Parameter analysis of ant algorythm

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Abstract. Ant algorithms are a method of probability directed search of global optimum in solving of the optimization problem. One of the application areas is transporting and logistics problem solving. The examples of such problems are a travelling salesman problem, routing tasks, shipment tasks etc. The given work is devoted to the research of the optimal parameter of the ant algorithm. The given method has appeared in the context of "natural scaling". A main drawback of the ant algorithm is a fitting compilation of optimal parameters. On-the-day researches give"positive" best values estimation of some parameters of the ant algorithm. As a rule the parameters are adjusted on the basis of the many experiments performance. Experimental researches showed some parameter dependence on the number of diagram dimensions. This affords to get out of the under test generation of the optimal algorithm parameters.
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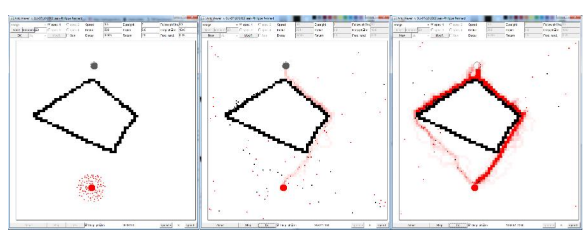
Introduction

The given article is devoted to the search of the ant algorithm (AA) optimal parameter points. AA belongs to the category of "swarm intelligence". Its author is Marko Dorigo, who made researches in this area in the middle 90-s of the XX century [1]. AA got spread currency in graphic problems solving. One of the popular applications is transport and logistics problems solving. To improve the quality of solutions and its compliment, new operators were used, and the agent's behavior was changing [2]. There existing various modifications of AA, hybrid algorithms, based on AA, bee, genetic algorithms, which are among them [3]. The main idea is in ant behavior modeling [4]. Each ant is revealed as a simple agent, conforming to the elementary rules, but in general, it is revealed as an intelligent multi-agent system. One of the main drawbacks of AA is a complicated and lengthy process of the algorithmic parameter setting. Some parameters, such as the amount of ants in the colony is nonlinearly depends on the size of a problem, for example, on the amount of nodes in a graph. In case of taking the natural analogy it is similar the necessity of a larger colony in searching of a larger square to cover it completely. For a small site territory the colony of moderate size is enough. It is important to find an optimal size of the ant colony, because its oversize in the algorithm causes not only the growth of the computing exercises, but also the redundancy in pheromone secreting, which has a negative impact on AA.

Ant algorithm

The main idea of the ant algorithm is the movement of ants modeling. The choice of the direction of ants' movement is performed on food sources proximity (for graph problems they are the verges to adjacent vertices vertexes) and pheromones level. Pheromones are special chemical compounds, naturally shed by ants. In arithmetic model pheromones are stored in some weight matrix $[tau]_{i,j}$ and are represented as real numbers. In actual model the ants shed pheromones as markers on their route. When the ants find the food source, the level of pheromones shed by them increases. The dual negative association is an exhaling of pheromones. The exhaling of pheromones increases on long routes, that is why with the passage of time the pheromones level increases on the shortest route.

The carried modeling is represented In Figure 1.



a) b) c)

Figure 1. Modeling of a pheromone path laying by the usage of software "Ants Viewer":
a) initialization; b) search; c) optimal route

Figure 1 shows various stages of the ant colony, modeled by the software "Ants Viewer" [5]. The depictured aim of the colony is the finding of the shortest route to the food source, which is showed by grey points. Obstructions are marked by black rails, the colony is marked below, and ants are marked by red points. Figure 1b) ants explore the field, leaving red traces of the pheromone. Figure 1c) clearly shows that on the shortest route the pheromone level is well over, consequently, ants "prefer" to follow this route.

The work [6] of Kureichik V.M. and Kazharov A.A. introduces a survey, research and modification of ant algorithms for solving of transportation problems, including the traveling salesman problem. In these problems the results quality, obtaining by means of AA, excels the results quality of other algorithms. Experimental researches were made for the traveling salesman problem.

Traveling salesman problem

To solve this problem we have got: a graph G=*(X,U)*, where $|X|$ = n is a vertex set (cities), $|U|$ = *m* is a branches set (possible ways between the cities). Values matrix *D(i, j)*, where $i, j \in 1, 2, \ldots, n$, represent a cost of moving from vertex x_i to x_j.

It requires finding the permutation φ of the elements of the set *X*, such as that the value of the objective function will be:

$$F(\varphi) - D(\varphi_1, \varphi_n) + \sum_{i=1}^{n-1} D(\varphi_i, \varphi_{i+1}) \rightarrow min$$

Hence, the criterion is the way length and the criterion of minimization is the aim of the problem. If the graph is not fully connected, in the matrix *D,* in the cells, corresponding the absent branches in the graph, the perpetuity is assigned with the result that the underpass with respect to the given branch is excluded [7].

Behavior of ants

Behavior of ants is determined by the algorithm selection of the following route point. A migration probability of an ant from the point *i* to the point *j* is determined by the following formula [4]:

$$\begin{cases} P_{ij,k}(t) = \dfrac{[\tau_{ij}(t)]^{\alpha} \cdot [\eta_{ij}(t)]^{\beta}}{\sum\limits_{l \in J_{i,k}} [\tau_{il}(t)]^{\alpha} \cdot [\eta_{il}(t)]^{\beta}}, j \in J_{i,k} \\ P_{ij,k}(t) = 0, j \notin J_{i,k} \end{cases}, \quad (1)$$

$$\eta_{ij} = 1/D_{ij},$$

where *[alpha]*, *[beta]* are parameters, which assume a track weight coefficient of the pheromone, are the coefficients of heuristics. The pheromone level in the time point *t* at the branch D_{ij} corresponds *[tau]*$_{ij}$*(t).* Parameters *[alpha]* and *[beta]* determine the contribution of two parameters and their influence on the equation.

They determine ants "greed". As *[alpha]*=0 an ant longs to chose the shortest branch, as *[beta]*=0 – the branch with most pheromone [8]. Easy to notice that given formula has the phenomenon of a "roulette wheel". Except listed parameters *m,* the quantity of ants in the colony is of the prime importance. Upon this parameter depends *[tau]*$_{ij}$*(t),* a cumulative size of emitted pheromones at the iteration *t* at the verge from the vertex *I* to the vertex *j.*

The work is devoted to the defining of optimal values *[alpha]*, *[beta], m*. This affords us to find out quasioptimal solutions for smaller period.

Experimental researches

Researches were performed on ECM with following characteristics: Intel(R) Core(TM) 2 Quad CPU Q8400 2.67 GHz, with working memory of 6 gilberts. Experimental researches were performed with standard benchmarks of the travelling salesman problem – graphs from 30 (Oliver's benchmark), vertexes 50, 75 and 98 (Eilon's benchmarks) as well as with tests berlin52, st70 and others [1]. Researches show that the optimal values of coefficients are *[alpha]*=1, *[beta]*=4, the evaporation rate may vary from 0.5 to 0.9 and depends on the assumed graph. A resulting solution and algorithmic parameters depend heavily on characteristics of the considered graph: quantity, condensation, vertex spread etc. The choice of size value *m* of an ant colony is entangled as far as its optimal value is not constant.

Figure 2 performs graphic presentation of objective function (OF) correspondence on algorithmic *[alpha]* and *[beta]* parameters. As is clear from the figure, the optimal values *[alpha]* and *[beta]* are 1 and 4 appropriately.

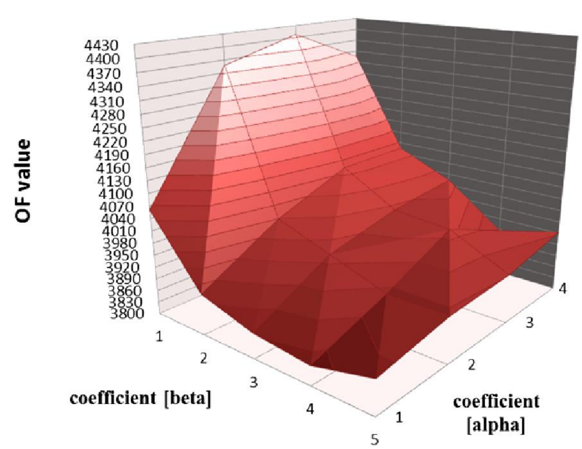


Figure 2. (OF) correspondence on algorithm *[alpha]* and *[beta]* parameters

As the researches showed it is important to increase the colony size with adding of the contribute in the size of the problem. If we enlarge the size of the colony in steps, we may see that as we attain a certain number, the solution quality starts to fall off. In real environment, to cover more area of searching it is necessary to involve more ants. With this, the excess amount of ants may cause an intensive growth of pheromone level, which will effect adversely on the optimal path search. To gain the size of the colony is possible in two cases: in execution time and dimension of a problem increase. In the 1st case we encounter time complexity of the algorithm, depending on the colony dimension which levels out by the execution of time increase of the algorithm. In 2nd case with the problem increase, increases the variation in routes and the stagnation removes, we need to increase the size of the colony. To find out dependence, manifold tests are made, which are represented in table 1. Columns in the table set the size of graphs (n = 30, 60, 90, 120), lines set the size the colony. OF values are recorded in cells.

Table 1. Dependence of the OF expectation value from the colony size

m	OF expectation value n=30	OF expectation value n=60	OF expectation value n=90	OF expectation value n=120
5	445.8			
10	433			
20	436.4	3243.4		
40	**423.6**	3224.2		
60	424.6	3202		
80	427	**3141.8**	3768.2	
100	425.2	3162.8		
120		3174.2	3721.2	
160			**3697.2**	4578.2
200			3723.6	
240			3723.8	4576.8
280			3716.4	
320				**4537**
400				4567.6
480				4565.2
560				4550
640				4557.6

Experimental researches, whose results are presented in table 1, were made for geometric travelling salesman problem with symmetric vertex incidence matrix. Vertex coordinates are generalized accidentally and the symmetric vertex incidence matrix was rated on the account of Euclidean distance between vertexes. Reference parameters of the ant algorithm for table 1 are the following:

- stop restriction – the algorithm doesn't improve more than 0.05%;
- *[alpha]* coefficient = 1;
- *[beta]* coefficient = 4;
- evaporation coefficient – 0.5.

The sizes of graphs, for which the experimental researches were made, are 30, 60, 90 and 120 vertexes appropriately. A life period of the colony was chosen by the means so as to the moment of the end of the period the algorithm won't show improvements more than 0.05%.

As we may see from table 1, the function of OF-dependency on the amount of ants is represented by the following diagram, showed on figure 3.

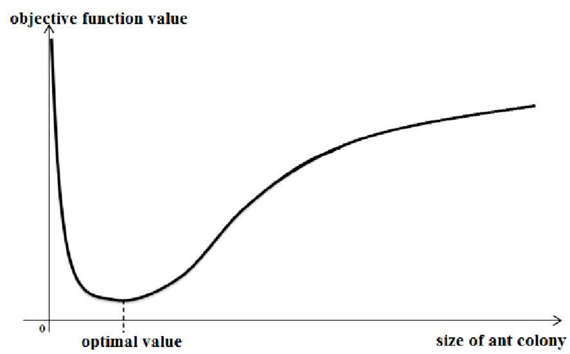


Figure 3. Generalized graph of OF-value dependency from the colony size

The increase of ants leads to the loss in value of OF to the certain moment until the amount of ants m gives the optimal value. With the further increase in the size of the colony, the algorithm starts giving the worst algorithmic solving. This is connected with that the excess amount of ants evolve such amount of pheromones that evaporization process influence becomes small to negliable. This way, with the passage of time edges of the graph become "oversaturated with pheromones". In the same time an excess amount of pheromones cannot be balanced by the increase of vaporization coefficient so as it causes entering the stagnating under the first iterations.

In this case the amount of evolved pheromones is not so high and they are almost completely vaporized. Figure 4 shows the dependency of optimal colony size (amount of ants) from the size of the problem (vertexes amount).

By coarse line the diagram of optimal size of the colony, available from experiments on the basis of table 1 is showed. Tine line reflects a trend line, an approximated second-order curve. Quadratic equation is used as a basis, i.e. rising of a problem order, the edges of graph amount rises quadratically. Detailed analysis and the relationship equation of the optimal amount of ants and vertexes amount in graphic model calculation.

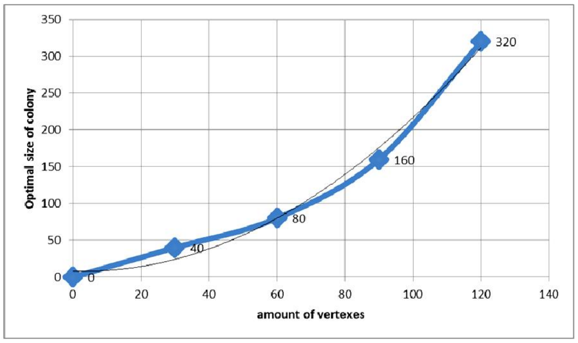


Figure 4. Optimal size of ant colony and the problem order relationship

To determine relationship between two variables and forecasting in mathematical statistics the method of analytic fitting [9], which involves building-up of regression equation, characterizing the relationship between the range level and temporary variable is used.

Choosing the kind of functional relationship between optimal value of the size of ant colony and vertexes amount, a finite differences method may be used [10] (the essential condition of given attitude is the interval congruence between range levels). Finite differences of the first order are differences between consequent levels of range:

$$\Delta^1_t = Y_t - Y_{t-1},$$

where Y is the optimal size of the value of the ant colony for graph with t vertexes amount.

Finite differences of the second order are differences between consequent finite differences of *thirst order*:

$$\Delta^2_t = \Delta^1_t - \Delta^1_{t-1}.$$

Finite differences of *j-order* are differences between consequent finite differences of *(j–1)-order*:

$$\Delta^j_t = \Delta^{j-1}_t - \Delta^{j-1}_{t-1}.$$

If an overall trend of the size of ant colony varying is expressed in linear equation of $Y = a_0 + a_1 t$, the finite differences of first order are constant: $\Delta^1_2 = \Delta^1_3 = \ldots = \Delta^1_n$ and second-order differences are equal to zero. The overall trend is expressed in square parabola and is given by: $Y = a_0 + a_1 t + a_2 t^2$ we obtain constant second-order finite differences.

If increasing frequency of the size of ant colony is roughly constant, the exponential is used to determine functional relationship between the amount of ants and amount of vertexes.

Choosing a form of equation you should proceed from the information volume you have. The more parameters the equation contains, the more experimental observations with the same degree of estimation liability are made.

The choice of the curve shape may be made on the basis of conventional quality criteria of regression equation, the sum of squared deviations of plurality level actual values from level values, calculated on the found functional relationship. Out of curves the one is chosen, to which the minimum criterion value corresponds. Another statistics criterion is the multiple determination coefficient R^2 [11].

Table 2. Computation of the coefficient of multiple determination

y_i	Δ^1_t	Δ^2_t	Rates of growth
0	–	–	–
40	40	–	0
80	40	0	2
160	80	40	2
320	160	80	2

The equation of trend is as follows $y = a_0 + a_1 t + a_2 t^2$. To find out the parameter of equation (table 3) by ordinary least squares (OLS) we use the following system of equations:

$$\begin{cases} a_0 n + a_1 \sum_{i=1}^{n} t + a_2 \sum_{i=1}^{n} t^2 = \sum_{i=1}^{n} y, \\ a_0 \sum_{i=1}^{n} t + a_1 \sum_{i=1}^{n} t^2 + a_2 \sum_{i=1}^{n} t^3 = \sum_{i=1}^{n} yt, \\ a_0 \sum_{i=1}^{n} t^2 + a_1 \sum_{i=1}^{n} t^3 + a_2 \sum_{i=1}^{n} t^4 = \sum_{i=1}^{n} yt^2 \end{cases} \quad (1)$$

in which the summing up of variables is made to this order vertex number $n=5$.

Table 3. Solution of equations system

t	y	t^2	y^2	$t \cdot y$	t^3	t^4	$t^2 y$
0	0	0	0	0	0	0	0
30	40	900	1600	1200	27000	810000	36000
60	80	3600	6400	4800	216000	12960000	288000
90	160	8100	25600	14400	729000	65610000	1296000
120	320	14400	102400	38400	1728000	207360000	4608000
300	600	27000	136000	58800	2700000	286740000	6228000

Substituting data of table 3 in system of equations (1), for our values we get the following equations:

$$5a_0 + 300a_1 + 27000a_2 = 600,$$

$$300a_0 + 27000a_1 + 2700000a_2 = 58800,$$

$$27000a_0 + 2700000a_1 + 286740000a_2 = 6228000,$$

We get $a_0 = 0.0222$, $a_1 = -0.13$, $a_2 = 8$. Equation of optimal value on the parameter of ant colony on vertexes amount dependence:

$$y = 0.0222t^2 - 0.13t + 8.$$

Empirical coefficients of trend are the only estimations of theoretical coefficients and the equation itself reflects only the general tendency in behavior of variables we consider. This way the optimal amount of ants' dependency equation from the amount of vertexes in graph n is described by the following dependency (2):

$$m(n) = 0.0222n^2 - 0.1333n + 8. \quad (2)$$

Determination index is calculated according to the formula (3):

$$R^2=1-\left(\sum_{i=1}^{n}(y_i - y_t)^2\right)/\left(\sum_{i=1}^{n}(y_i - y)^2\right); \quad (3)$$

Error!.

That is 99% of cases influence the data change. In other words, the closeness of functional dependency of the optimal value of the ant colony size from the number of vertexes is high. This way, a value of approximation assurance [11] is equal:

R^2 = 0.99. Let's check the equation assurance (2) at the graph with the amount of vertexes 150. As you can see from table 4, experimental researches confirmed an expected effect. In table cells FO are determined for each experiment. Best solutions are found for 480<m<500 and for m=488 the best solution is found as well. According to the (2) the optimal value of colony is equal to:

$$m(150) = 0.0222*150^2-0.1333*150+ 8 \approx 488.$$

Table 4. FO dependence on the colony size on the graph with 150 vertexes

Ants number	Number of the experiment					Average FO
	1	2	3	4	5	
320	4845	4776	4798	4852	4817	4817.6
480	4836	4834	4778	4747	4795	4798
488	4755	4799	4788	4762	4760	**4772.8**
500	4757	4803	4760	4761	4807	4777.6
520	4887	4729	4872	4900	4721	4821.8
640	4862	4934	4840	4794	4782	4842.4
800	4828	4882	4898	4809	4855	4854.4

Gratitudes

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Conclusion

Presented researches afford us to avoid the necessity of FO-parameter identification. Obtained results afford us to judge concerning the dependency of ant algorithm upon dimension of problem. The approximated formula for optimal size value of ant colony is identified. Using the identified values of colony size we may make the efficiency of FO in travelling salesman and other route problems solving.

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