Inverse Weibull Model for Software Reliability Data Analysis using MCMC method for Non-informative Set of Priors

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Abstract: Markov chain Monte Carlo (MCMC) method has been used to estimate the parameters of inverse Weibull (IW) model depended on a complete sample. In order to obtain Bayes estimates of parameters for the IW model using MCMC simulation method in OpenBUGS (an established software for Bayesian analysis using MCMC method), independent non-informative set of priors for IW parameters are assumed to obtain samples from the posterior density function. It has been shown that the MCMC is easy to implement computationally. In addition, the estimates always exist and statistically consistent and their probability intervals are convenient to construct. Applying MCMC to estimate parameters of IW is elaborated. A real software reliability data set is considered to illustrate the methods of inference discussed in this paper.

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1. Introduction

The inverse Weibull distribution (IWD) is found to be appropriate over Weibull distribution when data indicates the non-monotone hazard rate. There are various real life examples where data do not shows the monotone hazard rate. For example, [15] have studied breast cancer data and observed that the mortality increases initially, reaches to a peak after some time and then declines slowly i.e., associated hazard rate is modified bathtub or particularly unimodal.

Since last decade, the inverse Weibull distribution is increasingly attracting the attention of the researchers. The two-sample Bayesian prediction for inverse Weibull distribution from complete sample of observations have been discussed in [3]. Recently, [22] showed that the classical as well as Bayesian estimation procedures for the estimation of the unknown parameters of inverse Weibull distribution under censoring schemes. For more details on inverse Weibull distribution and related inferences, see [7, 9, 10, 26].

IWD plays an important role in many applications, including the dynamic components of diesel engines and several data sets such as the times to breakdown of an insulating fluid subject to the action of a constant tension, see [3,5, 8, 18-20] for more practical applications.

For instance, an interpretation of the IWD in the context of the load strength relationship for a component has discussed in [2]. Fitted IWD to the flood data is considered, for more details see [6, 18, 19, 22]. It has remarked that the IWD will be an appropriate model for analyzing such data, see [14].

In this paper, the authors proposed the IWD for modeling the software reliability data and obtaining the ML estimates with associated probability intervals. The Bayes estimation of the IW model is considered, when both parameters are unknown. It is observed that the Bayes estimates cannot be computed explicitly under the assumption of independent uniform priors for the parameters.

The authors developed the procedure to generate MCMC samples using Gibbs sampling technique from the posterior density function in OpenBUGS, based on the generated posterior samples. In addition, it can compute the Bayes estimates of the unknown parameters and construct highest posterior density credible intervals. Moreover, it can estimate the reliability function. All Statistical computations and functions for IW were built using R statistical software see, [12, 13, 17, 23-25]. Real set of data has been considered, in order to demonstrate how the proposed method can be used in practice for software reliability data.

2. Model Analysis

2.1 Probability density function (pdf)

The two-parameter inverse weibull (IW) model has the probability density function

$$f(x;\alpha,\lambda) = \alpha\lambda\, x^{-\alpha-1} e^{-\lambda x^{-\alpha}}, \alpha > 0, \lambda > 0, x \geq 0. \quad (1)$$

where α and λ are the shape and scale parameters, respectively. The probability density function with the two-parameter of the inverse Weibull model will be denoted by IW(α, λ).

Some of the typical IW density functions for different values of α and for λ = 1 are depicted in Figure 1.

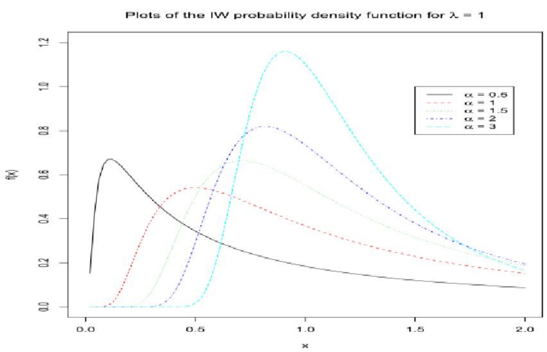

Fig 1. The PDF of IW model for λ=1 and different values of α.

From the graph above, it can be seen clearly that the density function of the inverse Weibull model can take different shapes.

2.2 Cumulative density function (CDF)

The distribution function of the inverse Weibull model with two parameters is given by

$$F(x;\alpha,\lambda) = e^{-\lambda x^{-\alpha}}, \alpha > 0, \lambda > 0, x \geq 0. \quad (2)$$

2.3 The Reliability function

The reliability (survival) function of inverse Weibull model is

$$R(x;\alpha,\lambda) = 1 - e^{-\lambda x^{-\alpha}}, \alpha > 0, \lambda > 0, x \geq 0. \quad (3)$$

2.4 The Hazard function

The hazard rate function of inverse Weibull model is

$$h(x;\alpha,\lambda) = \frac{\alpha\lambda\, x^{-\alpha-1} e^{-\lambda x^{-\alpha}}}{1-e^{-\lambda x^{-\alpha}}}. \quad (4)$$

The hazard rate is an increasing function. It has been graphed in Figure 2 for scale parameter λ =1 and different values of shape parameter α.

2.5 The cumulative hazard function

The cumulative hazard function H(x) of inverse Weibull model defined as

$$H(x) = -\log R(x). \quad (5)$$

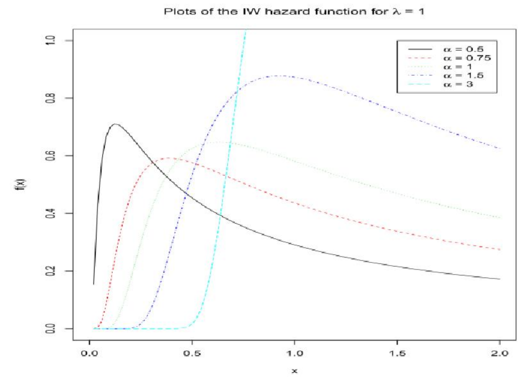

Fig 2. The hazard function of IW model for $\lambda = 1$ and different values of α.

2.6 The Failure rate average (fra) and Conditional survival function (csf)

Two other relevant functions useful in reliability analysis are failure rate average (fra) and conditional survival function (csf). The failure rate average of X is given by

$$FRA(x) = \frac{H(x)}{x} = \frac{\int_0^x h(x)\,dx}{x}, x > 0, \quad (6)$$

where H(x) is the cumulative hazard function. An analysis for FRA(x) on x permits to obtain the IFRA (increasing failure rate average) and DFRA (decreasing failure rate average) classes.

The survival function (s.f.) and the conditional survival of X are defined by

$$R(x) = 1 - F(x),$$

and

$$P(X > x+t \mid X > t) = R(x|t) = \frac{R(x+t)}{R(x)}, t > 0, x > 0, R(.) > 0, \quad (7)$$

respectively, where F(·) is the cdf of *x*. Similarly to h(x) and FRA(x), the distribution of *x* belongs to the new better than used (NBU), exponential, or new worse than used (NWU) classes, when R (x | t) < R(x), R(x |t) = R(x), or R(x | t) > R(x), respectively.

2.7 The Quantile function

The quantile function of inverse Weibull model is given by

$$= \left(\frac{-1}{\lambda}\log q\right)^{-\frac{1}{\alpha}}, 0 < q < 1. \quad (8)$$

2.8 The random deviate generation function

Let U be the uniform (0,1) random variable and F(.) a cdf for which F^{-1}(.) exists. Then

$F^{-1}(u)$ is a draw from distribution F(.). Therefore, the random deviate can be generated from IW(α, λ) by

$$x = (\frac{-1}{\lambda}\log u)^{-\frac{1}{\alpha}}, 0 < u < 1. \quad (9)$$

where u has the U(0, 1) distribution.

3. Maximum Likelihood Estimation, (Mle) And Information Matrix

For completeness purposes, in this section, we briefly discuss the maximum likelihood estimators (MLE's) of the two-parameter inverse Weibull model and discuss their asymptotic properties to obtain approximate confidence intervals based on MLE's, see [6].

Let $x = (x_1, x_2, \ldots, x_n)$ be an observed sample of size n from IW(α, λ), then the log-likelihood function L(α, λ) can be written as

$$\log L = n\log\alpha + n\log\lambda - (\alpha+1)\sum_{i=1}^{n}\log x_i - \lambda\sum_{i=1}^{n}x_i^{-\alpha}, \quad (10)$$

Therefore, to obtain the MLE's of α and λ, see [7], we can maximize (10) directly with respect to α and λ or solve the following two non-linear equations using Newton- Raphson method

$$= \frac{n}{\alpha} - \sum_{i=1}^{n}\log x_i + \lambda\sum_{i=1}^{n}x_i^{-\alpha}\log x_i = 0, \quad (11)$$

$$= \frac{n}{\lambda} - \sum_{i=1}^{n}x_i^{-\alpha} = 0. \quad (12)$$

3.1 Information Matrix and Asymptotic Confidence Intervals

Let us denote the parameter vector by $\underline{\delta} = (\alpha, \lambda)$ and the corresponding MLE of $\underline{\delta}$ as $\hat{\underline{\delta}} = (\alpha, \lambda)$ as then the asymptotic normality results in

$$(\hat{\underline{\delta}} - \underline{\delta}) \rightarrow N_2(0, (I(\underline{\delta}))^{-1}), \quad (13)$$

where I($\underline{\delta}$) is the Fisher's information matrix given by

$$I(\underline{\delta}) = -\begin{bmatrix} E(\frac{\partial^2 \log L}{\partial \alpha^2}) & E(\frac{\partial^2 \log L}{\partial \alpha\,\partial \lambda}) \\ E(\frac{\partial^2 \log L}{\partial \alpha\,\partial \lambda}) & E(\frac{\partial^2 \log L}{\partial \lambda^2}) \end{bmatrix}. \quad (14)$$

In practice, it is useless that the MLE has asymptotic variance $(I(\underline{\delta}))^{-1}$ because $\underline{\delta}$ is unknown. Hence, it approximate the asymptotic variance by "plugging in" the estimate value of the parameters [8]. The common procedure is to use observed Fisher information matrix O($\hat{\underline{\delta}}$) (as an estimate of the information matrix I($\underline{\delta}$) is given by

$$O(\underline{\delta}) = -\begin{bmatrix} \frac{\partial^2 \log L}{\partial \alpha^2} & \frac{\partial^2 \log L}{\partial \alpha\,\partial \lambda} \\ \frac{\partial^2 \log L}{\partial \alpha\,\partial \lambda} & \frac{\partial^2 \log L}{\partial \lambda^2} \end{bmatrix}_{|(\hat{\alpha},\hat{\lambda})} = -H(\underline{\delta})_{|\hat{\underline{\delta}}}, \quad (15)$$

where H is the Hessian matrix. The Newton-Raphson algorithm to maximize the likelihood produces the observed information matrix. Therefore, the variance-covariance matrix is given by

$$= \begin{pmatrix} Var(\hat{\alpha}) & Cov(\hat{\alpha},\hat{\lambda}) \\ Cov(\hat{\alpha},\hat{\lambda}) & Var(\hat{\lambda}) \end{pmatrix}. \quad (16)$$

Hence, from the asymptotic normality of MLEs, approximate 100(1-γ)% confidence intervals for α and λ can be constructed as

$\pm Z_{\gamma/2}\sqrt{Var(\hat{\alpha})}$ and $\hat{\lambda} \pm Z_{\gamma/2}\sqrt{Var(\hat{\lambda})}$,

where $Z_{\gamma/2}$ is the upper percentile of standard normal variate.

3.2 Computation of Maximum Likelihood Estimation

In this section, real set of data analysis for illustrating the methods proposed in the previous sections. The set of data is extract from [1], and it represents the survival times (in days) of guinea pigs injected with different doses of tubercle bacilli. It is known that guinea pigs have a high susceptibility to human tuberculosis and that is why they were used in this particular study. The regimen number is the common logarithm of the number of bacillary units in 0.5 ml. of challenge solution; i.e., regimen 6.6 corresponds to 4.0 x 10^6 bacillary units per 0.5 ml. (log (4.0 x 10^6) =6.6). Corresponding to regimen 6.6, there were 72 observations listed below:

12	15	22	24	24	32	32	33	34	38	38	43	44	48	52	53	54	54	55	56
57	58	58	59	60	60	60	60	61	62	63	65	65	67	68	70	70	72	73	75
76	76	81	83	84	85	87	91	95	96	98	99	109	110	121	127	129	131	143	146
146	175	175	211	233	258	258	263	297	341	341	376								

The mean, standard deviation and the coefficient of skewness are found as 99.82, 81.12 and 1.83, respectively. The measure of skewness indicates that the data are positively skewed where the coefficient of skewness is the unbiased estimator for the population skewness obtained by

$$K = \frac{\sqrt{n(n-1)}}{n-2} \cdot \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^3}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2\right)^{3/2}}$$

The inverse Weibull model is used to fit this data set which provide above. We have started the iterative procedure by maximizing the log-likelihood function given in (10). We have used *optim*( ) function in R with option Newton- Raphson method [23]. It has obtained $\hat{\alpha} =$1.414756, $\hat{\lambda} =$283.831339 and the corresponding log-likelihood value = -395.6491. The similar results are obtained using *maxLik* package available in R. An estimate of variance-covariance matrix, using (15) and (16), is given by

$$= \begin{pmatrix} 1024.7966598 & -0.9224821108 \\ -0.9224821108 & 0.0008935625 \end{pmatrix}$$

By using (16), we constructed the approximate 95% confidence intervals for the parameters of inverse Weibull model based on MLE's. Table 1 shows the MLE's with their standard errors and approximate 95% confidence intervals for α and λ.

Table 1. Maximum likelihood estimate, standard error and 95% confidence interval.

Parameter	MLE	Std. Error	95% Confidence Interval
alpha	1.414756	0.1174773	(1.184505, 1.645007)
lambda	283.831339	125.8085636	(37.251085, 530.411593)

4. Model Validation

In order to study the goodness of fit of the inverse Weibull model, it is important to compute the Kolmogorov-Smirnov (K-S) statistics between the empirical distribution function and the fitted distribution function when the parameters are obtained by method of maximum likelihood [25]. The authors found the result of K-S test is D= 0.1380984 with the corresponding p-value = 0.1166077. Therefore, the high p-value clearly indicates that IW model can be used to analyze this data set.

The plot of the empirical distribution function and the fitted distribution function in Figure 3 indicates that the graph is reasonable coincided match between the empirical distribution function and the fitted distribution function.

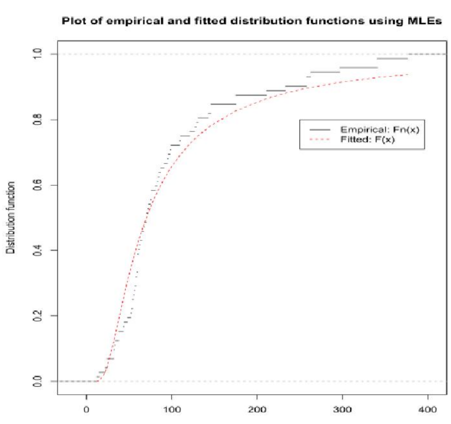

Fig 3. The graph of empirical distribution function and fitted distribution function.

From previous results, it is clear that the estimated IW model provides excellent fit to the given set of data.

The graphical methods widely used for checking whether a fitted model is in agreement with the data are Quantile- Quantile (Q-Q) and Probability-Probability (P-P) plots in model validation.

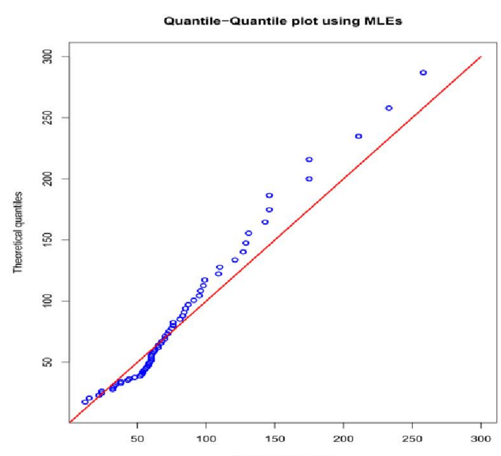

Fig 4. Quantile-Quantile (Q-Q) plot using MLEs as estimate.

Let $\hat{F}(x)$ be an estimate of F(x) based on $x_1, x_2, \ldots, x_n$. The scatter plot of the points $(p_{i:n})$ versus $x_{i:n}$, i = 1 , 2, . . . ,n , is called a Q-Q plot.

The Q-Q plot shows the estimated versus the observed quantiles. If the model fits the set of data well, the pattern of points on the Q-Q plot will roughly exhibit a 45-degree straight line. As can be seen from the approximately straight line pattern in Figure 4, the IW model fits the data well. This is also supported by the Probability-Probability(P-P) plot in Figure 5.

Let $x_1, x_2, \dots, x_n$ be a sample from a given population with estimated cdf $\hat{F}(x)$. The scatter plot of the points $\hat{F}(x_{1:n})$ versus $p_{i:n}$, i = 1 , 2, . . . ,n , is called a P-P plot. If the model fits the data well, the graph will be close to the 45- degree line [24]. Here we note that all the points in the P-P plot are inside the unit square [0, l] x [0, 1].

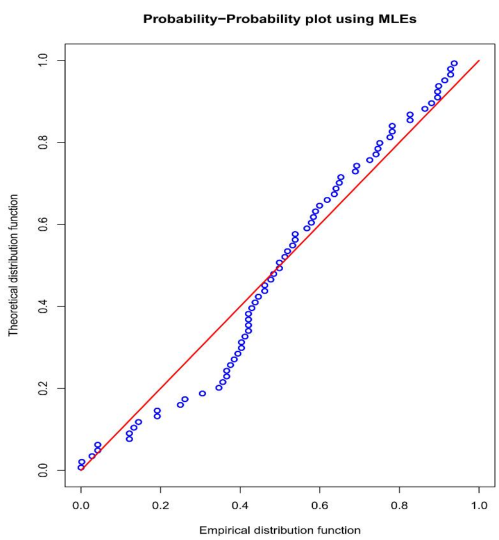

Fig 5. Probability-Probability(P-P) plot using MLEs as estimate.

As can be seen from Figure 4 and Figure 5 that the data do not deviate dramatically from the line.

5. Bayesian Estimation Using Markov Chain Monte Carlo (MSMC)

A Monte Carlo method is an algorithm that relies on repeated pseudo-random sampling for computation, and is therefore stochastic (as opposed to deterministic). Monte Carlo methods are often used for simulation. The union of Markov chains and Monte Carlo methods is called MCMC [21]. A Markov chain is a random process with a finite state-space and the Markov property, meaning that the next state depends only on the current state, not on the past [4].

The most widely used piece of software for applied Bayesian inference is the OpenBUGS [27]. The software offers a user-interface, based on dialogue boxes and menu commands, through which the model may then be analyzed using MCMC techniques. It is a fully extensible modular framework for constructing and analyzing Bayesian probability models for the existing probability models, [16]. As the IW model is not available in OpenBUGS, thus it requires incorporation of a module to estimate parameters of IW model. The Bayesian analysis of a probability model can be performed for the models defined in OpenBUGS. Recently, a number of probability models have been incorporated in OpenBUGS to facilitate the Bayesian analysis [11]. The readers are referred to [12, 23-25] for implementation details of some models.

5.1 Bayesian Analysis under Uniform Priors

The developed module is implemented to obtain the Bayes estimates of the IW model using MCMC method. The main function of the module is to generate MCMC sample from posterior distribution for non-informative set of priors, i.e. Uniform priors. It frequently happens that the experimenter knows in advance that the probable values of θ lie over a finite range [a, b] but has no strong opinion about any subset of values over this range. In such a case a uniform distribution over [a, b] may be a good approximation of the prior distribution, and its p.d.f. is given by

$$\pi(\theta) = \begin{cases} \dfrac{1}{b-a}, & 0 < a \le \theta \le b \\ 0, & otherwise. \end{cases}$$

The authors run the two parallel chains for sufficiently large number of iterations until convergence attained at the length of 40000 with 5000 the burn-in. Final posterior sample of size 7000 is taken by choosing thinning interval five i.e. every fifth outcome is stored. Therefore, we have the posterior sample $\{\alpha_{1i}, \lambda_{1i}\}$, i = 1,…,7000 from chain 1and $\{\alpha_{2}, \lambda_{2i}\}$, i = 1,…,7000 from chain 2. The chain 1 is considered for convergence diagnostics plots. The visual summary is based on posterior sample obtained from chain 2 whereas the numerical summary is presented for both the chains.

5.2 Convergence diagnostics

We started the simulation draws or chains at initial values for each parameter of priors. Because of dependence in successive draws, first draws were discarded as a burn-in to obtain independent samples. Therefore, we need to be sure that the chains have converged in MCMC analysis in order to make inferences from the posterior distribution. This were checked by several diagnostic analysis as follows.

5.3 History(Trace) plot

From the graph above, we can conclude that the chain has converged as the plots show no long upward or downward trends, but it looks like a horizontal band.

5.4 Autocorrelation plot

For this autocorrelation plot, it's clear that, the chains are hardly autocorrelated at all. The letter is good as our posterior sample contains more information about the parameters than when successive draws are correlated. The graph shows that the correlation is almost negligible. So we may consider the independent samples from the target distribution, i.e., posterior.

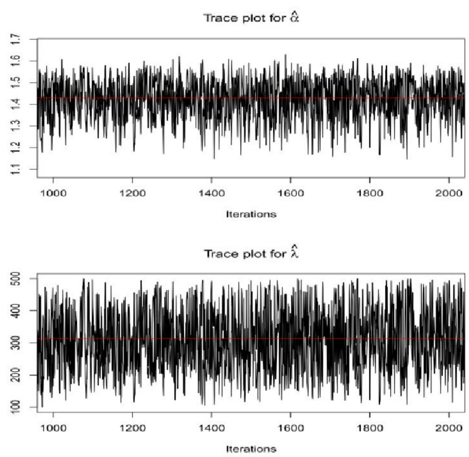

Fig 6. Sequential realization of the parameters α and λ.

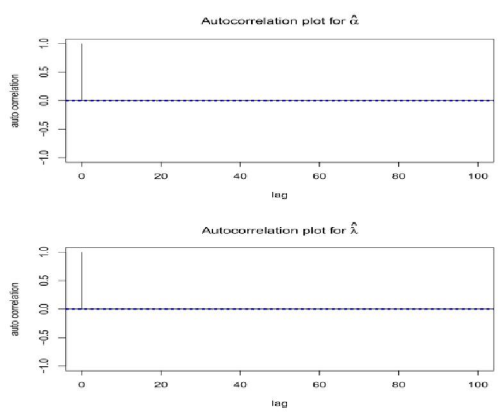

Fig 7. The autocorrelation plots for α and λ.

5.5 Visual summary by using Kernel density estimates

Histograms can provide insights on skewness, behaviour in the tails, presence of multi-modal behaviour, and data outliers; histograms can be compared to the fundamental shapes associated with standard analytic distributions.

Histogram and kernel density estimate of α and λ based on MCMC samples, vertical lines represent the corresponding MLE and Bayes estimate.

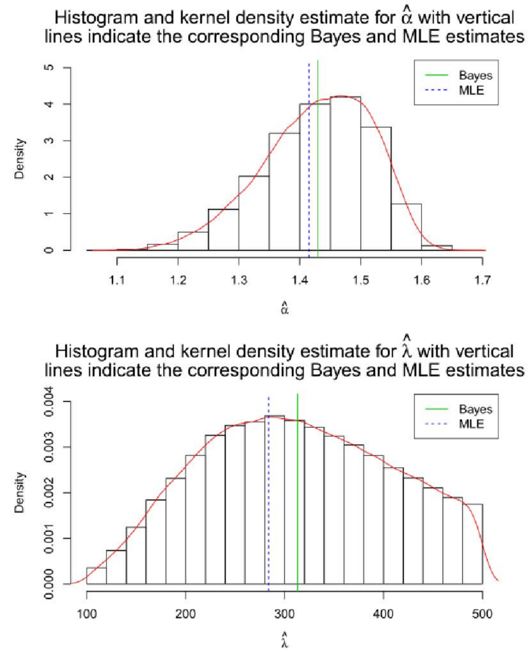

Fig 8. Histogram and kernel density estimate of α and λ.

Table 2: Numerical summaries based on MCMC sample of posterior characteristics for IW model under uniform priors

Characteristics	Chain 1		Chain 2	
	alpha	lambda	alpha	lambda
Mean	1.43	313.6	1.429	312.6
Standard Deviation	0.08912	94.3988	0.08954	94.86215
Naïve SE	0.0004764	0.5045828	0.0004786	0.5070595
Time-Series SE	0.0006651	0.7261442	0.0006284	0.6555530
Minimum	1.086	100.1	1.090	100.1
2.5^{th} percentile ($P_{2.5}$)	1.238	144.1	1.235	142.3
First Quartile (Q_1)	1.371	240.3	1.370	239.0
Median	1.438	310.5	1.437	309.1
Third Quartile (Q_3)	1.498	387.2	1.497	386.4
97.5^{th} percentile ($P_{97.5}$)	1.575	485.0	1.576	486.1
Maximum	1.679	500.0	1.662	500.0
95% Credible Interval	1.238, 1.575	144.1, 485.0	1.235, 1.576	142.3, 486.1
95% HPD Credible Interval	1.252, 1.584	163.8,499.9	1.245, 1.581	160.9, 499.7

5.6 Numerical Summary

Various quantities of interest and their numerical values based on MCMC sample of posterior characteristics for inverse Weibull model under uniform priors, have considered, in Table 2. The numerical summary is based on final posterior sample (MCMC output) of 7000 samples for alpha and lambda.

{$\alpha_{1i}, \lambda_{1i}$}, $i = 1,\ldots,7000$ from chain 1,
and
{$\alpha_{2i}, \lambda_{2i}$}, $i = 1,\ldots,7000$ from chain 2.

5.7 Running Mean (Ergodic mean) Plot

It can be studied the convergence pattern by calculating the running mean, i.e. the mean of all sampled values up to and including that at a given iteration. Time series (Iteration number) graph of the running mean for each parameter in the chain, is generated. The Ergodic mean plots for the parameters shown in figure 9 depict the convergence pattern.

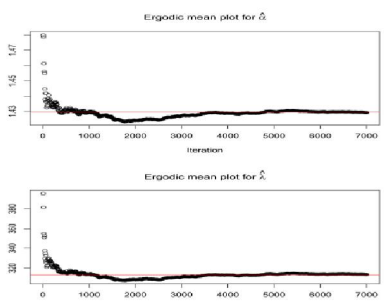

Fig 9. The Ergodic mean Plots for alpha and lambda.

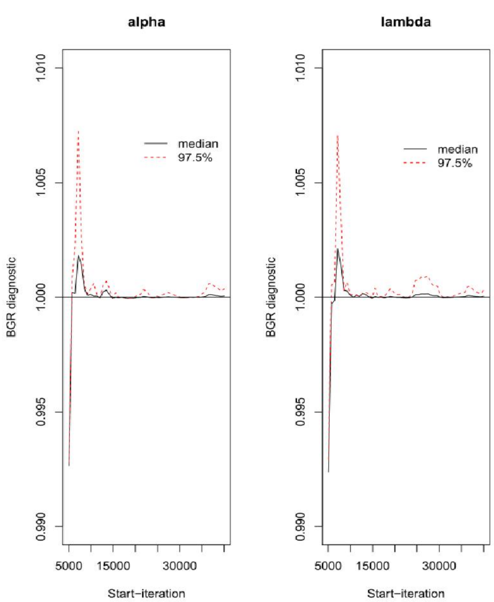

Fig 10. The BGR plots for alpha and lambda.

5.8 Brooks-Gelman-Rubin Diagnostic

It is clear from the graph in Fig 10 that the evidence for convergence comes from the black line being close to 1 on the y-axis and from the red line being stable (horizontal) across the width of the plot.

From the Figure 10, it is clear that convergence is achieved, so that it can obtain the posterior summary statistics.

5.9 Visual Summary by Using Box Plots

The boxes represent inter-quartile ranges and the solid black line at the Centre of each box is the mean, the arms of each box extend to cover the central 95 per cent of the distribution - their ends correspond, therefore, to the 2.5% and 97.5% quantiles. (Note that this representation differs somewhat from the traditional.)

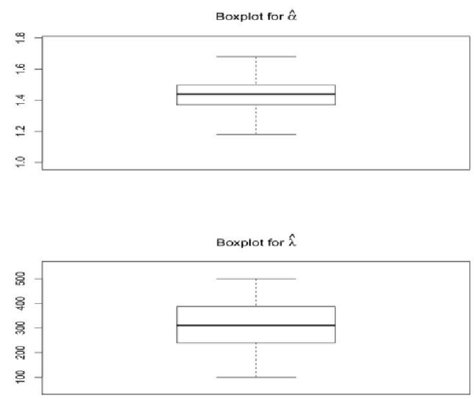

Fig 11. The boxplots for alpha and lambda.

6. Comparison with Mile

For the comparison with MLE, we have plotted three graphs. Fig 12 represents the density functions using MLEs and Bayesian estimates, computed via MCMC samples under uniform priors, are plotted.

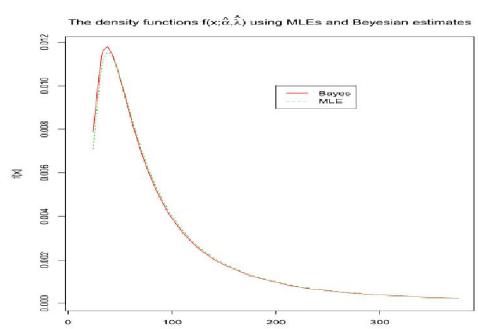

Fig 12. The density functions f(x, $\hat{\alpha}, \hat{\lambda}$) using MLEs and Bayesian estimates.

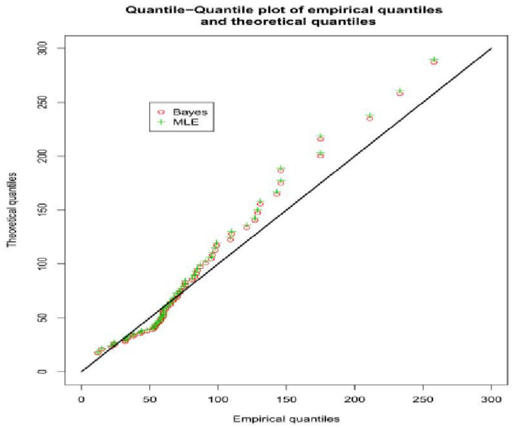

Fig 13. Q-Q plot of empirical quantiles and theoretical quantiles computed from MLEs and Bayesian estimates.

Whereas, Fig.13 represents the Quantile-Quantile (Q-Q) plot of empirical quantiles and theoretical quantiles computed from MLE and Bayes estimates.

The Fig.14 exhibits the estimated reliability function using Bayes estimate under uniform priors and the empirical reliability function.

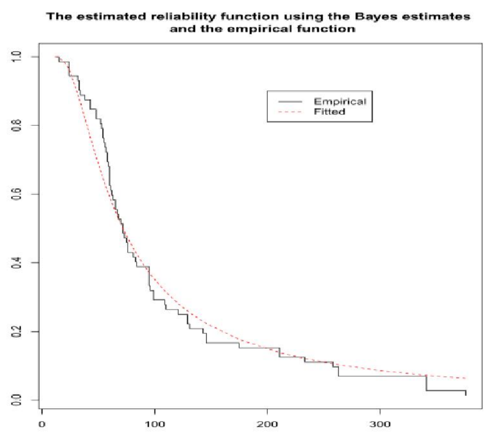

Fig 14. The estimated reliability function using Bayes estimate and the empirical reliability function.

Thus, It is clear from the above Figures 12, 13 and 14, the MLEs and the Bayes estimates with respect to the uniform priors are quite close and fit the data well.

7. Conclusion

The inverse Weibull model with shape parameter α and scale parameter λ has been discussed and estimate of its parameters obtained based on a complete sample using the Markov chain Monte Carlo (MCMC) method. The MCMC method has proven more effective as compared to the usual methods of estimation.

Bayesian analysis under different set of priors has been carried in to OpenBUGS to study the convergence pattern. A numerical summary based on MCMC samples of posterior characteristic for inverse Weibull model has been worked out under non-informative priors. A visual summary under different set of priors which include box plot, kernel density estimation and comparison with MLE has been attempted and it has been found that the proposed methodology is suitable for empirical modeling and best suited for data set, which is considered for illustration under uniform sets of priors.

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