Optimal control of labour potential of the region

Irina Vladimirovna Zaytseva[1,2], Eugene Andreyevich Semenchin[1], Yana Vitalyevna Vorokhobina[3], Marina Victorovna Popova[2,3]

[1]North-Caucasian Federal University, Stavropol, Kulakov prospect, 2, Stavropol, 355037, Russia
[2]Stavropol State Agrarian University, Stavropol, Zootekhnichesky lane, 12, Stavropol, 355017, Russia
[3]Nevinnomyssk State Humanitarian and Technical Institute, Mira Boulevard, 17, Nevinnomyssk, Stavropol region, 357108, Russia

Abstract. Human resources as a basis for economic development of a country become apparent in the labour potential of the population. Thereby, active employment policy, job hunting stimulation, training, professional skills, acquisition of a new profession is needed. When developing the regional employment policy, the labour potential must be used to get the best of quantitative and qualitative changes in the workforce, as well as methods to ensure their effectiveness. Different social and economic processes in the region influence change of the regional labour potential. Economic and mathematical modeling of the labour potential allows solving control problems at the regional level. The research highlights the key social and economic processes in the region that influence changes of its labour potential**.** The differential equation was stated to describe these changes and represent together with the conditions that are given to solve it, a mathematical model of the dynamics of the labour potential. The optimal control task of the amount of unemployed people of working age is studied within this model. The proposed economic and mathematical model of the labour potential of the region is tested on statistical data of the economy of Stavropol region.
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Introduction

Human resources as a basis for economic development of a country become apparent in the labour potential of the population. Changes in the economy are impossible without effective realization of the labour potential of a country and its regions [1].

The analysis of the regional problems of labour economy enables to make a conclusion that the regional labour market is characterized by imbalances of demand and supply of the skilled labour. Thereby, active employment policy, job hunting stimulation, training, professional skills, acquisition of a new profession is needed. When developing the regional employment policy, the labour potential must be used to get the best of quantitative and qualitative changes in the workforce.

The basis of the labour potential is labour. The labour potential is defined as an interconnected complex of quantitative and qualitative characteristics of the population to perform labour activity and ensure achievements of production objectives under specific social and economic conditions, taking into account scientific and technical advance [2]. In the research works of foreign scientists one can find the analysis of a salary increase impact on the scale of the labour supply. The substitution effect causes a labor supply increase, while the income effect leads to its reduction. In the research [3] positive correlation between the labour supply volume and the size of net salaries for the USA conditions is found out.

In Sweden, the reduction of the net salary for the particular level of its gross value resulted in a reduction of the labour supply [4].

In the research [5] questions of the labour division are considered, the problems with the costs of knowledge coordination and record are studied.

In the research works [6, 7] a mathematical model of the labour market self-organization for several economic sectors is developed. It enables to trace certain trends of the labour market functioning. The mathematical model also enables to analyze the obtained information on the stable and unstable states of the labour market for n different economic sectors and make a forecast of its state [8].

Meanwhile, the factors, which influence the labour potential formation, during the transition process from the labour power to the labour resources.

The concept “labour potential” should be considered as an extension of the concepts “labour power” and “labour resources”. The categories “labour power” and “labour resources” are similar in their meanings, but have differences in the content. The difference between these concepts is that the “labour power” is an individual person’s ability to work, and “labour resources” is considered as the population size of working age, which has necessary physical and mental abilities, professional training and qualifications to work in the social production. "Labour potential" contains both existing resources and hidden resources. "Labour resources" contain only stock which is unused [9].

In the research [10] it is stated that the number of labour resources is a dynamic value, which is divided into indicators of natural reinforcement and natural leave. At this moment indicators of natural reinforcement include: the number of working age people, the number of people before and after working age, the number of people from the educational institution system and army. Indicators of natural leave include: the number of people of retirement age, the number of working age people who died, the number of people who joined the army or entered educational institutions at full-time course of study.

In this research a differential equation is suggested. It takes into account changes in the basic social and economic processes taking place in the region and influencing the changes in labour potential of the region. This equation with the given initial conditions is a mathematical model of the labour potential dynamics. This model enables to study the task of optimal control of the amount of working age unemployed people, who did not find work.

The main part

The change of the labour potential of the region is influenced by different social and economic processes in the region, which include as follows [10]:

1) the number of working age unemployed people, who found a job;
2) the value of population emigration;
3) the value of population immigration;
4) the number of people before and after the working age;
5) the number of working age people who died;
6) the number of retired people;
7) the number of working age people who joined the army or entered educational institutions at full-time course of study.

Let $a(t)$ – function, which values at every instant are t, $t \in [t_0, T]$, they coincide with the volume of the labour potential at this moment: difference between the values $a(t+\Delta t)$, $(t+\Delta t) \in [t_0, T]$, $\Delta t \geq 0$, and $a(t)$ characterizes change of the labour potential value during a period of time Δt. We denote during a period of time $t \in [t_0, T]$ by

$b_1(t)$ – the number of working age unemployed people, who found a job;

$b_2(t)$ – the number of working age unemployed people, who did find a job;

$c(t)$ – the number of working age people who died;

$d(t)$ – the number of people before and after the working age who found a job;

$e_1(t)$ – quantity of immigrants, $e_2(t)$ – quantity of emigrants in the region;

$g(t)$ – the number of working age people who retired and do not work (e.g., military personnel);

$h(t)$ - the number of working age people who joined the army or entered educational institutions at full-time course of study.

Then, it is obvious,

$$a(t+\Delta t) - a(t) \approx [k(t)a(t) + b_1(t) + b_2(t) - c(t) + d(t) + e_1(t) - e_2(t) - g(t) - h(t)]\Delta t, \quad (1)$$

where $k(t)$, $t \in [t_0, T]$ – some given coefficient which depends on time and can take on both positive and negative values.

Let's divide the left and the right side of the equality (1) by Δt, as a result we will obtain:

$$\frac{a(t+\Delta t) - a(t)}{\Delta t} \approx k(t)a(t) + b_1(t) + b_2(t) - c(t) + d(t) + e_1(t) - e_2(t) - g(t) - h(t). \quad (2)$$

Passing to the limit in (2) at $\Delta t \to 0$ and assuming that the derivative exists in the left side of this equality, and all functions in the right side are continuous, we have:

$$\frac{da(t)}{dt} = a(t) + b_1(t) + b_2(t) - c(t) + d(t) + e_1(t) - e_2(t) - g(t) - h(t), \quad (3)$$

where $t \in [t_0, T]$

To the ***differential equation*** (3) add the initial condition $a(t_0) = a_0$, which is satisfied with the solution $a(t)$ of this equation (suppose that a_0 - the volume of labour potential at the initial moment t – is known).

Suppose variable value $b_2(t)$ can be regulated, and others values ($k(t)$, $b_1(t)$, $c(t)$, $d(t)$, $e_1(t)$, $e_2(t)$, $g(t)$, $h(t)$) – cannot be regulated on $[t_0, T]$, which one can built in an explicit form by means of regression analysis methods using known statistic data about these functions. Assume that $b_2(t)$ satisfies the conditions (limits):

$$\alpha_1 \leq b_2(t) \leq \alpha_2,$$

$\alpha_1 = const > 0$, $\alpha_2 = const > 0$, - permissible correspondingly minimum and maximum levels of unemployment in the region (note that $\alpha_1 \neq 0$, as unemployment, for example, hidden, always exists, even under the most favorable economic conditions).

We will try to choose the value $b_2(t)$ in such a way that on $[t_0, T]$ its total value (integral)

$$\int_{t_0}^{T} b_2(t)dt \qquad (4)$$

is minimal.

This task is the task of dynamic object optimal control. Let's find its solution using Pontryagin's maximum principle.

We introduce Hamilton's function

$$H=(k(t)a(t)+b_1(t)+b_2(t)-c(t)+d(t)+e_1(t)-e_2(t)-g(t)-h(t))\cdot\varphi_1(t)-b_2(t)\cdot$$

Then

$$\frac{d\varphi_1}{dt}=-\frac{\partial H}{\partial a}\varphi_1=-\varphi_1,$$

$$\varphi_1(t)=e^{-(t-t_0)}.$$

Consequently,

$$H=e^{-(t-t_0)}(a(t)+b_1(t)+b_2(t)-c(t)+d(t)+e_1(t)-e_2(t)-g(t)-h(t))-b_2(t).$$

H reaches a maximum on $b_2(t)$ at $b_2^{(0)}(t)=\alpha_1$. Inserting the optimum $b_2^{(0)}(t)=\alpha_1$ into the equation (3) and solving the obtained equation with the initial condition $a(t_0)=a_0$, we will have:

$$a^{(0)}(t)=\left[a_0+\int_{t_0}^{t} b^{(0)}(s)\cdot\exp\{-(s-t_0)\}ds\right]\cdot\exp\{(t-t_0)\}, \qquad (5)$$

where

$$b^{(0)}(t)=b_1(t)+\alpha_1-c(t)+d(t)+e_1(t)-e_2(t)-g(t)-h(t). \qquad (6)$$

By the specified function type $a(t)$ from (5)-(6) one can easily make a forecast on the labour potential volume at the moment T: it equals to the value $a(T)$.

The value $b_2(t)$ can be selected in a different way, if instead of criterion (4) the following criterion will be selected

$$\int_{t_0}^{T} b_2^2(t)dt. \qquad (7)$$

Expression (7) is total square of deviation on [0, T] values $b_2(t)$ from 0 (zero). Therefore, in practice it is more convenient to set the criterion (7) instead of the criterion (4): it is not required to determine the upper and lower limits. α_1, α_2.

As before we will determine $b_2^0(t)$ ***from the minimum condition (7), i.e.,***

As in the past will be determined from the minimum condition (7), i. e.

$$\int_{t_0}^{T} b_2^2(t)dt \to \min_{b_2(t)}, \qquad (8)$$

***if* a(t) *satisfies the equation (3) with the initial condition* a(0)=** $a_0(t)$.

In the equation (3) we will make a change

$$u(t)=b_1(t)+b_2(t)-c(t)+d(t)+e_1(t)-e_2(t)-g(t)-h(t). \qquad (9)$$

Because from (9) it follows that

$$b_2(t)=u(t)-b_1(t)+c(t)-d(t)-e_1(t)+e_2(t)-g(t)-h(t), \qquad (10)$$

then denoting

$$r(t)=b_1(t)-c(t)+d(t)+e_1(t)-e_2(t)-g(t)-h(t), \qquad (10`)$$

we will pass from the control task (3), (8) to the task

$$\frac{d}{dt}a(t)=a(t)+u(t), \qquad (11)$$

$$\int_{t_0}^{T}(u(t)-r(t))^2dt \to \min_u. \qquad \textit{(12)}$$

When solving the task (11), (12) we use theorem 3.1 from [11].

We introduce Hamilton's function for consideration

$$H(a,u,\varphi)=\varphi_0(u-r)^2+\varphi_1(u+a).$$

If $\varphi_0=0$, from the condition

$$\frac{dH}{du}=0 \qquad (13)$$

it follows that $\varphi_1=0$, i.e., at the same time $\varphi_0=0,\ \varphi_1=0$, that is impossible. Therefore (see theorem 3.1 from [11]) $\varphi_0<0$. One can suppose $\varphi_0=-1$.

From condition (13) we find

$$u_0(t)=r+\frac{\varphi_1}{2}. \qquad (14)$$

As

$$\frac{d}{dt}\varphi_1(t)=-\frac{\partial H}{\partial a}=-\varphi_1,$$

then

$$\varphi_1=ce^{-t},\ c=const. \qquad (15)$$

From (9), (10`), (14), (15) it follows that

$$b_2^0(t)=\frac{1}{2}e^{-(t-t_0)}.$$

We insert (14), (15) in (11), find the equation for the optimum $a^{(0)}(t)$:

$$\frac{d}{dt}a^{(0)}(t) = k(t)a^{(0)}(t) + r(t) + \frac{1}{2}e^{-(t-t_0)},$$

$$a(t_0) = a_0, \qquad (16)$$

where *r(t)* is given by the expression (10`).

Solution of the task (16) has the same form as before (see (5)), but in this case

$$b^{(0)}(t) = r(t) + \frac{1}{2}e^{-(t-t_0)},$$

where *r(t)* is given by (10`).

Example 1. On the basis of statistical data presented in the statistical books on Stavropol region (see, for example, [12]) till the moment of time $t_0 = 2011$ (i.e. till 2011 year) the following has been identified:

$\alpha_1 = 87900$, $\alpha_2 = 177700$, $a_0 = 2787000$, $k(t) \approx 0{,}001$, $b_1(t) = 82400$, $c(t) = 7681$, $d(t) = 509$, $e_1(t) = 2900$, $e_2(t) = 1271$, $g(t) = 5700$, $h(t) = 6000$

(these values are practically the same during 5 years). Then, according to (6), $b(t) = 153057$. According to (5)

$a^{(0)}(2012) = 2\,942\,922$, $a^{(0)}(2013) = 3\,099\,000$, $a^{(0)}(2014) = 3\,255\,234$, $a^{(0)}(2015) = 3\,411\,624$.

Example 2. We suppose the same quantities and their values are from Example 1. Let us find $a^{(0)}(2012)$, $a^{(0)}(2013)$, $a^{(0)}(2014)$, $a^{(0)}(2015)$, if the criterion (7) is considered instead of criterion (4). In this case

$a^{(0)}(2012) = 2\,854\,978$, $a^{(0)}(2013) = 2\,923\,024$, $a^{(0)}(2014) = 2\,991\,139$, $a^{(0)}(2015) = 3\,059\,321$.

Thus, in Examples 1 and 2 by the specified function type *a(t)* from (5)-(6) the volume of the labour potential is forecasted from 2012 to 2015. Using the criterion (7) instead of the criterion (4) did not result in significant changes of the obtained values. The obtained values of the labour potential volume show an increasing number of the labour potential in Stavropol region from 2012 to 2015.

Conclusions

The last decades of the 20th century and the beginning of the new millennium is characterized by the global change in attitude to the human capital and potential. In modern society a human being is the main goal and an active subject of all social and economic processes [13].

Researches of the formation and use of the labour potential of the region are descriptive and not numerous. However, they must be expanded and different methods should be used: economic, social, mathematical, etc., which enable to estimate the impact of the labour potential on economic development and to justify methods of its control [14].

The complex nature of the problem of modeling of social and economic state dependencies of the region to support decision making in the key parameters control, in particular, the labour potential, requires unified modeling methods which are used to study indicators and factors of the regional researches [15, 16].

The authors have developed economic and mathematical model of the labour potential optimal distribution of the region into economic sectors. The model is tested on the statistical data for Stavropol region [17]. The model allows using statistical data on its labour resources and economic indicators, to distribute the labour potential of the region under consideration in the optimal way. Parameters of the model are the total investment amount per employee in the industry and the amount of profit it brings to the industry. They are calculated according to the statistics provided by the state statistical bodies of the region. Utility function is used as the target function in the presented model, it is defined on the great number of sets of human resources in different economic sectors.

In the research [18] the task of optimal distribution of the labour potential of the region into economic sectors is solved with mathematical methods. The developed model takes into account the number of industries in the region, the state of its economy, the income that one employee of *i* industry brings to the region if its sector of the economy will be in the *j* state (*i=1, ..., n; j=1, ..., m*). Having composed the matrix of consequences, with a minimum income per an employee of Stavropol region, one can plan the distribution of the labour resources into economic sectors of the region.

The organizational component of the labour potential of the region is essential for functioning of regional enterprises, workforce, households, individual workers, because it determines conditions to form and implement population potentials, professional growth, to achieve high performance, to meet needs. Since the labour potential to a greater degree than the other economic resources reflects the potential of the region, its importance in the regional social and economic complex control constantly grows. Economic and mathematical modeling of the labour potential enables to solve problems of control at the regional level. Solution of the optimal control task of the unemployed people of working age, who did not find work, enables to explore the processes of formation and use of the labour potential of the region and also brings into focus analysis of peculiarities of these processes development under conditions of a market economy in Russia within the specific region.

The authors are going to develop a system of economic and mathematical methods of analysis and forecasting of the labour potential of a region.

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Corresponding Author:
Dr.Zaytseva Irina Vladimirovna
North-Caucasian Federal University, Stavropol
Kulakov prospect, 2, Stavropol, 355037, Russia

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