integral representation for the EM fields. The fields can be expressed in a compact form in terms of the dyadic Green's function for this problem

4. Discussions

A unified general approach to the problem of radiation of arbitrary sources in a stratified medium is presented. The model of the medium is considered to consists of N horizontally stratified layers and an upper half-space. First, all layers are assumed to be isotropic, then the more general case of a uniaxial medium is considered where all layers posses both tensor permittivities and tensor permeabilities which in general can be complex. All axes of anisotropy are considered perpendicular to the boundaries separating the different layers.

Starting by defining two types of dyadic Green's functions which are dual to each other, namely an electric type dyadic Green's function $\overline{\overline{G}}$ and a magnetic type dyadic Green's function $\overline{\overline{\Gamma}}$ (Stratton, 1941). These Green's functions are resolved into their TE and TM parts. The resulting integrals are expressed in cartesian coordinates in terms of Weyl-type integral. A simple procedure to obtain the fields in any arbitrary layer is described. Tractable forms are shown to be easily deduced from the physical picture of the waves radiated from the primary sources and the multiple reflections from the stratified medium. The dyadic Green's function in the field region is properly represented by extracting the delta function singularity. Recursion relations for appropriately defined reflection and transmission coefficients are presented.

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