Model of asset portfolio improvement in structured investment products

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Abstract. Type of the investment portfolio model depends on a market trend. For a highly volatile market it is important to develop the algorithm of taking investment decisions, which is efficient regardless of the current trend and minimizes the negative risk effect. The authors propose to use Structured Investment Products as a strategy of investment portfolio management based on the combination of options, shares, bonds, swaps, etc. The authors have developed the mathematical model of the Asset Portfolio Improvement, the methodology of Structured Investment Products development and the algorithm of managing Structured Investment Products portfolio depending on option type. The model developed allows outlining the strategy of building and managing the investment portfolio with the regulated risk level, which raises the efficiency of investments in market instrument.


Keywords: Structured Investment Products (SIP), portfolio, risk, profit, mathematical model

Introduction

According to Charles H. Dow, giving the ground for the theory and methodology of technical analysis, three types of stock market trends can be distinguished:

1. an uptrend is a trend in which every following maximum of the stock market index is higher than the previous and every following minimum is higher than the previous;
2. an downtrend is a trend in which every following maximum of the stock market index is lower than the previous and every following minimum is lower than the previous;
3. a sideways trend is a trend in which minimums and maximums of the stock market index lay close to the charted horizontal line [1].

In an uptrend classic models of investment portfolios, based on the improvement of profitability and risk of stocks and shares, are efficient. However, they are inefficient not only in a downtrend (without short selling), but also in a sideways trend.

The present-day situation in the Russian stock market can be referred to a highly volatile sideways trend: the graphic analysis of the dynamics of the MICEX index from April 1, 2011 to April 1, 2013 allows distinguishing 15 micro trends changing one another. Moreover, the magnitude of the MICEX index as of April 1, 2013 is close to that of August 2011. [2]

Hereafter the acquisition of Russian enterprises shares in accordance with the MICEX index structure, starting with August 2011, would be an inefficient investment from the point of profitability and the acquisition of the corresponding portfolio in April 2011 would lead to a negative profit of 22 – 23% of the total investment. This problem becomes complicated if seen from the methodological point of view. In agreement with the adjustment to the Capital Asset Pricing Model (CAPM), the MICEX index can be used as a market portfolio. The market portfolio stands for a portfolio consisting of all stocks in which every stock ratio refers to its relative market value [3]. Investing in the market portfolio, investors who share the same views do not get enough capital return, which can be explained by the fact that they take all market risks contained in the investment portfolio.

The solution to this problem is to develop the mathematic-economic model and algorithm of taking investment decisions, which are efficient regardless of the current trend in the stock market and minimizing negative effect of the market risk. The development of the concept is a relevant task from the theoretical point of view as well as from the practical one.

The authors of the article have worked out the mathematical model of the Asset Portfolio Improvement in Structured Investment Products (SIPs). Structured products offer exposure to a wide range of markets with varying features and product structures. “The large diversity of terms, payout schedules and risk profiles in structured products demonstrates one of the key benefits of these products – they offer investors precision in constructing investment portfolios to suit specific investment strategies” [4]. The benefits and peculiarities of different types of structured products are analyzed in [5], [6], [7], [8]. The elaborated model allows outlining the strategy of building and managing the investment portfolio with the regulated level of the market risk, which raises the efficiency of investments in the instrument of the stock and forward markets.

Methodology

The notion of the market risk is formalized in CAPM. The beta coefficient is used as the measure for
the market risk in CAPM and is calculated for the stock or the investment portfolio and shows the changing market portfolio profitability. The positive value of the beta coefficient demonstrates the asset growth interest in the market portfolio growth. The negative value shows how much the asset price will rise in case the index falls by the measurement unit and vice versa. [9]

The beta coefficient for asset $x$ is calculated by the formula:

$$\beta_x = \frac{\text{cov}(r_x, r)}{\sigma^2}$$

where:

- $r_x$ – profitability of asset;
- $r$ – profitability of market portfolio;
- $\text{cov}(r_x, r)$ – profitability covariance;
- $\sigma^2$ – variance of market portfolio $i$.

Varieties of investment strategies which allow managing the risk at the moment of building the investment portfolio are usually referred to the structured investment products (SIPs) by stock market professionals.

One of the definitions claims that SIP “is a type of investment specifically designed to meet an investor's financial needs by customizing the product mix to adhere to the investor's risk tolerance. SIPs are generally created by varying the amount of exposure to risky investments and often include the use of various derivatives” [10]. SIP stands for a strategy of investment portfolio management built by the combination of option transaction, shares, bonds, swaps, etc. Structured investment products are supposed to have the system of peculiar requirements for their asset formation and management. The key peculiarity of structured investment products is that they can combine the features of either shares (theoretical possibility of the unlimited profit) or instruments with fixed profitability or bank deposits (risk limit) [11].

The combination of some various investment instruments within SIPs allows fixing the risk from zero value to 10 – 15%. In addition, the profitability of such products is lower than the profitability of the direct investment in the basic asset and, as a rule, it is supposed to have rather conservative levels. Mainly, this effect is achieved by the use of the optional approach to investment decision-making [12]. Potential SIP buyers are often conservative private investors and investment trusts.

Historically, the primary SIP market tends to increase in the period of serious corrections in stock markets or during the growth of the market volatility.

In particular the latest peak of SIP emission was in the period of 2007 – 2009, when the global financial crises resulted into the fall of all key stock indicators and the growth of interest rates. Before that, extreme points came up in early 2000s when there was the bankruptcy of IT companies in the USA and the fall of the high technology index NASDAQ [13]. This factor can be explained by the fact that the SIP emission is closely connected with product marketing. That is why the volume of structured products tends to increase in the appropriate periods of time. The first SIP in Russia came up only in 2005 [14].

**SIPs as a strategy of market risk management**

The development of the structured investment product is aimed at maximizing investor’s asset value. The profit on the structured product is comprised of the interest return taken from the risk-free asset investment and the revenue on the option. Moreover, if the options are not exercised, investor’s asset value shrinks by the sum equal to the option expenses decreased by the interest return return (Figure 1).

**Figure 1 SIP profit generation**

The SIP value is calculated by the formula:

$$A_t = -\text{risk} \cdot I + \sum_{i=1}^{n} \text{REVENUE}_t + B \cdot (1 + r_f)$$

where:

- $A_t$ – SIP value at $t$ time;
- $I$ – volume of initial investments;
- $\text{risk}$ – fixed risk level expressed as a percentage investment volume, $\text{risk} = \text{const};$
- $r_f$ – risk-free rate;
- $t$ – time;
- $B$ – sum invested in risk-free instruments:

$$B = \frac{I \cdot (1 - \text{risk})}{1 + r_f}.$$  

$$\text{REVENUE}_t = \text{revenue on option } i$$

providing that it is exercised at $t$ time:  

$$\text{REVENUE}_t = (\max[0, P_t - S_t - \text{opt}]) \cdot \text{opt}$$

where:

- $P_t$ – basic asset value $i$ at time $t$;
\[ P_{10} \] basic asset value \( t \) at time \( 0 \),
\[ S_{1} \] option strike with basic asset \( t \),
\[ \text{opt}_{t} \] quantity of acquired options with basic asset \( t \).

The quantity of acquired options depends on the options value and the sum and can be expressed by the following equation:
\[ \sum_{t=1}^{n} (P_{\text{opt}_{t}} \cdot \text{opt}_{t}) = I - P \]

where:
\[ P_{\text{opt}_{t}} \] value of option with basic asset \( t \). The option value is calculated according to the Black-Scholes model.

It must be pointed out that the market value for options can diverge from the theoretical value. That is why it is necessary to find out the maximum variance coefficient.

As the risk level is the constant defined at the moment of the SIP development and the magnitude depends on the risk value, the task of maximizing SIP asset value at \( t \) time comes down to solve the problem of maximizing

To maximize \( \text{REVENUE}_{\text{opt}} \), it is necessary to meet the following system of requirements:
\[ \begin{align*}
\text{opt}_{t} & \rightarrow \max \\
\sum_{t=1}^{n} \text{REVENUE}_{\text{opt}_{t}} & \rightarrow \min
\end{align*} \]

To maximize \( \sum_{t=1}^{n} \text{REVENUE}_{\text{opt}_{t}} \), the solution to the problem of the basic asset portfolio improvement is necessary. This solution represents the modified Markowitz problem concerning additional constraints focused on minimizing option value and taken after the analysis of the Black-Scholes model.

It is important to note that according to the economic logic, the risk-free rate should be the lowest SIP profitability limit: hereby the requirement for the excess return is formalized by the investment in risky assets.

The task of the basic asset improvement is formalized in the following way:
\[ \begin{align*}
\left( \sum_{i=1}^{n} \sum_{j=1}^{n} x_{i} \cdot x_{j} \cdot \sigma_{ij} \right)^{1/2} & \rightarrow \min; \\
\sum_{i=1}^{n} \left( \frac{P_{10} - 1}{P_{10}} \right) \cdot x_{i} & \geq \frac{r_{f}}{k}; \\
x_{i} & \geq 0; \\
\sum_{i=1}^{n} x_{i} & = 1;
\end{align*} \]

The basic asset portfolio yield must be higher than the level \( \frac{r_{f}}{k} \) that is the result of the SIPs profitability which mathematically can be presented by the contribution coefficient:
\[ \text{SIPs\ profitability} = \frac{r_{f}}{k} - 1 = \sum_{t=1}^{n} \left( k_{t} \cdot \frac{r_{t}}{P_{10}} - 1 \right) \geq \frac{r_{f}}{k}. \]

The model of the structured investment product differentiates if the structured investment product is aimed at lowering a basic asset value, i.e. an optional part of SIP consists of put-options. The revenue on options is calculated as follows:
\[ \text{REVENUE} = \max_{t} \left[ (P_{10} - S_{1}; 0) \cdot \text{opt}_{t} \right]. \]

The problem of the basic asset portfolio improvement is presented as follows:
\[ \begin{align*}
\left( \sum_{i=1}^{n} \sum_{j=1}^{n} x_{i} \cdot x_{j} \cdot \sigma_{ij} \right)^{1/2} & \rightarrow \min; \\
\sum_{i=1}^{n} \left( \frac{P_{10} - 1}{P_{10}} \right) \cdot x_{i} & \geq \frac{r_{f}}{k}; \\
x_{i} & \geq 0; \\
\sum_{i=1}^{n} x_{i} & = 1; \\
P_{it} & \leq S_{i} \leq P_{10}.
\end{align*} \]

The market risk of structured investment products expressed by the beta coefficient can be described as follows:
\[ \exists P_{\text{opt}} < S \Rightarrow \beta_{\text{SIP}} = 0; \]
\[ \exists P_{\text{opt}} \geq S \Rightarrow \beta_{\text{SIP}} = k, k \geq 0. \]

Results. On the basis of the model given above the unified SIP development methodology is worked out:

Step 1. Defining basic assets. The investor or the person in charge of choosing the basic assets must rely not only on the expected potential of the value growth but also on the criteria of liquidity and availability of liquid optional contracts on the given assets.
Step 2. Defining investments. As a rule the SIP value varies from 1 mio.rub. to more, depending on the issuer’s policy.

Step 3. Defining risks. In most cases the risk varies from 0 to 10%, though SIP constructions with higher risks are possible.

Step 4. Acquiring assets.

The first part of investment assets is targeted at acquiring risk-free assets. More often the capital is allocated among top-rated bonds with the low duration level, which minimizes the price risks and liquidity risks or the bank deposit. That is more typical for the SIP issuer affiliated to the bank or when the partner agreement is available. It is necessary to mention that bonds often look more preferable owing to their liquidity. Whereas the withdrawal of funds from the deposit account may lead to the fine equal to the sum of the interest charged or will result in higher temporary expenses.

The second part of investment assets is targeted at the acquisition of options.

Step 5. Selling risk-free assets. Before the date of the planned option expiration bonds shall be sold at the stock exchange (or assets shall be withdrawn from the deposit account).

Step 6. Making a decision on the option exercise.

At the moment of the expiration, the option In-the-Money is exercised, the profit is fixed. The option Out-of-the-Money is not exercised, the negative profit is fixed, equal to the total price of options adjusted to the revenue from bonds or the deposit account.

It must be noted that it is important to differentiate the patterns of SIP asset management in relation to the time of options expiration (Figure 2).

Figure 2 Algorithm of managing SIP portfolio depending on option type

Peculiarities of SIP practical use

Practicability of the SIP use is mainly determined by the current state of the economy: inflation and interest rates. Moreover, the expected profitability of SIPs correlates with the profitability of the basic asset. The basic asset price changes are also affected by the combination of internal and external factors (mostly dealing with shares).

Thus, there is no single recipe of a structured investment product relevant to any economic environment. However, changes of SIP integral parts, on condition the external structure and philosophy are preserved, are able to maintain the relevance of these investment strategies practically in every historical period.

As a result, depending on market expectations investments may be reinvested in SIPs consisting of call or put options and futures on different shares/primary goods/currency pairs, etc.

Experimentally key disadvantages of structured investment products may be revealed.

Firstly, the investment strategy cannot be fully schemed. It means there still exists a risk of making wrong management decisions while managing the product assets.

Secondly, taking into account that in order to hold the fixed risk, a significant share of assets needs to be invested in riskless instruments, whose profitability, as a rule, is not high, a lot of SIPs are characterized by a low level of the contribution coefficient in the basic asset growth. This problem can be solved if previously to broaden the scope of investment instruments. For example, if to make use of the Gordon model, it is possible to replace risk-free assets with shares with high and easily predicted dividends [15]. However, with this approach it is impossible to end the beta coefficient in the zero value.

Discussion

In the process of SIP asset management it is possible to manage the market component of the overall risk in the investment portfolio constituting the SIP. Nevertheless, risks connected with the human factor and liquidity still influence the efficiency of SIPs [16]. “It is difficult to rationalize investor demand for structured equity products within any plausible normative model of the behavior of rational investors” [5].

Beta coefficient was chosen as a mathematical basis of market risk modelling despite of the existing critics of this approach, for example in [17].

Structured investment products are progressive approach to capital management. However, there remain a number of potentially controversial issues, the answers to which may serve
as a ground for further expansion of the study of perspective:

1) marketing of SIPs;
2) assessment of intrinsic value of SIPs. SIP issuer’s revenue is not restricted only to commission fees: these products have their own value. The SIP’s value depends on a number of factors: transactional and commission expenses of the investment bank, payment for trading and analytical department participation, net value of generated product cash flows and their mobility allowing for the future uncertainty of asset value. increase in the participation rate of the underlying asset growth while preserving the requirements to the maximum allowable loss [7], [18];
3) increasing of the participation rate in the underlying asset growth while preserving the requirements to the maximum allowable loss;
4) SIP role in the product line of an investment bank [19].

Conclusion

The improvement of structured investment products steps over the boundaries of the classical brokerage. The SIP’s issuer takes the opportunity of holding specialized promotion campaigns that highlight the brand and offer unusual investment decisions, which can influence the growth of the client database at the expense of target accounts of classical banks.

The elaborated model allows forming different structured investment products which vary by the class of the used assets. It is important that some structured products are effective only in some time spans, i.e. the investor has to review his inputs every three months or twice a year.

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