Teaching geometric probability using technology of creative workshops

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Abstract. This article describes use of creative technology workshops for teaching geometric probability in algebra courses in profile schools. Emphasis is laid on the design of individual topic, which promote not only the acquisition of requisite skills and knowledge in the target subject, but also support self-learning and improved cognitive performance in each pupil.

Keywords: elements of probability theory, geometric probability, creative workshops

1. Introduction
The principal aim of modern secondary education is to prepare pupils who can (a) easily adapt to their society, (b) successfully continue their education and (c) prosper in their future professions, even in periods of rapid change and economic globalization. One approach toward realizing these goals has been use of profile teaching in schools. This approach attempts to create favourable conditions for encouraging competitive pupils, providing individualized instruction, nurturing each pupil’s dedication and promoting advancement to institutions of higher education. Achieving these goals requires pupils to master many mathematical topics such as statistics, theories of probability and modern ideas associated with analyses of stochastic processes.

The latter poses special problems. For example, results from a recent survey of 30 teachers and 320 pupils at schools in Kazakhstan show that the greatest difficulty in the study of stochastic processes is posed by topics on ‘geometric probability’. The reasons for this appear to be insufficient numbers of teachers experienced with geometric probability and non-uniform developments of the topic in textbooks. These problems are magnified by the general trend toward increasing the science and technical content of school curricula while neglecting the simultaneous development of pupils’ cognitive abilities needed for those pupils to successfully embrace the added content.

We have studied these issues with the following goals in mind: (a) identify and address those cognitive problems that hamper the understanding and use of theories of probability and (b) design a sequence of creative technology workshops for teaching geometric probability. In section 2 we introduce a basis for studying geometric probability, in section 3 we summarize the features of creative technology workshops and in section 4 we lay out a design for such lessons applied to teaching geometric probability.

2. The basis for studying geometric probability
A methodological basis for studying geometric probability rests on the legacy of such scientists as Pascal, Fermat, Huygens, Bernoulli, Markov, Bayes and Laplace [1], who were early pupils of the theory of probability and statistics. That legacy is enhanced by the work of Puasson, Gnedenko [2], Avdeeva, Valieva, Firsov, Gleman and Varga [3], who study the teaching of probability and statistics in schools. It has also been influenced by scholars and educators of Kazakhstan, such as Bektaev [4], Zhangbyrbayev, Satybaldiev, Rahymbek [5] and Berikhanova [6].

In his manual for teachers, Mordkovich identified the subject of ‘probability and geometry’ as one that connects a new topic ‘probability’ to a standard school course ‘geometry’ [7]. In geometric probability, various probabilistic models are viewed stochastically; usually, situations having an infinite number of possible outcomes are linked to explorations over finite numbers of elementary events. The result is that every pupil encounters new stochastic material connected to familiar geometric and elementary statistical ideas. Through direct study of this subject, pupils can discover gaps in their understanding of length, area and volume of bodies, solutions of inequalities, graphs of functions and, in some problems, the calculus of Newton and Leibniz. In this way, the repetition and reinforcement of already known facts, the principles of algebra and the
The benefits of studying geometric probability in courses on algebra and mathematical analysis are clearly stated in Okunev’s manual [8]:

- The material lends itself to visualization, which is helpful to most pupils and particularly important for those with visually dominant creative thinking.
- Pupils’ knowledge is enriched by our attempts to quantify accidental events.
- Geometry, probability theory and mathematical statistics are integrated so that a pupil can get a deeper understanding of each.
- Classical, statistical and geometric probabilities are generalized to axiomatic probability, which makes learning more economical.

An analysis of the content for geometric probability provided in textbooks and manuals shows that this topic is typically taught in class’s: 8, 9, 11. However, different authors introduce concepts in geometric probability in different ways. Further, many textbooks fail to define geometric probability; instead, they merely describe the subject by analysing example situations. Some of the different definitions are given in Table 1; the most common is the last.

(Table 1 here, omitted)

3. Features of creative technology workshops

To overcome the difficulties cited in section 1 and thereby realize the benefits cited in section 2, we propose to use creative technology workshops to teach geometric probability. In this section we summarize the prominent features of such creative workshops.

‘Pedagogical Workshops’ were created in France in the 1920s by psychologists Paul Langevin, Henri Wallon, Jean Piaget and others. Representatives of the ‘Groupe Francais d'Education Nouvelle’, founded by Henri Bassis, engaged in the development and promotion of those creative workshops within the pedagogical community. Since the late 1990s, teachers in Kazakhstan have begun to learn and use such creative workshops. The creative workshops, which can be used at any stage in education, embrace the following educational objectives:

- Create conditions for self-actualization and self-learning.
- Provide opportunities for pupils to construct their own knowledge and create a whole image of the world.
- Develop capabilities for self-assessment and self-correction.

- Allow learners the right to be wrong and to participate in collaborations.

The creative workshops typically have these characteristics:

1. They start from a statement of the workshop’s interesting topic.
2. They attempt to isolate three main groups of conceptions within each pupil: the need to communicate, the need to control and direct cognitive processes and fascination with science and the scientific process.
3. They encourage pupils to acknowledge their ignorance and try to stimulate the desire to know what is unknown. Pupils are prompted to construct their own understanding by solving a task posed by themselves and by working in pairs.
4. They encourage teachers to generalize beyond the immediate topic to discuss research, tell science stories and introduce pupils to theories of cognition, philosophy and science methodology [9].

Workshops have entered the theory and practice of teaching mathematics in school. Textbook authors interpret ‘creative workshops’ as one technique for teaching mathematics [10]. These workshops feature a combination of learning activities such as individual, pair, group and collective activities in the classroom. Such activities allow pupils to be creative in learning what is unknown, and they can increase the pupils’ self-confidence.

Klahan and Yuenyong [11] have observed that technology can be viewed through activities; specifically, designing, making and using or evaluating. Designing refers to both traditional acts of analysis-synthesis-evaluation and to modern approaches needed for complex nonlinear processes. Candidates for making include materials, energy and information. Using and evaluating refers to pupil interactions with their new understanding that promotes further growth.

Design of technology creative workshops for geometric probability

We now outline how technology workshops (Section 3) can be applied to teaching geometric probability (Section 2). The following framework is designed for pupils who study specialized mathematics. Teachers organizing a technology creative workshop should proceed as follows:

1. A title is chosen for the workshop.
2. The first problems broached by the workshop should be suggested and formulated independently by the pupils. The teacher may need to refine and supplement some problem statements before pupils begin trying to solve the problems.
3. Pupils then solve their problems individually. This is done in the classroom, and
pupils are allowed to solve the problems in any order they prefer. Then, the pupils are grouped into pairs that discuss and critique their solutions.

4. Larger groups are organized to discuss the problems and their solutions.

5. Now pupils are assigned a set of tasks that are carefully sequenced to lead pupils to the discovery and synthesis of new knowledge.

6. The newly acquired knowledge is exercised by having pupils solve another series of problems, either in groups or individually.

7. The teacher comments on completed tasks.

8. The teacher reviews and summarizes the results of the workshop.

We illustrate this process with the following three workshops designed to help teach geometric probability:

4. Workshops on the topic “Geometric probability”

4.1 Workshop 1. Cognition theory.

_Aim:_ To formulate the concept of ‘geometric probability’ and continue the development of probabilistic statistical thinking in pupils.

_Expected results:_ Pupils will understand the basic notions of geometric probability.

_Workshop program_ (45 min):

1. **Introduction.** (2 min)

2. **Individual work**
   - Appeal to pupils’ existing knowledge and personal experience. (10 min)
   - 1.1. Write the title of the workshop in a notebook.
   - 1.2. Write down problems that are suggested to you by the topic and for which you would like to see solutions.
   - 1.3. Formulate a definition for ‘geometric probability’ by recording examples 1.2 and 1.3 from the textbook into your notebook.

3. **Work in pairs**
   - Allow pupils to discuss their results with another person who is also interested in the topic. (5 min)
   - 2.1. Discuss the results of tasks 1.2 and 1.3.
   - 2.2. Draw conclusions.

4. **Teacher works with the entire class**
   - Here the teacher tries to guide pupils to a common view of problem solving in the topic area. (8 min) The teacher should emphasize the following facts:
     * In situations having an infinite number of outcomes, classical probability theory does not apply.
     * The classical definition of probability can only be used for situations that have equally possible outcomes.

4. **Group work**
   - Pupils demonstrate to each other alternative ways of problem solving in the topic area. (5 min)

5. **Teacher comments**
   - Here the focus is on key concepts, applications of those concepts and how errors are discovered. (3 min)

6. **General workshop discussion**
   - The teacher summarizes class progress and formulates new problems. (4 min) For example, the class should now recognize this descriptive definition of geometric probability: The geometric probability that a randomly chosen point in plane X will lie within figure A is the ratio of the area of A, S(A), to the area of the plane, S(X), provided X contains the whole of figure A.

7. **Assign a home task**

4.2 Workshop 2. I learn and study, I am discovering new things.

Aim: To understand this theoretical concept: a ‘geometric model of the initial problem situation.’ We also want to identify the main stages in problem solving via geometric probability and continue the development of probabilistic and statistical thinking in pupils.

_Expected results:_ Pupils will be able to create a geometric model of an initial problem situation and will have the skills to explore the model by themselves.

_Workshop program_ (45 min):

1. **Teacher works with entire class**
   - The teacher reviews the two principal facts articulated in Step 3 of Workshop 1:
     - In situations having an infinite number of outcomes, classical probability theory does not apply.
     - The classical definition of probability can only be used for situations that have equally possible outcomes.

2. **Individual work**
   - Pupils individually confront the problem of this workshop: ‘Find a geometric model of the problem situation’.
2.1 Example: Two spies have decided to meet at a fountain. Each of them might be there from 12:00 to 13:00 (1 hour). According to their instructions, the spy to arrive first at the fountain should wait for 15 minutes; if the other does not appear in that time, he should leave. What is the probability of their meeting?

Write down the solution of this example

2.2 Write down the problems you want to solve by this tutorial.

2.3 We must define what is meant by a geometric model of the problem situation.

3. Work in pairs

Pupils exchange their results of the individual work.

3.1. Discuss the results of the tasks.

3.2. Draw conclusions.

3.3. Solve this problem: What is the probability of your meeting with a friend, if you have agreed to meet at a certain place between 12:00 and 13:00 and the first to arrive should wait for five minutes before leaving?

4. Group work

Tasks to build and study models.

Group 1: Randomly draw a triangle. What is the probability that it is acute triangle?

Group 2: Randomly draw a triangle. What is the probability that it is isosceles?

Group 3: Randomly draw a triangle. What is the probability that it is right triangle?

Group 4: Randomly draw a triangle. What is the probability that it is obtuse triangle?

5. Teacher comments to class

The focus is on key concepts, application of those concepts and how errors are discovered.

6. Workshop discussion

The teacher summarizes class progress and formulates new problems.

7. Assign a home task.

4.3 Workshop 3. Learn to solve problems through study.

Aim: To create topics and practical devices that continue the development of probabilistic and statistical thinking in pupils.

Expected results: Problem solving according to the types of tasks presented in the textbook and in control work.

The study can be organized in groups, and then the pupils perform individual tests.

5. Conclusions and discussion

The use of workshop technology allows the teacher to organize various kinds of self-study and individual work for pupils. Acquisition of knowledge and development of skills are regulated, while, at each stage, difficulties and obstacles are identified. In addition, pupils are engaged in systematic individual and group study that should increase their desire to learn. In a relatively short time (2–3 lessons), a teacher can not only teach a new subject but can also develop the skills needed to use the subject matter.

Technology workshops applied to geometric probability allow pupils to develop probabilistic and statistical thinking. Pupils gain understanding of causality and the need for qualitative and quantitative assessments of possible outcomes in particular situations. Pupils also gain the specific knowledge, abilities and skills needed to make those assessments. Moreover, in constructing the workshops, teachers may find that geometric probability can serve as an effective organizing principle for illustrating intra- and inter-disciplinary connections in mathematics.

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