An A Posteriori Approach for Decision-Making with Multiple Stochastic Objectives

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Abstract: Many approaches that integrate simulation models with stochastic multiple objective optimization techniques have been proposed, many of which use evolutionary algorithms. These approaches generate a finite set of Pareto optima, and this Pareto optimal set often contains a very large number of solutions, which could be overwhelming to the decision-maker. In this paper, an innovative approach that effectively reduces the number of the non-dominated Pareto solutions while considering the stochastic nature of the objective functions after the optimization process is proposed. A detailed description of the proposed a posteriori approach and a numerical example that demonstrates the performance are provided.

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1. Introduction

It is widely known that simulation is a powerful tool that enables decision-makers in business and industry to improve organizational performance. The ability to model a physical process on the computer, incorporating the uncertainties that are inherent in all real dynamic systems, provides an enormous advantage for analysis and decision-making.

Decision-makers frequently use simulation within their organizations to evaluate and compare proposed, often complex and mathematically intractable, designs of their systems and processes with the goal of optimizing a particular performance objective. However, most real-world decisions involve the simultaneous, non-trivial optimization of multiple, and often conflicting, objectives. Due to the “satisficing” of the objectives within the context of the decision, often a large set of compromise, or trade-off, solutions that seek to balance the set of objectives are identified. After which, the best trade-off solution is selected according to the decision-maker or set of decision-makers’ preferences and existing and future physical, technological and financial constraints.

Applications of the optimization of multiple objectives, in research and in practice, typically involve using metaheuristic search procedures in deterministic settings, with the procedures generating a large set of Pareto optima (i.e., non-dominated solutions) that characterize the efficient frontier in the objective space from which the decision-maker must select the most preferred solution. However, the success of these search procedures is not as consistent in noisy environments where the objective functions are stochastic such as when using simulation as the evaluator of the individual objective functions. Evolutionary algorithms (EAs) are popular and have become common approaches to solving multiobjective optimization and generating the set of Pareto optimal solutions, as they are generally believed to be able to handle fairly well with either deterministic or stochastic objective functions since promising areas of the search space are sampled several times.

The multiobjective optimization problem involves two stages of algorithm decision-making: (1) the optimization stage and (2) the post-Pareto analysis stage (Aguirre & Taboada, 2011). The first stage focuses on obtaining a set of non-dominated solutions. An EA-based simulation optimization approach requires a large number of simulation evaluations due to the stochastic components not only of simulation model but also because of the stochastic features of EAs before a satisfactory solution can be found (Syberfeldt, Ng, John, & Moore, 2009). Since there is variation in the output from a simulation optimization approach, appropriate statistical techniques, such as point and confidence interval estimation, must be used to determine the precision of the key performance measures. In addition, the efficient frontier in the stochastic objective space that is characterized by the set of Pareto optimal solutions is not as clearly defined as that under deterministic objective functions.

The second stage involves the reduction of the trade-off solutions in the set of Pareto optima. In order to be adequately representative of the possibilities and trade-offs, non-dominated solutions under stochastic objectives may be too large for decision-makers to practically consider. Therefore, some intelligent means of reducing and organizing the non-dominated set of solutions in the presence of stochastic objectives is required.

Most of the existing work focuses primarily on the first stage. Nevertheless, the second stage of decision-making is as important as finding the set of non-dominated solutions (Aguirre & Taboada, 2011). Reducing the number of solutions to select from is not a simple task and can be overwhelming when presented with an extraordinarily large set of potential compromise, or trade-off, solutions. In this paper, an innovative *a posteriori* approach that effectively deals with noise in objectives and aims to reduce the number of the non-dominated Pareto solutions effectively is presented. The proposed approach uses statistical analysis on the Pareto optimal solutions in order to reduce the number of solutions to a set of representative solutions with priority that is presented to the decision-maker for final selection.

The remainder of this paper is organized as follows. The next section presents a brief overview of decision-making in the presence of multiple objectives. In Section 3, previous related work is discussed. The proposed approach is described in Section 4. Explanation of the *a posteriori* approach and a numerical example are presented in Section 5. Application of the simulation optimization to the case study is given in Section 6. This paper is concluded and suggestions for future work are presented in Section 7.

2. Decision-Making Considering Multiple Objectives

Many real-world problem decision-making situations seek trade-off, or compromise, solutions rather than to seeking a single global optimal solution, as these critical decisions often involve multiple, often conflicting, objectives that must be addressed simultaneously. Decision-makers frequently use simulation within their organizations to model, evaluate and compare proposed, often complex and mathematically intractable, designs of their systems and processes with the goal of optimizing a particular performance objective, or set of performance objectives.

Multiple objective decision problems, unlike single objective decision problems, address a number of objective functions to be minimized or maximized. There are many mathematical programming techniques for multiobjective optimization. Most of the recent work focuses on the approximation of the Pareto optimal solution set (Abraham, Jain, & Goldberg, 2005). In other words, instead of identifying a single global solution, multiobjective optimization results in a number of trade-off (or, compromise) solutions among the set of objectives. This set of solutions is considered the set of Pareto efficient solutions (Coello Coello, 2006), as shown in Figure 1. A Pareto optimal solution is called non-dominated if none of the objective functions can be improved without degrading one or more of the other objective values (Winston, 2003).

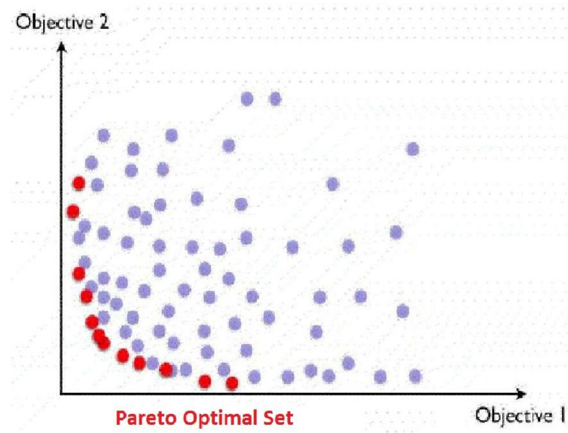

Figure 1. Two-objective space assuming the two objectives are to be minimized. The red points represent the Pareto frontier

3. Previous Related Work

Multiobjective optimization solution approaches can be broadly categorized as non-Pareto-based techniques and Pareto-based techniques. There are several multiobjective optimization approaches that are Pareto-based with the intention to generate the Pareto frontier. However, it is until within the last two decades that researchers and practitioners have realized of the potential of using evolutionary algorithms in this area as this family of stochastic optimization metaheuristic search methods can effectively generate a set of Pareto optima (Coello, 2001). These algorithms have proven themselves as general, robust and powerful search mechanisms. Particularly, they possess several characteristics that are desirable for real-world problems involving multiple conflicting objectives, and intractably large and highly complex search spaces (Wang, Zhang, Gao, & Li, 2008). Evolutionary algorithms deal simultaneously with a set of possible solutions allowing the identification of several members of the Pareto optimal set in a single run of the algorithm. Furthermore, Evolutionary algorithms are less susceptible to the shape or continuity of the Pareto front. For instance, they can easily deal with discontinuous or concave Pareto fronts.

However, evaluating and prioritizing large sets of candidate solutions is a particularly difficult task for decision-makers who utilize the sophisticated Pareto-based approaches. The most common approach is to use information from the decision-maker and decision-maker preferences to reduce the number of solutions. The articulation of decision-maker preferences may be done either before (*a priori* methods), during (interactive methods), or after (*a posteriori* methods) to aid the decision-making, and in effect, the optimization process. Several studies have proposed ways to reduce the number of Pareto solutions to a reasonable number based on prior information known by the decision-maker. However, multiobjective decision-making approaches are widely used to select the most appropriate solution among the other available solutions (Noghin, 2011).

4. A Proposed A Posteriori Approach for Reducing the Number of the Non-Dominated Pareto Solutions

In this proposed methodology, an innovative approach that effectively reduces and then prioritizes the set of Pareto solutions while considering the stochastic nature of m objective functions is developed. Figure 2 shows the general logic of the proposed approach.

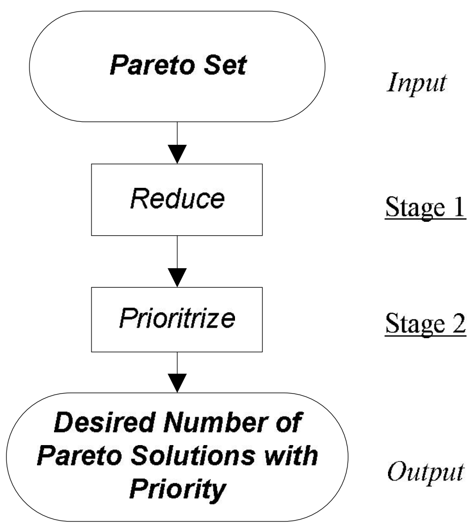

Figure 2. General logic of the proposed approach

First, the proposed approach begins with a given set of P Pareto optima compromise (or, trade-off) solutions as input. The reduction of the candidate set of compromise solutions is performed while considering the statistical precision of the performance measures under study and preferences on objectives by the decision-maker. Second, the reduced set of solutions is prioritized to assist the decision-maker in identifying the most appropriate compromise solution.

5. Numerical Example and Computational Study

In this example, a two-objective, two-variable minimization problem is considered. The proposed approach begins after the optimization process with a given set of P Pareto optima solutions. Figure 3 shows the original Pareto optimal front generated by using a simulation multiobjective optimization approach that uses multiobjective evolutionary algorithms and discrete-event simulation. Each point on the curve (as shown in Figure 3) is generated after running n = 100 independent simulation replications. As such, the points along the Pareto frontier are the mean objective values across the replications, and each has an associated standard deviation along each dimension in the objective space.

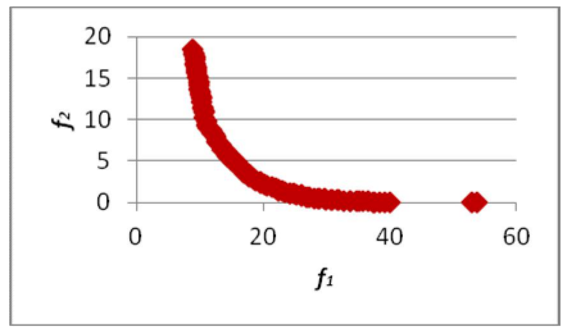

Figure 3. Objective space for the original mean objective functions – 2 objectives to minimize and 100 Pareto solutions

Using the standard deviations, the precision of the mean objective values of the Pareto points is represented by the confidence interval along each objective dimension computed using

$$\bar{x} \pm t_{\frac{\alpha}{2},n-1}\left(\frac{s}{\sqrt{n}}\right), \qquad (1)$$

where $\bar{x}$ is the mean objective value from the n independent replications, s is the standard deviation of the objective value from the n independent replications, α is the level of significance, and $t_{\alpha/2,n-1}$ is the upper $\alpha/2$ critical value for the t-distribution with n-1 degrees of freedom. Stage 1 of the proposed approach starts after the optimization process with computing the upper and lower confidence limit for each Pareto point using Eq. 1 (Mendenhall & Sincich, 2012). Here, for illustration, a level of significance α = 10% is assumed. In addition, the preferred objective, f_2 for this example, is identified by the decision-maker.

First, the set of f_2 solutions is sorted from largest to smallest and then the overlapping confidence intervals among the whole set are identified. On the other hand, the marginal error values associated to f_1 is calculated. One solution is selected among each set of overlapping confidence intervals for the set of f_2 solutions by identifying the smallest marginal error value associated to f_1. The first iteration reduces the original set of 100 Pareto optimal solutions to 36 solutions, as shown in Figure 4. Second, the previous step is repeated to ensure that there are no more overlapping confidence intervals for f_2 solutions. In the example, this second iteration reduces the previous set of 36 Pareto solutions to 29 solutions, as shown in Figure 5. Third, the first step is repeated to make sure that there are no more overlapping confidence intervals for f_2 solutions. For the example, there are no more overlapping confidence intervals for the f_2 objective.

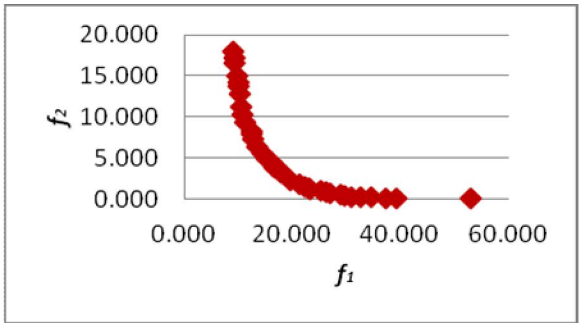

Figure 4. Set of 36 Pareto optimal solutions generated after the first iteration for the f2 objective

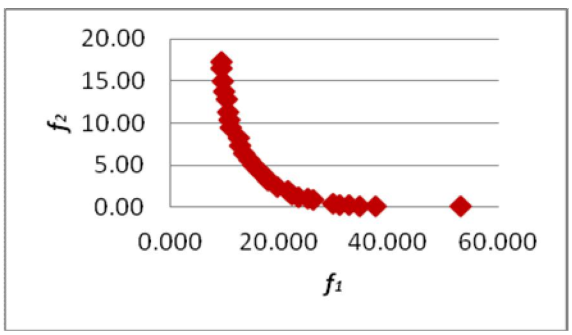

Figure 5. Set of 29 Pareto optimal solutions generated after the second iteration for the f2 objective

Now, the first step is performed for the f_1 objective in order to check for overlapping confidence intervals for the f_1 objective solutions. After the first iteration for the f_1 objective, the previous set of 29 Pareto solutions is reduced to 26 solutions, as shown in Figure 6. The previous step is repeated for the f_1 objective to make sure that there are no more overlapping confidence intervals for f_1 objective solutions. For the example, there are no more overlapping confidence intervals for the f_1 solutions. Now, the final reduced set of Pareto P^* is considered for Stage 2, which is the set of solutions with no overlapping confidence intervals, as shown in Figure 6.

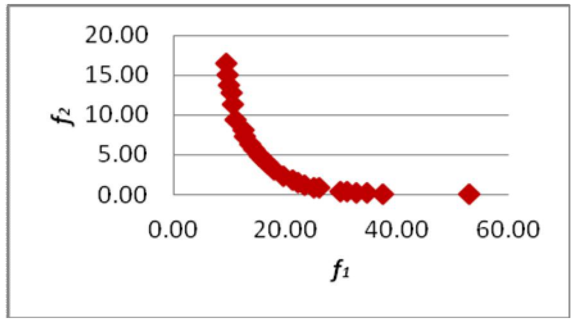

Figure 6. Set of 26 Pareto optimal solutions generated from the first iteration for the f1 objective

Stage 2 of the proposed approach prioritizes the representative solutions identified in Stage 1. Many researchers have used the popular swing-weighting approach among the other multicriteria decision-making approaches in the presence of multiple objectives. Using swing weights, the decision-maker determines the solutions that are the most important, then the second most important, etc. In addition, the decision-maker also specifies the degree of importance of each solution relative to the others. The importance values are then normalized to sum to 1.0 (Clemen & Reilly, 2004; Weber, Eisenführ, & Von Winterfeldt, 1988). The swing-weighting approach is used in the proposed approach. Considering the current example, assuming a small f_2 objective value is desired first, and then a small f_1 objective value is desired second. Table 1 shows the prioritized solutions using the swing-weighting approach. Table 2 summarizes the assessment of the swing weights.

Table 1. The feasible solutions with priority

Priority	f_1	f_2
1	9.3392	16.5312
3	9.7002	15.0016
5	9.9405	13.7149
6	10.2172	12.8005
7	10.5606	11.2652
8	11.1592	9.3721
9	12.4009	8.1761
10	12.7776	7.2942
11	13.5532	6.3289
13	14.4148	5.6274
14	15.2220	4.9464
15	16.0767	4.4184
16	16.7845	3.9056
18	17.8300	3.1504
19	19.6535	2.3465
21	21.3838	1.8605
22	22.3329	1.5400
24	23.4215	1.2334
25	25.1981	0.9443
26	26.1460	0.8195
23	29.7536	0.4388
20	30.9444	0.3129
17	32.7454	0.2248
12	34.6728	0.1596
4	37.4140	0.0879
2	53.0004	0.0467

Table 2. The assessment of swing weights

Attribute Swing from Worst to Best	Consequence to Compare		Rank	Rate	Weight
(Benchmark)	53.000	16.531	3	0	0.000
f_1	9.339	16.531	2	75	0.429
f_2	53.000	0.047	1	100	0.571
				175	1.000

The overall utility for different feasible solutions is determined as shown in Eqs. 2-27. The value of the corresponding weight or the relative utility shows how the prioritized solutions are computed. Eqs 2 and 27 show how the weight values shown on Table 2 are calculated for f_1 and f_2. The decision-maker identifies the prioritized solutions based on the value of the corresponding weight or the relative utility. With largest value of the weight, the solution is considered the most important whereas the smallest value of the weight place the solution with the least important.

U (9.3392, 16.5312) = H(0) + B(1) = 0.5714 (2)
U (9.7002, 15.0016) = H(0.96) + B(0.00) = 0.4144 (3)
U (9.9405, 13.7149) = H(0.94) + B(0.00) = 0.4046 (4)
U (10.2172, 12.8005) = H(0.91) + B(0.00) = 0.3938 (5)
U (10.5606, 11.2652) = H(0.88) + B(0.00) = 0.3814 (6)
U (11.1592, 9.3721) = H(0.84) + B(0.00) = 0.3615 (7)
U (12.4009, 9.3721) = H(0.75) + B(0.01) = 0.3260 (8)
U (12.7776, 7.2942) = H(0.73) + B (0.01) = 0.3169 (9)
U (13.5532, 6.3289) = H(0.69) + B(0.01) = 0.2995 (10)
U (14.4148, 5.6274) = H(0.65) + B(0.01) = 0.2824 (11)
U (15.2220, 4.9464) = H(0.61) + B(0.01) = 0.2683 (12)
U (16.0767, 4.4184) = H(0.58) + B(0.01) = 0.2683 (13)
U (16.7845, 3.9056) = H(0.56) + B(0.01) = 0.2453 (14)
U (17.8300, 3.1504) = H(0.52) + B(0.01) = 0.2330 (15)
U (19.6535, 2.3465) = H(0.48) + B(0.02) = 0.2150 (16)
U (21.3838, 1.8605) = H(0.44) + B(0.03) = 0.2015 (17)
U (22.3329, 1.5400) = H(0.42) + B(0.03) = 0.1965 (18)
U (23.4215, 1.2334) = H(0.40) + B(0.04) = 0.1925 (19)
U (25.1981, 0.9443) = H(0.37) + B(0.05) = 0.1871 (20)
U (26.1460, 0.8195) = H(0.36) + B(0.06) = 0.1856 (21)
U (29.7536, 0.4388) = H(0.31) + B(0.11) = 0.1953 (22)
U (30.9444, 0.3129) = H(0.30) + B(0.15) = 0.2146 (23)
U (32.7454, 0.2248) = H(0.29) + B(0.21) = 0.2409 (24)
U (34.6728, 0.1596) = H(0.27) + B(0.29) = 0.2826 (25)
U (37.4140, 0.0879) = H(0.25) + B(0.53) = 0.4106 (26)
U (53.0004, 0.0467) = H(1) + B(0) = 0.4286 (27)

6. Application of Simulation Optimization to the Case Study

In this section a comparison between the results of the problem generated by using the *a posteriori* approach, and the simulation framework for the (s, S) inventory with backlogging model integration with the NSGA II are illustrated. However, for the simulation optimization approach, the population size is four assuming the desired number of representative solutions is four for this case study. Table 3 and Figure 7 show the feasible solutions for the different approaches.

Table 3. The feasible solutions for different approaches

A Posteriori Approach		Simulation Optimization Approach (Population Size of 4)	
f1	*f2*	*f1*	*f2*
53.000	0.047	40.721	0.050
37.414	0.088	23.164	1.375
9.700	15.002	11.219	9.618
9.339	16.531	8.966	18.925

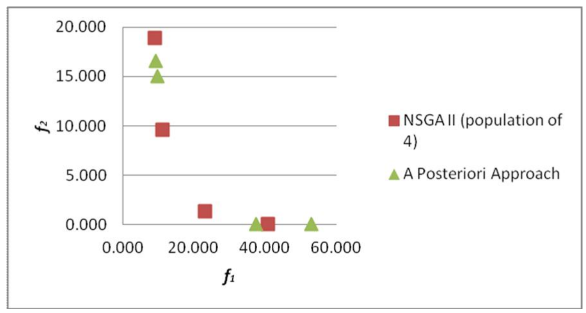

Figure 7. The feasible solutions for the different approaches

Table 3 and Figure 7 show the feasible solutions for the different approaches. Assuming a lower value of *f2* is desired first, and then a lower value of *f1* is desired second. The results of the *a posteriori* approach and the simulation optimization approach show similar spread in the representative solutions along the Pareto frontier. In addition, the lowest value of 0.04 for *f2* with *a posteriori* approach among the candidate solutions is improved compared to the simulation optimization approach value for *f2* of 0.05.

7. Conclusions and Future Work

The objective of this study is the improvement of the decision-making selection process in the presence of multiple stochastic objectives. The *a posteriori* proposed approach reduces a large set of trade-off solutions after the optimization process to a manageable number of representative solutions while considering the stochastic nature of the objective functions. Prioritization in support of the representative solutions is considered to assist the decision-maker in identifying the most appropriate solution. With the *a posteriori* approach, preference information is applied by the decision-maker after the optimization process. The results discussed herein show the promise of the proposed approach. The *a posteriori* approach compared to the simulation optimization approach (assuming a population size of four) show better results for the interest of decision-maker. It is important to now note that a pressing area to consider for future research is to enhance the proposed approach with identifying the final number of representative solutions taking into consideration the decision-maker preferences.

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