Moment Inequality for Exponential Better than Used in Convex Average Class of Life Distributions with Hypothesis Testing Application

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Abstract: The exponential better than used in convex average EBUCA class of life distribution is considered. A moment inequality is derived for EBUCA distributions which demonstrate that if the mean life is finite, then all moments exist. Based on this inequality, a new test statistic for testing exponentiality against EBUCA is introduced. It is shown that the proposed test is simple, enjoys good power and has high relative efficiency for some commonly used alternatives. Critical values are tabulated for sample sizes $n = 5(1)40.$ A sets of real data is used as a practical applications of the proposed test in the medical science.

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1. Introduction and Definitions

The aging life is usually characterized by non-negative random variable $X \geq 0$ with distribution function (cdf), F and survival function (sf), $\bar{F} = 1 - F$. Associated with X is the notion "random remaining life" at age t, denoted by X_t where X_t has sf as

$$\bar{F}_t(x) = \frac{\bar{F}(x+t)}{\bar{F}(t)}, \quad x, t \geq 0 \qquad (1.1)$$

Note that $X_t \overset{st}{\leq} X$, of $\bar{F}_t(x) \leq \bar{F}(x)$ (st denote the stochastic ordering) if and only if $\bar{F}$ is an exponential distribution. Comparing X and X_t in various forms and types create classes of aging useful in many biomedical, engineering and statistical studies, see Barlow and Proschan (1981). It is well known that the relation $X_t \overset{st}{\leq} X$ or $\bar{F}_t(x) \leq \bar{F}(x)$ defines the class of new better than used (NBU).

On the other hand, the relation $E(X_t) \leq E(X)$ defines the class of new better than used in expectation (NBUE), harmonic new better than used in expectation (HNBUE), decreasing mean residual lifetime (DMRL), exponential better than used (EBU) and exponential better than used in convex (EBUC).

Many test statistics have been developed for testing exponentiality against various aging alternatives. Testing exponentiality against the classes of life distribution has received a good deal of attention. For testing against new better than used (NBU) we refer to Hollander and Proschan (1972) and Koul (1977) among others. For testing against new better than used in expectation (NBUE), we refer to Hollander and Proschan (1975) and Ahmad *et al.* (1999) among others. For harmonic new better than used in expectation (HNBUE), we refer to Klefsjo (1982) and Hendi *et al.* (1998). For decreasing mean residual lifetime (DMRL), we refer to Hollander and Proschan (1975), Ahmad and Li (1992) among others. For exponential better than used (EBU), we refer to Hendi *et al.* (2005). And For testing against exponential better than used in convex (EBUC), we refer to Hendi and ALghufily (2009).

Moments inequalities for some classes of life distribution have been appeared in beginning of this decade and hence have been used in testing. Ahmad (2001) used the new testing against IFR, NBU, and NBUE. Testing against IFRA, NBUC and DMRL based on moments inequality have been studied by Ahmad and Mugdadi (2002). Abu-Youssef (2002) used the same technique for testing against DMRL. For testing against HNBUE see AL-Ruzaiza *et al.* (2003). Elbatal (2009) used the moments inequality for testing against RNBRU. And for testing against EBELC see Abdul-Moniem (2011).

The classes EBUC and EBUCA may be defined on basis of a variability definitions due to ALghufily (2008), which is the following:

Definition (1.1): F belongs to EBUC iff

$$\int_{x+t}^{\infty} \bar{F}(u)\,du \leq \mu \bar{F}(t) e^{x/\mu}, \quad x, t > 0 \quad (1.2)$$

Using the above definition to introduce exponential better than used in convex average order (EBUCA).

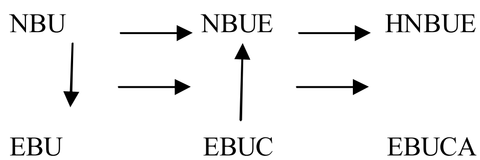
Definition (1.2): F belongs to EBUCA iff

$$\int_0^{\infty}\int_{x+t}^{\infty} \bar{F}(u)\,du\,dx \leq \mu \bar{F}(t) \int_0^{\infty} e^{x/\mu} dx, \quad x, t > 0 \quad (1.3)$$

Putting $v(x+t)=\int_{x+t}^{\infty}\overline{F}(u)\,du$. then (1.3) becomes

$$\int_0^{\infty} v(x+t)\,dx \le \mu^2 \overline{F}(t), \qquad (1.4)$$

The implication among the above classes of life distributions are:



The purpose of this paper is to give a moment inequality for the EBUCA class. The moments developed in section 2 are used to construct test statistics for the class EBUCA in section 3. In that section we obtained Monte Carlo null distribution critical points for sample sizes 5(1)40. This test statistic has exponentially high efficiencies and power for some of the well known alternatives relative to other tests. Finally in section 4 we apply the proposed test to real practical data in medical science.

2. Moment Inequality

We state and prove the following result.

Theorem 2.1 if F is EBUCA, then

$$\frac{\mu_{r+3}}{(r+2)(r+3)} \le \mu^2 \mu_{r+1}, \qquad (2.1)$$

where

$$\mu_{(r)} = E(X^r) = r\int_0^{\infty} x^{r-1}\,\overline{F}(x)dx \qquad (2.2)$$

Proof:

F is EBUCA, then (1.4) holds. Multiplying both sides in (1.4) by t^r, r>0 and integrating over (0,∞) with respect to t, then

$$\int_0^{\infty}\int_0^{\infty} t^r v(x+t)dxdt \le \mu^2 \int_0^{\infty} t^r\,\overline{F}(t)dt \qquad (2.3)$$

Using integrating by parts, the left hand said (L.H.S) of (2.3) will be

$$L.H.S = \int_0^{\infty}\int_0^{\infty} t^r v(x+t)dxdt$$

Let y=x+t, and z=t, we get

$$L.H.S = \int_0^{\infty}\int_0^{y} v(y)\,z^r dz\,dy$$

$$= \frac{1}{r+1}\int_0^{\infty} v(y)\,y^{r+1}\,dy$$

Hence $v(y) = \int_y^{\infty}\overline{F}(u)du = E[(Y-y)I(Y>u)],$ thus L.H.S may be written as follows:

$$L.H.S = \frac{1}{r+1}E\int_0^{\infty}(Y-y)\,y^{r+1}\,dy$$

$$= \frac{1}{r+1}E[\frac{Y^{r+3}}{r+2} - \frac{Y^{r+3}}{r+3}]$$

$$= \frac{1}{r+1}E(Y^{r+3})[\frac{1}{r+2} - \frac{1}{r+3}]$$

i.e

$$L.H.S = \ = \frac{\mu_{r+3}}{(r+1)(r+2)(r+3)} \qquad (2.4)$$

Also, the right- hand side of (2.3) is equal to

$$R.H.S = \mu^2 \int_0^{\infty} t^r\,\overline{F}(t)dt,$$

Thus

$$R.H.S = \mu^2 \frac{\mu_{r+1}}{r+1} \qquad (2.5)$$

Substituting by (2.4)and (2.5) in (2.3), hence the result (2.1) now follows.

3. Testing The EBUCA Class

3.1 Test Procedure

Let $X_1, X_2, ..., X_n$ represent a random sample from a population with distribution F. We wish to test the null hypothesis H_0 : F is exponential with mean μ against H_1 :F is EBUCA and not exponential. Using theorem (2.1), we may use the following as a measure of departure from H_0 in favor of H_1:

$$\delta_E = \mu^2 \mu_{r+1} - \frac{\mu_{r+3}}{(r+2)(r+3)}, \qquad (3.1)$$

Note that under $H_0 : \delta_E = 0$, while under $H_1 : \delta_E > 0$. Thus to estimate δ_E by $\hat{\delta}_E$, let $X_1, X_2, ..., X_n$ be a random sample from F and μ is estimated by $\overline{X}$, where $\overline{X} = \frac{1}{n}\sum X_i$ is the usual sample mean. Then $\hat{\delta}_E$ is given by using (3.1) as

$$\hat{\delta}_{E_n} = \frac{1}{n(n-1)...(n-r-2)} \sum_{i=1}^{n}\sum_{j=1}^{n}\left[X_i^{r+1} \prod_{k=1}^{2} X_{j_k} - \frac{X_i^{r+3}}{(r+2)(r+3)} \right]$$
(3.2)

But

$$\hat{\Delta}_{E_n} = \frac{\hat{\delta}_{E_n}}{\bar{X}^{r+3}} \quad (3.3)$$

Setting

$\phi(X_i, X_{j_1}, X_{j_2}) = X_i^{r+1} X_{j_1} X_{j_2} - \frac{X_i^{r+3}}{(r+2)(r+3)}$, then $\hat{\Delta}_{E_n}$ in (3.3) is a classical U-statistic, cf. Lee (1990). The following theorem summarizes the large sample properties of $\hat{\Delta}_{E_n}$.

Theorem 3.1. As $n \to \infty, \sqrt{n}(\hat{\Delta}_{E_n} - \Delta_E)$, is asymptotically normal with mean 0 and variance

$$\sigma^2 = \mu^{-2r-6} Var\left\{ X_1^{r+1}\mu^2 + 2X_1\mu\mu_{r+1} - \frac{X_1^{r+3}}{(r+2)(r+3)} - 2\frac{\mu_{r+3}}{(r+2)(r+3)} \right\},$$
(3.4)

Under $H_0 : \Delta_E = 0,$ the variance σ_0^2 is given by

$$\left[\sigma_0^2 = (2r+2)! + 4(r+2)(r+1)! + \frac{(2r+6)! + 8[(r+3)!]^2}{(r+2)^2(r+3)^2} - \frac{2(2r+4)!}{(r+2)(r+3)} - (20+4r)[(r+1)!]^2 \right] \quad (3.5)$$

Proof:

Since $\hat{\Delta}_{E_n}$ and $\frac{\hat{\delta}_{E_n}}{\mu^{r+3}}$ have the same limiting distribution, we use $\sqrt{n}(\hat{\Delta}_{E_n} - \Delta_E)$ Now this is asymptotically normal with mean 0 and variance $\sigma^2 = Var[\phi(X_1)]$, where

$$\phi(X_1) = E\left(\phi(X_1, X_{2_1}, X_{2_2})/X_1\right) + E\left(\phi(X_{2_1}, X_1, X_{2_2})/X_1\right) + E\left(\phi(X_{2_2}, X_{2_1}, X_1)/X_1\right)$$
(3.6)

Note that

$$E[\phi(X_1, X_{2_1}, X_{2_2})/X_1] = X_1^{r+1}\mu^2 - \frac{X_1^{r+3}}{(r+2)(r+3)}$$
(3.7)

$$E[\phi(X_{2_1}, X_1, X_{2_2})/X_1] = X_1\mu\mu_{r+1} - \frac{\mu_{r+3}}{(r+2)(r+3)}$$
(3.8)

And

$$E[\phi(X_{2_2}, X_{2_1}, X_1)/X_1] = X_1\mu\mu_{r+1} - \frac{\mu_{r+3}}{(r+2)(r+3)}$$
(3.9)

Set $\phi(X_1)$ to be the sum of the right hand side of (3.7),(3.8) and that of (3.9). Thus

$$\phi(X_1) = X_1^{r+1}\mu^2 + 2X_1\mu\mu_{r+1} - \frac{X_1^{r+3}}{(r+2)(r+3)} - 2\frac{\mu_{r+3}}{(r+2)(r+3)}$$
(3.10)

Then (3.4) follows.

Under H_0

$$\phi(X_1) = X_1^{r+1} + 2X_1(r+1)! - \frac{X_1^{r+3}}{(r+2)(r+3)} - 2(r+1)!$$
(3.11)

Hence (3.5) follows. The Theorem is proved.

When r = 0,

$$\delta_{E1} = \mu^3 - \frac{1}{6}\mu_3 \quad (3.12)$$

In this case $\sigma_0^2 = 10$, and the test statistic is and

$$\hat{\delta}_{E_{1n}} = \frac{1}{n(n-1)(n-2)} \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\left[X_i X_j X_k - \frac{X_i^3}{6} \right]$$
(3.13)

$$\hat{\Delta}_{E_{1n}} = \frac{\hat{\delta}_{E_{1n}}}{\bar{X}^3} \quad (3.14)$$

3.2 Monte Carlo Null Distribution Critical values

In this section we use Monte Carlo simulation method to find the critical points of the measure given in (3.14) for sample sizes 5(1)40 and based on 1000 replications. It is clear from Table (3.1) that the critical values change slowly as n increases and behave like normal for large sample size. Also, the critical values in Table (3.1) increases as the level of significant increases.

3.3 The Power estimates

We calculate the power estimate of the statistic $\hat{\Delta}_{E_{1n}}$ in (3.14) at level 95% upper percentile and for following alternatives distributions:

i) The linear failure rat family:

$$\bar{F}_1(x) = \exp\left(-x - \frac{\theta}{2}x^2\right), \quad \theta > 0,\ x \geq 0$$

The Makham family:

$$\bar{F}_2(x) = \exp[-x - \theta(x + e^{-x} - 1)], \quad \theta > 0,\ x \geq 0$$

The Weibull family:

$$\bar{F}_3(x) = \exp(-x^{\theta}), \quad \theta > 0,\ x \geq 0$$

All these alternative distributions are IFR (for an appropriate restriction on θ) they all belong to wider class. Moreover, all these distribution reduce to exponential distribution for (i) and (ii) when the value $\theta = 0$ and for (iii) when the value $\theta = 1$. Table (3.2) displays the power estimate for $\hat{\Delta}_{E_{1n}}$ test statistic with respect to these distributions. The estimates are based on1000 simulated samples of size 10,20 and 30 at level 95% upper percentile.

Table (3.1): Critical values of $\hat{\Delta}_{E_{1n}}$

n	1%	5%	10%	90%	95%	99%
5	-1.83238	-0.32672	-0.03550	1.77920	2.82063	5.95206
6	-2.35108	-0.40027	-0.08074	1.85956	2.56796	5.71912
7	-2.25305	-0.46969	-0.12102	1.31544	1.79062	4.23534
8	-2.09101	-0.56434	-0.15649	1.33548	1.94028	3.44891
9	-2.54397	-0.62547	-0.18200	1.18529	1.66927	2.76958
10	-3.59582	-0.63582	-0.28096	1.01725	1.49677	3.40802
11	-2.53083	-0.79468	-0.28323	1.01864	1.47210	2.60973
12	-2.23502	-0.54662	-0.20208	0.87189	1.37917	2.25489
13	-2.37739	-0.87166	-0.43448	0.93298	1.37352	2.20668
14	-2.35406	-0.71336	-0.31486	0.88554	1.22039	2.28772
15	-2.80908	-0.76775	-0.30584	0.90328	1.11454	1.93806
16	-2.42074	-0.89948	-0.40881	0.83246	1.16078	1.94603
17	-2.30671	-0.87638	-0.39452	0.83846	1.19083	1.92082
18	-2.19052	-0.67124	-0.29398	0.75374	1.08480	2.08989
19	-2.79121	-0.73590	-0.35234	0.69678	0.95742	1.81973
20	-2.66603	-0.91918	-0.42214	0.74602	0.95603	1.48212
21	-2.07895	-0.76676	-0.36460	0.73970	0.91138	1.48607
22	-2.00371	-0.83233	-0.38720	0.65043	0.84561	1.39481
23	-2.42023	-0.96643	-0.42168	0.66727	0.90187	1.48137
24	-2.43669	-0.83594	-0.36578	0.58519	0.76914	1.30803
25	-3.09942	-0.68886	-0.39708	0.64729	0.84751	1.26803
26	-1.80176	-0.76338	-0.39321	0.64028	0.85413	1.24436
27	-2.13363	-0.86304	-0.41420	0.60959	0.81524	1.41560
28	-2.02121	-0.77687	-0.43473	0.55761	0.71666	1.33047
29	-2.15574	-0.78476	-0.39013	0.65616	0.84376	1.31038
30	-1.73304	-0.63646	-0.35049	0.58457	0.76211	1.07752
31	-1.90722	-0.67857	-0.35822	0.57278	0.73072	1.12261
32	-2.58974	-0.76264	-0.38828	0.59514	0.74379	1.12293
33	-2.61048	-0.81816	-0.42122	0.53567	0.67144	0.95865
34	-1.95412	-0.71222	-0.39667	0.54671	0.68720	1.11895
35	-2.08193	-0.78461	-0.40382	0.50379	0.63482	0.95667
36	-1.91167	-0.74702	-0.43382	0.55387	0.73678	1.00468
37	-1.52007	-0.66158	-0.32030	0.54067	0.68728	0.97243
38	-1.86049	-0.69478	-0.41492	0.50313	0.66101	1.05507
39	-1.60343	-0.76881	-0.41749	0.50826	0.62593	1.12496
40	-1.93253	-0.73665	-0.44209	0.47925	0.60808	0.92844

Table (3.2): power estimate for $\hat{\Delta}_{E_{1n}}$

Distribution	θ	Sample size		
		n=10	n=20	n=30
F_1:Linear failure rat	2	0.784	0.919	0.905
	3	0.844	0.938	0.928
	4	0.852	0.954	0.964
F_2 :Makham	2	0.527	0.589	0.490
	3	0.566	0.616	0.552
	4	0.597	0.658	0.572
F_3 :Weibull	2	0.993	1.000	1.000
	3	1.000	1.000	1.000
	4	1.000	1.000	1.000

From above table it noted that the power of the test increases by increasing the value of the parameter θ and the sample size n as it was expected.

3.4 Pitman asymptotic efficiency

In this section we calculate the Pitman asymptotic efficiency (PAE) of our test $\hat{\Delta}_{E_n}$ for the three alternatives (i),(ii) and (iii) presented above. The efficiency of δ_E is equal to $\frac{1}{\sigma_0}(\frac{\partial}{\partial\theta}\delta_E\ )_{\theta=\theta_0}$.

Not that

$$\frac{\partial}{\partial\theta}\delta_E\bigg|_{\theta\to\theta_0} = \frac{1}{\mu_{\theta_0}^{r+3}}\left[\mu'_{r+1_{\theta_0}}\mu^2{}_{\theta_0} + 2\mu_{\theta_0}\mu'_{\theta_0}\mu_{r+1_{\theta_0}} - \frac{\mu'_{r+3_{\theta_0}}}{(r+2)(r+3)}\right] \quad (3.15)$$

We compare our test to other classes. Here we choose the test δ presented by Abdul-Moniem (2011) for EBELC class. The comparison is achieved by using Pitman asymptotic relative efficiency (PARE) which can be defined as follows:

Let T_{1n} and T_{2n} be two test statistics for testing $H_0: F_\theta \in \{F_{\theta_n}\}\theta_n = \theta + cn^{-1}$, where c an arbitrary constant, then the asymptotic relative efficiency of T_{1n} relative to T_{2n} can be defined as:

$$e(T_{1n}, T_{2n}) = \frac{\{\mu'_1(\theta_0)/\sigma_1(\theta_0)\}}{\{\mu'_2(\theta_0)/\sigma_2(\theta_0)\}},$$

Where $\mu'_i(\theta_0) = \left\{\lim_{\theta\to\infty}[\frac{\partial}{\partial\theta}E(T_{in})]\right\}_{\theta=\theta_0}$ and

$\sigma_i(\theta_0) = \lim_{n\to\infty} Var(T_{in}), i = 1,2$ is the null variance.

Table (3.3) contains PAE's of the above alternatives (i),(ii) and (iii) by using (3.15). Also we give PARE of $\hat{\Delta}_{E_n}$ and δ

Table (3.3): Efficiencies of $\hat{\Delta}_{E_n}$ and δ

Distribution	$\hat{\Delta}_{E_n}$	Δ	$e(\hat{\Delta}_{E_n}, \delta)$
F_1:Linear failure rat	1.581	0.949	1.666
F_2:Makham	0.257	0.198	1.298
F_3:Weibull	1.054	0.791	1.332

The Efficiencies in Table (3.3), shown clearly that our test statistic $\hat{\Delta}_{E_n}$ perform well for F_1, F_2 and F_3 than the procedure $\hat{\Delta}_{E_n}$ of Abdul-Moniem (2011) and more efficient.

4. Applications

Example 1:

The following data represent 39 liver cancer's patients taken from El Minia Cancer Center Ministry of Health in Egypt [see Attia et al. (2004)]. The ordered life times (in day) are: 10, 14, 14, 14, 14, 14, 15, 17, 18, 20, 20, 20, 20, 20, 23, 23, 24, 26, 30, 30, 31, 40, 49, 51, 52, 60, 61, 67, 71, 74, 75, 87, 96, 105, 107, 107, 107, 116, 150.

It is found that the $\hat{\Delta}_{E_{1n}}$ test statistic for the set data by using (3.14) is 0.4796 which is less than the critical value in Table (3.1) at 95% upper percentile. Thus we reject H_1 which states the data set has EBUCA property.

Example 2:

We calculate the $\hat{\Delta}_{E_{1n}}$ test statistic for the data set of 40 patients suffering from blood cancer (Leukemia) from one of Ministry of Health in Saudi Arabia [see Aboummah *et al.* (1994)]. The ordered life times (in day) are: 115, 181, 255, 418, 441, 461, 516, 739, 743, 789, 807, 865, 924, 983, 1024, 1062, 1063, 1165, 1191, 1222, 1222, 1251, 1277, 1290, 1357, 1369, 1408, 1455, 1478, 1549, 1578, 1599, 1603, 1605, 1607, 1696, 1735, 1799, 1815, 1852.

From the above set of data, we have the computed values of $\hat{\Delta}_{E_{1n}}$ given (3.14) is 0.7517 which is greater than the critical value in Table (3.1) at 95% upper percentile. Thus we accept H_1 which states the data set has EBUCA property.

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