Global stabilization for Noval chaotic dynamical system

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Abstract: This paper introduces a simple feedback control strategy for chaotic systems is investigated using the Noval system as an example. Based on the theory of nonlinear differential equations and Gerschgorin theorem, a control scheme is proposed for global stabilizing the unstable equilibria of Noval chaotic dynamical system. Using a suitable designing to the feedback gain matrix which depends on a few algebraic inequalities, chaotic orbits are suppressed and dragged to the target (system's equilibria). Numerical simulation results are presented to verify our control method.
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1. Introduction

Chaos, is an interesting phenomenon in nonlinear dynamical systems, in the last three decades, chaos has been extensively studied within the scientific, engineering and mathematical communities [1- 6].

A chaotic system is a nonlinear deterministic system that displays complex, noisy-like and unpredictable behavior. where one usually expects the system to behave in a predictable way. However, some properties of chaotic sets favor applications where the desired behavior is a periodic oscillation. The sensitive dependence on initial conditions and the presence of a dense set of unstable periodic orbits embedded in chaotic sets leads to the concept of chaos control, where small perturbations are sufficient to stabilize one of the many unstable periodic states. Moreover, since trajectories on chaotic attractors come arbitrarily close to any of the embedded unstable periodic orbits due to ergodicity, there is no need to apply external forces to drive the system to the proximity of the desired state and the control effort from then on is ideally very low, constrained by the noise level. A wealth of numerical and experimental applications of chaos control have been conducted since the introduction of the concept by Ott et al. [1]. In these troublesome cases chaos should be suppressed as much as possible or totally eliminated. Therefore controlling chaos has become one of the most considerable research area in the nonlinear problems ranging from biology, physics and chemistry to economics.

Various control algorithms have been proposed in recent years to control chaotic systems including OGY method [1], linear feedback [1-16], nonfeedback method [17-19], adaptive control [20-24] backstepping method [25-28] and sliding mode control [29-32],. These control algorithms can be used to stabilize a desired unstable periodic orbit (UPO) embedded within a chaotic attractor.

Our aim in this study is to design a suitable feedback control depending on Gerschgorin theorem [33]. This control scheme capable to stabilize chaotic systems globally to the desired equilibrium points.

The rest of the paper is organized as follows. In Section 2, a mathematical description of the control method. In Section 3, the stabilization problem of Noval chaotic dynamical system is investigated and numerical simulation results are also given in Section 4. Finally, in Section 5 the conclusion of the paper is given.

2. Simple global stabilization criterion of chaotic systems:

Simple global stabilization criterion, based on Lyapunov stability theory and Gerschgorin's Theorem [33] is used to design a successful scheme to control chaos. The proposed control method is designed to achieve global stabilization of the unstable equilibria of some dynamical systems. This control technique is efficient and ease of implementation in most real systems. In order to apply this technique to a chaotic system, we rewrite the dynamical system in the form:

$$\dot{x} = Ax + g(x) + u \qquad (1)$$

where $x \in R^n$ is the state vector, $u \in R^n$ is a control input controller vector, $A \in R^{n\times n}$ is a constant matrix, and $g(x)$ is a continuous differential nonlinear function. Let x_e be unstable equilibrium solution of the uncontrolled system (1) ($u = 0$), then x_e satisfies

$$Ax_e + g(x_e) = 0 \qquad (2)$$

Our object is to design a controller $u = k(x_e - x)$ such that the solutions of (1) are converged to x_e. If we define the error vector as $e = x - x_e$ then by subtracting (2) from (1) we obtain the corresponding error dynamical system:

$$\dot{e} = (A-K)e + g(x) + g(x_e) \qquad (3)$$

where $K = diag(k_1, k_2, ..., k_n)$ with $k_i \in R, i = 1,2,...,n,$ is a feedback gain matrix. Let's assume that the function g satisfies

$$g(x) - g(x_e) = M_{x,x_e} e, \qquad (4)$$

where M_{x,x_e} is a bounded matrix and its elements depend on x, x_e. Hence the system (3) can be put in the simple form

$$\dot{e} = (A - K + M_{x,x_e})e \qquad (5)$$

In order to apply the control scheme we need the following essential theorem to derive the conditions of stabilizing the zero solution of the system (4) and its linear part:

Theorem 1: If the feedback gain matrix K is chosen such that

$$\lambda_i \le \mu < 0 \qquad i = 1,2,...,n \qquad (6)$$

where μ is a negative constant and λ_i *are the eigenvalues of the matrix* $(A - K + M_{x,x_e})^T P + P(A - K + M_{x,x_e})$ *with a positive definite symmetric constant matrix P, then the error dynamical system (5) is globally exponentially stable about the origin. [34, 35]*

Our object is to design a suitable feedback gain matrix K such that Theorem 1 is satisfied. Based on the well known Gerschgorin's Theorem [33] in matrix theory we get the following result:

Remark 1: *Choose* $P = diag(p_1, p_2, ..., p_n)$ *and let* $P(A + M_{x,x_e}) + (A + M_{x,x_e})^T P = [\bar{a}_{ij}]$ *and let* $R_i = \sum_{j=1, j\ne i}^{n} |\bar{a}_{ij}|$. *If a suitable feedback matrix K is chosen such that*

$$k_i \ge \frac{1}{2p_i}(\bar{a}_{ij} + R_i - \mu) \quad ,i = 1,2,...,n, \qquad (7)$$

then (6) is satisfied which implies that the zero solution of (4) is globally stable.

Remark 2: *We can simplify the inequalities (7) by taking* $P = I$, *which implies*

$$k_i \ge \frac{1}{2}(\bar{a}_{ij} + R_i - \mu) \quad ,i = 1,2,...,n \qquad (8)$$

Thus the inequalities (7) are sufficient to design a control scheme that makes chaotic orbits of the system (1) converges asymptotically to one of the unstable equilibria of the uncontrolled system ($u = 0$). Hence by choosing a suitable feedback matrix K the inequalities (8) are satisfied. To show the benefit of the proposed control scheme we apply it to Noval chaotic dynamical systems.

3. Stabilizing unstable equilibria of Noval system:

Now we will apply the previous criterion of chaos control to stabilize the unstable equilibria of Noval chaotic dynamical system [36] which is given by the autonomous differential equations:

$$\begin{aligned} \dot{x} &= y - x \\ \dot{y} &= ay - xz \\ \dot{z} &= xy - b \end{aligned} \qquad (9)$$

where $(x, y, z) \in R^3$ and *a* and *b* are real constant parameters.

The divergence of the flow (9) is given by.

$$\nabla.F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = -1 + a < 0,$$

where
$F=(F_1,F_2,F_3)=(y-x,ay-xz,xy-b).$

Hence the system is dissipative under the condition $a<1$.

The system of differential equations (9) has two equilibrium points:

$$E_1=(x_1,y_1,z_1)=(\sqrt{b},\sqrt{b},a)\ and\quad E_2=(x_2,y_2,z_2)=(-\sqrt{b},-\sqrt{b},a). \quad (10)$$

At the values *a=0.5* and *b=0.5*, chaotic behaviour of the Noval system (9) can be observed (see Fig. 1).

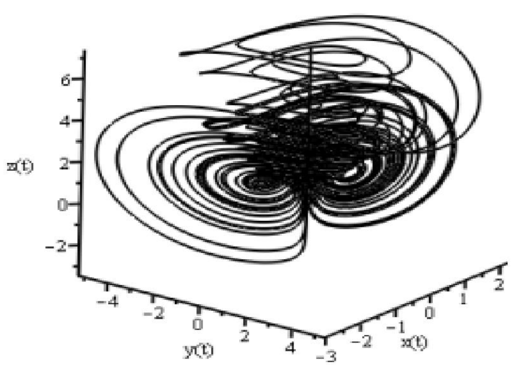

Figure 1: The chaotic attractor of Noval dynamical system at *a=b=0.5* in 3-dimensional.

Since Noval system (9) is a dissipative system thus the solutions of the system of equations (9) are bounded as $t\to\infty$ for $a<1$. Consequently, there exist a positive constants S such that $|x(t)|\le S<\infty, |y(t)|\le S<\infty\ and\ |z(t)|\le S<\infty$ hold for all $t\ge 0$.

The controlled Noval system is described by the following equations:

$$\begin{aligned}\dot{x}&=y-x+u_1\\ \dot{y}&=ay-xz+u_2\\ \dot{z}&=xy-b+u_3\end{aligned}\quad (11)$$

Let (x_e, y_e, z_e) be unstable equilibrium of (8) which we are going to drage the solutions of (8) to it (x_e, y_e, z_e) satisfies the system (8) i. e.

$$\begin{aligned}0&=y_e-x_e\\ 0&=ay_e-x_ez_e\\ 0&=x_ey_e-b\end{aligned}\quad (12)$$

Subtracting (12) from (11), we get :

$$\begin{aligned}\dot{e}_x&=e_y-e_x-k_1e_x,\\ \dot{e}_y&=a e_y-xz+x_e y_e-k_2e_y,\\ \dot{e}_z&=xy-x_e y_e-k_3e_z.\end{aligned}\quad (13)$$

where $e_x=x-x_e, e_y=y-y_e, e_z=z-z_e$, $u_1=-k_1e_x, u_2=-k_2e_y and\ u_3=-k_3e_z$. By using the form (3), the system (13) can be rewritten as

$$\dot{e}=(A-K)e+g(x)+g(x_e)$$

where

$$A=\begin{bmatrix}-1&1&0\\0&a&0\\0&0&0\end{bmatrix}, K=\begin{bmatrix}k_1&0&0\\0&k_2&0\\0&0&k_3\end{bmatrix},$$

$$g(x)=\begin{bmatrix}0\\-xz\\xy\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix} and\ X_e=\begin{bmatrix}x_e\\y_e\\z_e\end{bmatrix}$$

$$g(x)-g(x_e)=\begin{bmatrix}0\\-xz\\xy\end{bmatrix}-\begin{bmatrix}0\\-x_ez_e\\x_ey_e\end{bmatrix}=\begin{bmatrix}0&0&0\\-z_e&0&x\\y_e&x&0\end{bmatrix}\begin{bmatrix}e_x\\e_y\\e_z\end{bmatrix}=M_{x,x_e}e$$

where

$$M_{x,x_e}=\begin{bmatrix}0&0&0\\-z_e&0&x\\y_e&x&0\end{bmatrix}$$

Then the condition (ii) of Theorem 1 is satisfied while the condition (i) comes from choosing K as follows

$$A+M_{x,x_e}=\begin{bmatrix}-1 & 1 & 0\\ -z_e & a & x\\ y_e & x & 0\end{bmatrix}.$$

Hence,

$$(A+M_{x,x_e})+(A+M_{x,x_e})^T=\begin{bmatrix}-2 & 1-z_e & y_e\\ 1-z_e & 2a & 0\\ y_e & 0 & 0\end{bmatrix}$$

according to Theorems 1 and 2 , the equilibrium x_e of the controlled Noval system is asymptotically stable if the following inequalities hold:

$$\begin{aligned}
&k_1 \ge \frac{1}{2}(2+|1-z_e|+|y_e|-\mu)\\
&k_2 \ge \frac{1}{2}(2a+|1-z_e|-\mu) \qquad (14)\\
&k_2 \ge \frac{1}{2}(|y_e|-\mu)
\end{aligned}$$

then the zero solution of linear system

$$\dot{e}=(A-K+M_{x,x_e})e,$$

is asymptotically stable, it follows that the equilibrium x_e of the controlled system (11) is globally asymptotically stable. Since the trajectories of the chaotic dynamos systems are bounded, then the inequality (14) holds for large enough values of k_1, k_2 *and* k_3. According to Theorem 1 and remark 1 the two coupled dynamical systems (11) and (12) are globally asymptotically stabilized.

4. Numerical Results

By using MAPLE 15, to solve the systems of differential equations (11) and (12). The parameters are chosen to *a=0.5* and *b=0.5* in all simulations so that the Noval system exhibits a chaotic behavior if no control is applied (see Figure 1). From the Fig. 1 it can be seen that the solutions $x(t), y(t)$ *and* $z(t)$ are bounded and satisfy the inequalities:

$-3<x<3, -3<y<3$ *and* $-2<z<5.$

Choosing $\mu=0.1$. If we take the control parameters $k_1=3, k_2=2$ *and* $k_3=1$ then the Noval system convergence to $E_1=(\sqrt{b},\sqrt{b},a)$ see Figure 2 and if $k_1=3, k_2=2$ *and* $k_3=1$ then the Noval system convergence to $E_2=(-\sqrt{b},-\sqrt{b},a)$ see Figure 3.

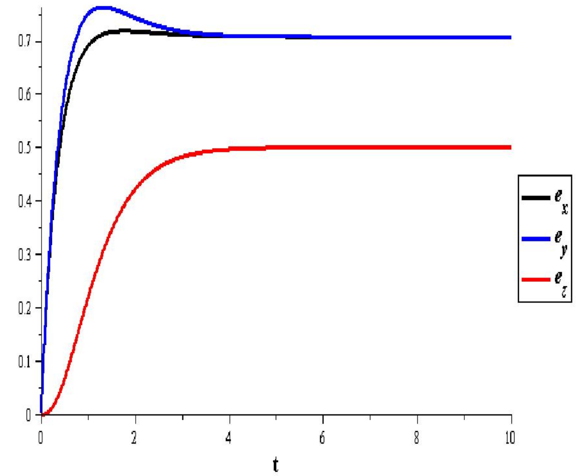

Figure 2: The time response of states for the Noval system convergence to E_1 when $k_1=3, k_2=2$ and $k_3=1$.

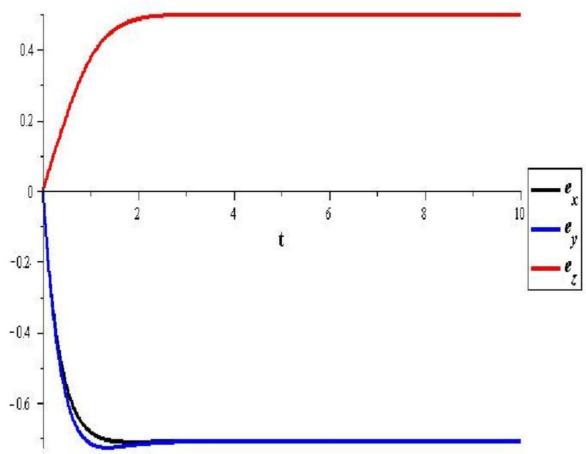

Figure 3: The time response of states for the Noval system convergence to E_2 when $k_1=3, k_2=2$ and $k_3=1$.

5. Conclusion

In this paper, by using feedback linearizing technique, and Gerschgorin theorem, the Noval chaotic dynamical system is successfully globally stabilized. This method can be applied to similar chaotic systems by a suitable choice of the feedback gain matrix. Numerical simulations are used to verify the effectiveness of the proposed control techniques.

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