A New Approach for Geometric Properties of DNA Structure in E^3

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Abstract: In this paper, we research the geometric properties of the double-helix of DNA for Bishop frame in 3-dimensional Euclidean space. In addition, Bishop frame vector fields of strand-helix and Bishop curvatures are found. Future we obtained the circle of curvature or osculating circle and the axis curvature. Moreover, we introduced Bishop spherical images and give some interesting corollaries.

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1. Introduction

Gregor Mendel, an Austrian monk, was the first one who expressed the principles of heredity in the 1860s. He defined the laws of homogenous and heterogenous inhertance as dominant and recessive traits. After that time, in 1946 deoxyribonucleic acid (DNA) in bacteria was shown to carry hereditary information by Edward Tatum and Joshua Lederberg at Yale University [15]. James Watson and Francis Crick in Cambridge worked on the structure of DNA that is based on the parameters provided by Maurice Wilkins and Rosalind Franklin and obtained with X ray crystallography. Ultimately they suggested a helical model of DNA in 1953 [31-35]. DNA is a critically important part of cells. DNA molecule is the material of the genes responsible for coding the genetic massages which are vital for the cellular functions. Each molecule of DNA has a deoxyribose backbone in which each deoxyribose is attached in order to one of four nucleic acids: Adenine or guanine, thymidine or cytosine. The basic block of DNA is a nucleodide, composed of three major parts: A deoxyribose sugar, a phosphate group and nucleic acid base [36-40]. The strands of DNA as two seperate parts are twisted around each other in clockwide direction, thus forming a right handed double helix with the nucleic acids on the inside and nuclear bases invariably paired by hydrogen bonds, adenine with thymidine and cytosine with guanine [41]. The DNA molecule which is actually microns in size measures nearly 2m long when stretched out. It is hard to imagine how such a long molecule fits into a cell, even into nucleus of a cell. Watson and Crick also proposed a tightly coiled two stranded helix as an explanation of this event. The cellular DNA is extremely compacted to form supercoils as a tertiary configuration. The formation of supercoils results in noteworthy consequences. Firstly, a means of condensing the large DNA into the cellular nucleus is possible. Secondly, supercoiling process takes a part during transcription and replication process of a cell [28]. Lastly, supercoils are one step in the process of DNA catenation or knotting [30]. The curving of DNA strand is called supercoiling that has two type [3,17]. The first of these, known as an interwound supercoil (or plectoneme), leads to an interwoven structure where the strand wraps upon itself with many sites of apparent self-contact. The second of these solenoidal supercoil which is similar to a coiled spring or telephone cable possesses no self-contact. There are some enzymatic reactions on the DNA molecule. One is DNA replication accomplished by DNA polymerase. Others are the effects of DNA topoisomerases and gyrase influencing the cellular functions [7,27,29].

Most remarkable deformation incident function on a scale at which the internal double-helix of DNA is inapplicable and a long strand reacts as though it were a selender elastic rod or fibre. Calladine and Drew described its thickness and length by extending it great number of times [32]. Many biologists and mathematicians have studied on the mechanics of a long elastic fibre modeling a single DNA molecule [42-43]. The results may light the way for molecular biologist to understand and control the spatial writhing of a molecule.

What attracts biologists and mathematicians is the supercoiling of DNA that is long fibre representing which the double helix adopts a configuration. Coleman and Swigon discovered that the helical angle varies along the ply and at the un-looped and there is a visible separation followed by a discrete point contact [8]. A play is formed by the interwound configuration of this rubber rod. One can observe a ply physically by twisting a long rubber rod of circular cross-section. By imposing the twist, the ends join in each other, the rod bends locally and it forms ply-plus-loop. Present authors and others [43-46] have made extensive experimental and theorotical studies of initial buckling and localized post bucling and they prior to self contact. The study of Thomson may be used in the static dynamic analogy and it has been covered rods of circular and non circular cross-section [24]. Meantime Maddoks and co-workers have studied the symmetry and bifurcation properties of closed but non-contacting rod [16]. Closed loop which is often formed of a molecule of DNA is called a plasmid. Calladine-Drew reproduced an electron micrograpy of a plasmid which shows negatively supercoiled, interwound DNA as got from E.coli bacteria [47].

The paper is organized as follows. Section 2 we give basic concepts, §3 we call attention to mathematical of DNA structure and introduces geometric properties of the double helix structure of DNA in 3-dimensional space for Bishop frame. In §4 gives a spherical images of a double helix structure of DNA molecule for Bishop frame.

2. Basic Concept

The Frenet frame is constructed for the curve of 3-time continuously differentiable non-degenerate curves. Curvature can equal to zero at some point on the curve. That is; the second derivative of the curve may vanish. In this situation, we can use an alternative frame. We use the tangent vector and two relatively parallel vector fields to construct this alternative frame such that the normal vector field along the curve is relatively parallel if its derivative is tangential. We call this frame a parallel frame along the reason for the name parallel is because the normal component of the derivatives of the normal vector field is zero. The advantages of the parallel frame and the comparable parallel frame with the Frenet frame in Euclidean 3-space was given and studied in [48-49].

In three dimensional Euclidean space {T,N,B} denote Frenet -Serret frame along the curve α.For an arbitrary curve α with first and second curvature, κ and τ in E^3, the following Frenet-Serret equations is given in [10].

$$\begin{bmatrix} T' \\ N' \\ B' \end{bmatrix} = \begin{bmatrix} 0 & k_1 & 0 \\ -k_1 & 0 & k_2 \\ 0 & -k_2 & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}$$

for a parametrized unit length curve α. Curvature functions are defined by

$$\kappa = \kappa(s) = \|T'(s)\| \quad \text{and} \quad \tau(s) = -\langle N, B' \rangle$$

Torsion of the curve α is given

$$\tau = \frac{\det(\alpha', \alpha'', \alpha''')}{\kappa^2} \tag{1}$$

The basic concept of this frame is to take the unique tangent vector and choose any convenient basis in the plane perpendicular to at each point such that the derivatives of depend on only and not each other. A parallel frame is not unique, in contrast to a Frenet frame. In three dimensional Euclidean space the alternative parallel frame equations are

$$\begin{bmatrix} T' \\ N_1' \\ N_2' \end{bmatrix} = \begin{bmatrix} 0 & k_1 & k_2 \\ -k_1 & 0 & 0 \\ -k_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} T \\ N_1 \\ N_2 \end{bmatrix} \tag{2}$$

for a parametrized unit length curve. The relation between Frenet Frame and Bishop Frame is as follows

$$\begin{bmatrix} T \\ N \\ B \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} T \\ N_1 \\ N_2 \end{bmatrix}$$

and the relation Frenet curvature is as follows

$$k_1 = \kappa\cos\theta$$

$$k_2 = \kappa\sin\theta$$

$$\kappa = \sqrt{k_1^2 + k_2^2}$$

also

$$\theta(s) = \arctan\left(\frac{k_2}{k_1}\right), \quad \tau(s) = \frac{d\theta(s)}{ds}$$

so that k_1 and k_2 correspond to a Cartesian Coordinate system for the polar coordinates κ,τ with $\theta = \int \tau(s)ds$. The functions $k_1(s)$, $k_2(s)$ are called Bishop curvatures. Thus, the relation between Frenet frame and Bishop frames can be written as

$$T = T$$

$$N_1 = N\cos\theta - B\sin\theta$$

$$N_2 = N\sin\theta + B\cos\theta$$

It is well known that for a unit speed curve with non vanishing curvatures the following propositions hold [10,14].

Proposition 1. Let α be a regular curve with curvature κ and τ. α is a general helix if and only if

$$\left(\frac{\kappa}{\tau}\right) = \text{constant}.$$

Proposition 2. Let α be a regular curve with curvatures κ and τ. α is a clindrical helix if and only if

$$\sigma = \left[\frac{\kappa^2}{(\kappa^2 + \tau^2)^{\frac{3}{2}}}\left(\frac{\tau}{\kappa}\right)'\right] = 0$$

3.Geometric Properties of DNA Structure

3.1 Geometric Properties DNA According to Frenet Frame

Turgut -Vanlı and Kandıra [26] give the geometric properties of the double helix structure of DNA in 3-dimensional space for Frenet frame. We operate in this idea parallel transport frame which is called Bishop frame and introduces the geometric properties of the double helix structure of DNA for this alternative frame.

Definition 1. Stump et all. described the vector position (path) of strand-helix as follow

$$R(t) = (r\cos t)i + (r\sin t)j + (\cot\beta)tk$$
$$= (r\cos t, r\sin t, (\cot\beta)t)$$

where β is superhelix angle and r is standoff distance [21]. In addition, $x = r\cos t$ and $y = r\sin t$ describe a circle of radius r, but $z = (\cot\beta)t$ increases (or decreases) indirect to t.

Turgut-Vanlı and Kandıra [26] give the mathematical properties of the double helix structure of DNA in 3-dimensional space as follow

(i) The variable arc-length along the strand is

$$s(t) = t\sqrt{(r^2+\cot^2ß)}$$

(ii) The parametrizasyon of the strand in terms of s is

$$R(s)=\begin{pmatrix} r\cos\left(\frac{s}{(r^2+\cot^2ß)}\right), \\ r\sin\left(\frac{s}{(r^2+\cot^2ß)}\right), \\ \cot\beta\left(\frac{s}{(r^2+\cot^2ß)}\right) \end{pmatrix} \quad (3)$$

(iii) The curvature of the strand is a constant and given by

$$\kappa = = \|R''(s)\| = \frac{r}{(r^2+\cot^2ß)}$$

The radius of curvature is given by

$$\rho = \frac{1}{\kappa} = \frac{(r^2+\cot^2ß)}{r}$$

(iv) The unit tangent vector to the curve of strand is given by

$$T(s) = \frac{1}{\sqrt{r^2+\cot^2\beta}}\begin{pmatrix} -r\sin\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right), \\ r\cos\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right), \\ \cot\beta \end{pmatrix}$$

(v) The unit normal vector to the curve of strand is given by

$$N = \left(-\cos\left(\frac{s}{\sqrt{r^2+\cot^2ß}}\right), -\sin\left(\frac{s}{\sqrt{r^2+\cot^2ß}}\right), 0\right)$$

(vi) The unit binormal vector to the curve of strand is given by

$$B(s) = \frac{1}{\sqrt{r^2+\cot^2\beta}}\begin{pmatrix} \sin\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right)\cot\beta, \\ \cos\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right)\cot\beta, \\ r \end{pmatrix}$$

3.2. Geometric Properties DNA According to Bishop Frame

In this section we introduce the geometric properties of the double helix structure of DNA in 3-dimensional space according to Bishop frame as follow.

(i) The unit tangent vector to the curve of strand according to s parameter is given by

$$T(s) = \frac{1}{\sqrt{r^2+\cot^2\beta}}\begin{pmatrix} -r\sin\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right), \\ r\cos\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right), \\ \cot\beta \end{pmatrix}$$

(ii) Let R(s) is vector position of strand-helix. Then N_1 (s) bishop vector to the curve of strand is given by

$$N_1(s)=\begin{pmatrix} -\cos\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right)\cos\theta \\ -\sin\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right)\sin\theta\left(\frac{\cot ß}{\sqrt{r^2+\cot^2\beta}}\right), \\ -\sin\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right)\cos\theta \\ +\cos\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right)\sin\theta\left(\frac{\cot ß}{\sqrt{r^2+\cot^2\beta}}\right), \\ \frac{-r\sin\theta}{\sqrt{r^2+\cot^2\beta}} \end{pmatrix}$$

(iii) Let R(s) is vector position of strand-helix. Then N_2 (s) bishop vector to the curve of strand is given by

$$N_2(s)=\begin{pmatrix} -\cos\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right)\sin\theta \\ +\sin\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right)\cos\theta\frac{\cot\beta}{\sqrt{r^2+\cot^2\beta}}, \\ -\sin\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right)\sin\theta \\ -\cos\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right)\cos\theta\frac{\cot\beta}{\sqrt{r^2+\cot^2\beta}}, \\ \frac{r\cos\theta}{\sqrt{r^2+\cot^2\beta}} \end{pmatrix}$$

(iv) Let R(s) is strand helix curve given with arclenth parameter. Then the Bishop curvatures k_1, k_2 of strand is given by

$$k_1=\frac{r\cos\theta}{r^2+\cot^2\beta}$$

$$k_2=\frac{r\sin\theta}{r^2+\cot^2\beta} \quad (4)$$

Corollary 3. Let R(s) be a strand helix curve of E^3 given with arclenth parameter. Then we can give a relationship θ and k_1, k_2 as follows

$$\theta=\arccos\left(\frac{k_1\left(r^2+\cot^2\beta\right)}{r}\right)$$

or

$$\theta=\arcsin\left(\frac{k_2\left(r^2+\cot^2\beta\right)}{r}\right)$$

(v) Let m be the center of osculating circle of the strand

$$m=R(s)+\rho(s)N(s)$$

Hence we obtain *m* according to Bishop frame

$$m=R(s)+\frac{\cos\theta}{\sqrt{k_1^2+k_2^2}}N_1+\frac{\sin\theta}{\sqrt{k_1^2+k_2^2}}N_2$$

$$=R(s)+\frac{k_1}{k_1^2+k_2^2}N_1+\frac{k_2}{k_1^2+k_2^2}N_2$$

and

$$\rho=\frac{1}{\sqrt{k_1^2+k_2^2}}$$

where

$$k_1=\frac{r\cos\theta}{r^2+\cot^2\beta} \quad \text{and} \quad k_2=\frac{r\sin\theta}{r^2+\cot^2\beta}$$

(vi) Let d_t be the axis curvature of osculating circle of the strand we know that

$$d_t(\mu)=m+\mu B$$

So we obtain d_t according to Bishop frame

$$d_t(\mu)=R(s)+\frac{\cos\theta}{\sqrt{k_1^2+k_2^2}}N_1(s)+\frac{\sin\theta}{\sqrt{k_1^2+k_2^2}}N_2(s)+\mu\left(-\sin\theta N_1(s)+\cos\theta N_2(s)\right)$$

$$=R(s)+\frac{k_1}{k_1^2+k_2^2}N_1(s)+\frac{k_2}{\sqrt{k_1^2+k_2^2}}N_2(s)$$

$$=R(s)+\left(\frac{k_1}{k_1^2+k_2^2}-\mu\sin\theta\right)N_1(s)+\left(\frac{k_2}{k_1^2+k_2^2}+\mu\cos\theta\right)N_2(s)$$

(vii) Let γ(η) be the equation of osculating circle of the strand we know that

$$\gamma(\eta)=R(s)+\rho(s)N(s)+\rho(s)\cos\frac{\eta}{\rho(s)}\left(-N(s)\right)+\rho(s)\sin\frac{\eta}{\rho(s)}T(s)$$

Similary, we obtain γ(η) acording to Bishop frame

$$\gamma(\eta)=R(s)+\frac{k_1}{k_1^2+k_2^2}N_1(s)+\frac{k_2}{\sqrt{k_1^2+k_2^2}}N_2(s)-\frac{1}{\sqrt{k_1^2+k_2^2}}\cos\left(\sqrt{k_1^2+k_2^2}\right)\eta\cos\theta N_1(s)-\frac{1}{\sqrt{k_1^2+k_2^2}}\cos\left(\sqrt{k_1^2+k_2^2}\right)\eta\sin\theta N_2(s)+\frac{1}{\sqrt{k_1^2+k_2^2}}\sin\left(\sqrt{k_1^2+k_2^2}\right)\eta T(s)$$

So we get

$$\gamma(\eta)=R(s)+\frac{1}{k_1^2+k_2^2}\sin\left(\sqrt{k_1^2+k_2^2}\right)\eta T(s)+\frac{1}{k_1^2+k_2^2}\left(k_1-\sqrt{k_1^2+k_2^2}\cos\left(\sqrt{k_1^2+k_2^2}\right)\eta\cos\theta\right)N_1(s)+\frac{1}{k_1^2+k_2^2}\left(k_2-\sqrt{k_1^2+k_2^2}\cos\left(\sqrt{k_1^2+k_1^2}\right)\eta\sin\theta\right)N_2(s).$$

4.Spherical Images of a Reguler Curve for Bishop Frame

Yılmaz and Turgut introduced a new version Bishop frame and translating type-2 Bishop frame vectors to the center of unit sphere of there-dimensional Euclidean space [35]. We apply this idea for DNA strand helix curve according to Bishop frame.

4.1 T Bishop Spherical İmage

Definition 2. Let $R=R(s)$ be a DNA strand helix curve in E^3. If first vector field of Bishop frame translate to the center of the unit sphere S^2, a spherical image $\varphi=\varphi(s)$ is obtained. This curve is called T type-1 Bishop spherical images. Let $\varphi=\varphi(s)$ be T Bishop spherical image of a regular curve $R = R(s)$. We shall analyse relationship between Bishop and Frenet-Serret invariants. first, we differentiate $\varphi = \varphi(s)$ and use equations (2)

$$\varphi' = \frac{d\varphi}{ds_\varphi}\frac{ds_\varphi}{ds} = k_1 N_1 + k_2 N_2 (s)$$

Here, we shall denote differention according to s by a dash, and differantion according to s_φ by a dot. Taking the norm of both sides the equation above, we have

$$T_\varphi = \frac{k_1 N_1 + k_2 N_2 (s)}{\sqrt{k_1^2 + k_2^2}} \qquad (5)$$

and

$$\frac{ds_\varphi}{ds} = \sqrt{k_1^2 + k_2^2}$$

by substituting (4) into (5) we can get

$$T_\varphi = \cos\theta N_1 + \sin\theta N_2$$

and differentiate with respect to s

$$T_\varphi' = \frac{\cot\beta}{\sqrt{(r^2 + \cot^2\beta)}}\left(-\sin\theta N_1 + \cos\theta N_2\right) - \frac{r}{r^2 + \cot^2\beta} T \qquad (6)$$

and

$$\dot{T}_\varphi = \frac{\cot\beta\sqrt{(r^2 + \cot^2\beta)}}{r}\left(-\sin\theta N_1 + \cos\theta N_2\right) - T$$

Since, we have the first curvature and the principal normal of φ

$$\kappa_\varphi = \left\|\dot{T}_\varphi\right\| = \sqrt{\frac{\cot^2\beta\left(r^2 + \cot^2\beta\right)}{r^2} + 1}$$

and

$$N_\varphi = \frac{\cot\beta\sqrt{r^2 + \cot^2\beta}}{\sqrt{\cot^2\beta\left(r^2 + \cot^2\beta\right) + r^2}}\left(-\sin\theta N_1 + \cos\theta N_2\right) - \frac{r}{\sqrt{\cot^2\beta\left(r^2 + \cot^2\beta\right) + r^2}} T$$

Cross product of $T_\varphi \times N_\varphi$ gives us the binormal vector field of Tangent Bishop spherical images

$$B_\varphi = \frac{r}{\sqrt{\cot^2\beta\left(r^2 + \cot^2\beta\right) + r^2}}\left(-\sin\theta N_1 + \cos\theta N_2\right) + \frac{\cot\beta\sqrt{r^2 + \cot^2\beta}}{\sqrt{\cot^2\beta\left(r^2 + \cot^2\beta\right) + r^2}} T$$

We express the torsion of the tangent Bishop spherical image from equation (1)

$$\tau_\varphi = \frac{\begin{pmatrix} -k_1\begin{Bmatrix} 3k_2'\left(k_1k_1' + k_2k_2'\right) \\ -\left(k_1^2 + k_2^2\right)\left[k_2'' - k_2\left(k_1^2 + k_2^2\right)\right] \end{Bmatrix} \\ +k_2\begin{Bmatrix} 3k_1'\left(k_1k_1' + k_2k_2'\right) \\ -\left(k_1^2 + k_2^2\right)\left[k_1'' - k_1\left(k_1^2 + k_2^2\right)\right] \end{Bmatrix} \end{pmatrix}}{\left[k_1^2\left(\frac{k_2}{k_1}\right)'\right]^2 + \left(k_1^2 + k_2\right)^3} \qquad (7)$$

by substituting (4) into (7) we can get

$$\tau_\varphi = 0$$

Remark 1. It is well known that T tangent vector which according to Bishop frame is equal to the T tangent vector in Frenet frame. Therefore, in E³ the spherical image of tangent indicative curve for Frenet frame is a circle.

Hence, the spherical image of tangent indicative curve for Bishop frame is also a circle.

4.2 N_1 Spherical İmage

Definition 3. Let $R = R(s)$ be a DNA strand helix curve in E³. If we translate of N_1 vector field of Bishop frame to the center of the unit sphere S², we get a spherical image, $\gamma = \gamma(s)$ which is called N_1 Bishop spherical images.

Let $\gamma = \gamma(s)$ be N_1 Bishop spherical image of a regular curve $R = R(s)$. We shall investigate relations among Bishop and Frenet-Serret invariants. First, we differentiate

$$\gamma' = \frac{d\gamma}{ds_\gamma}\frac{ds_\gamma}{ds} = -k_1 T$$

Taking the norm of both sides the equation above, we have

$$T_\gamma = T \qquad (8)$$

and

$$\frac{ds_\gamma}{ds} = k_1$$

So equation (8) one can get

$$T_\gamma(s) = \frac{1}{\sqrt{r^2 + \cot^2\beta}}\begin{pmatrix} -r\sin\left(\frac{s}{\sqrt{r^2 + \cot^2\beta}}\right), \\ r\cos\left(\frac{s}{\sqrt{r^2 + \cot^2\beta}}\right), \\ \cot\beta \end{pmatrix}$$

Differentiate with respect to s

$$T_\gamma'(s)=\frac{-r}{r^2+\cot^2\beta}\begin{pmatrix}\cos\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right),\\ \sin\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right),\\ 0\end{pmatrix}$$

and differentiate with respect to s_γ

$$\dot{T}_\gamma=\frac{1}{\cos\theta}\begin{pmatrix}\cos\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right),\\ \sin\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right),\\ 0\end{pmatrix}$$

Since, we have the first curvature and the principal normal of γ

$$\kappa_\gamma=\left\|\dot{T}_\gamma\right\|=\frac{1}{\cos\theta} \tag{9}$$

and

$$N_\gamma=\left(\cos\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right),\sin\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right),0\right)$$

Cross product of $T_\gamma\times N_\gamma$ gives us the binormal vector field of Tangent Bishop spherical images $R=R(s)$ curve

$$B_\gamma=\frac{1}{\sqrt{r^2+\cot^2\beta}}\begin{pmatrix}-\sin\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right)\cot\beta,\\ \cos\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right)\cot\beta,\\ -r\end{pmatrix}$$

We express the torsion of N_1 Bishop spherical image from equation (1)

$$\tau_\gamma=-\frac{k_1\left(\frac{k_2}{k_1}\right)'}{k_1^2+k_2^2} \tag{10}$$

by substituting (4) into (10) we get

$$\tau_\gamma=-\frac{\cot\beta\sqrt{r^2+\cot^2\beta}}{r\cos\theta} \tag{11}$$

Corollary 4. Let $\gamma=\gamma(s)$ be N_1 Bishop spherical image of the DNA starnd helix curve. Then $\gamma(s)$ is a general helix.

Proof. If substituting (9) and (11) into Proposition 1 we get

$$\frac{\kappa_\gamma}{\tau_\gamma}=-\frac{r}{\cot\beta\sqrt{r^2+\cot^2\beta}}$$

so $\gamma=\gamma(s)$ regular curve is a general helix.

Corollary 5. N_1 Bishop spherical image of the DNA strand helix curve is a clindrical helix. In other saying N_1 Bishop spherical image curve be a interwound DNA.

Proof. If substituting (9) and (11) into Proposition 2 we get

$$\sigma=\left[\frac{\kappa_\gamma^{\,2}}{\left(\kappa_\gamma^{\,2}+\tau_\gamma^{\,2}\right)^{\frac{3}{2}}}\left(\frac{\tau_\gamma}{\kappa_\gamma}\right)'\right]=0$$

Hence □ γ is a clindrical helix.

4.3 N_2 Spherical İmage

Definition 4. Let $R=R(s)$ be a DNA strand helix curve in E³. If we translate of N_2 vector field of Bishop frame to the center of the unit sphere S², we get a spherical image, $\phi=\phi(s)$ which is called N_2 Bishop spherical images.

Let $\phi=\phi(s)$ be N_2 Bishop spherical image of a regular curve $R=R(s)$. We shall investigate relations among Bishop and Frenet-Serret invariants. First, we differentiate

$$\phi'=\frac{d\phi}{ds_\phi}\frac{ds_\phi}{ds}=-k_2T$$

Taking the norm of both sides the equation above, we have

$$T_\phi=T \tag{12}$$

and

$$\frac{ds_\phi}{ds}=k_2$$

So equation (12) one can get

$$T_\phi(s)=\frac{1}{\sqrt{r^2+\cot^2\beta}}\begin{pmatrix}-r\sin\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right),\\ r\cos\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right),\\ \cot\beta\end{pmatrix}$$

Differentiate with respect to s

$$T_\phi'(s)=\frac{-r}{r^2+\cot^2\beta}\begin{pmatrix}r\cos\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right),\\ -r\sin\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right),\\ 0\end{pmatrix}$$

and differentiate with respect to s_ϕ

$$\dot{T}_\phi=\frac{1}{\sin\theta}\begin{pmatrix}\cos\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right),\\ \sin\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right),\\ 0\end{pmatrix}$$

Since, we have the first curvature and the principal normal of ϕ

$$\kappa_\gamma=\left\|T_\gamma\right\|=\frac{1}{\sin\theta} \tag{13}$$

and

$$N_\phi=\begin{pmatrix}\cos\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right),\\ \sin\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right),\\ 0\end{pmatrix}$$

Cross product of $T_\phi\times N_\phi$ gives us the binormal vector field of Tangent Bishop spherical images $R=R(s)$ curve

$$B_\phi=\frac{1}{\sqrt{r^2+\cot^2\beta}}\begin{pmatrix}-\sin\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right)\cot\beta,\\ \cos\left(\frac{s}{\sqrt{r^2+\cot^2\beta}}\right)\cot\beta,\\ -r\end{pmatrix}$$

We express the torsion of N_2 Bishop spherical image from equation (1)

$$\tau_\phi=-\frac{k_1\left(\frac{k_2}{k_1}\right)'}{k_1^2+k_2^2} \tag{14}$$

by substituting (4) into (14) we get

$$\tau_\phi=-\frac{\cot\beta\sqrt{r^2+\cot^2\beta}}{r\cos\theta} \tag{15}$$

Corollary 5. Let $\phi=\phi(s)$ be N_2 Bishop spherical image of the DNA starnd helix curve. Then $\phi(s)$ is a general helix.

Proof. If substituting (13) and (15) into Proposition 1 we get

$$\frac{\kappa_\phi}{\tau_\phi}=-\frac{r}{\cot\beta\sqrt{r^2+\cot^2\beta}}$$

so $\phi=\phi(s)$ regular curve is a general helix.

Corollary 6. N_2 Bishop spherical image of the DNA strand helix curve is a clindrical helix. In other saying N_2 Bishop spherical image curve be a interwound DNA.

Proof. If substituting (13) and (15) into Proposition 2 we get

$$\sigma=\left[\frac{\kappa_\phi^2}{\left(\kappa_\phi^2+\tau_\phi^2\right)^{\frac{3}{2}}}\left(\frac{\tau_\phi}{\kappa_\phi}\right)'\right]=0$$

Hence □ ϕ is a clindrical helix.

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