Method of an evaluation of formation exposing with preliminary data

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Abstract. In the presented paper we discussed a reasonableness of a certain mathematical model of nonequilibrium phase transitions of water in porous media without a consideration of moisture migration, which also describes a determination of basic functional spaces. Nowadays, several methods of oil extraction from productive stratum are known. But contemporary experience requires further improvement. Those problems have attracted attention not only oil industry specialists, but also professionals in other industries for a long time. However, mathematical modeling methods of complex filtration processes are developing in two directions, as strict models development, which are the most sensitive in a context of laws and engineering models using simplified schemes of multiphase fluids filtering in porous media. The main goal of management theory and practice is a formalization of patterns of object's functioning. Mathematical model of objects and processes during a creation of management systems possess a special significance, as well as in an examination of patterns' change of existing processes functioning of an object possess a special importance. That fact is related with an increase of a role of modeling for a discovery of those patterns and a study of complex phenomena. In the most cases, in a case of dynamic processes, data associated with their design, is known, as well as basic qualitative relationships of those processes functioning. In such cases, it is necessary to create an object's control mathematical model based on input and output variables in a context of their normal functioning. However, many technological objects are non-linear and an obtainment of adequate models for them is problematic. Recently, the most common method is secondary method of oil extraction from reservoirs. But in time when the last phase in a development of oil and gas comes, surfactants are starting to be implemented in order to destruct filtration channels. The specified method and its corresponding mathematical models are more widely implemented in Kazakhstan. That fact is related with hardly recoverable nature of oil deposts.

[Tang Jian-Gang, Mukhambetzhanov S., Nulahemaiti B., Baishemirov Z. **Method of an evaluation of formation exposing with preliminary data.** *Life Sci J* 2014;11(10s):321-326] (ISSN:1097-8135). http://www.lifesciencesite.com. 59

Keywords: mathematical modeling techniques, Porous media, Fluid filtration

Introduction

For starters, in order to discuss the task, firstly, considering a determination of main function spaces. $L_q(\Omega)$ - Banach space, which consists of all measurable functions, summarized by Ω with degree of $q \geq 1$. Norm is determined by the equation: $\|u\|_{q,\Omega} = (\int\limits_{\Omega} |u(x)|^q dx)^{1/q}$.

Measurability and summability is understood everywhere in Lebesgue meaning (Cannon, J. R., D. B. Henry and D. B. Kotlov, 1974). Elements $L_q(\Omega)$ are classes of mutually equivalent Ω functions. $L_{q,r}(Q_T)$ - Banach space, which consists of all measurable by Q_T functions with a final norm:

$\|u\|_{q,r,Q_T} = (\int\limits_0^T (\int\limits_{\Omega} |u(x,t)|^q dx)^{r/q} dt)^{1/r}$ 0, in which connection $q \geq 1$ and $r \geq 1$.

$L_{q,q}(Q_T)$ will be defined by - $L_q(Q_T)$, and norm $\|\cdot\|_{q,q,Q_T}$ -through $\|\cdot\|_{q,q,Q_T}$. $a_q^l(\Omega)$ with ℓ as integer - Banach space, which consists of all elements of $Lq(\Omega)$, having generalized constants of all types up to order of ℓ inclusive (Rakhmatullin, R. Sh and I. M. Gallyamov, 1986), summarized by Ω with degree of q (Rabinovich, N. R., 1989). Norm $a_q^l(\Omega)$ is determined by the equation:

$$\|u\|_{q,\Omega}^{(l)} = \sum_{j=0}^{l} << u >>_{q,\Omega}^{(j)} \qquad (1.1)$$

$$<< u >>^{(j)}_{q,\Omega} = \sum_{(j)} \left\| D^j_x u \right\|_{q,\Omega} . \qquad (1.2)$$

Symbol D^j_x means any derivative $u(x)$ to x of j order, and $\sum_{(j)}$ means a summarization by all possible derivatives u of order j. $\overset{0\ l}{a_q}(\Omega)$ – subspace of space $\overset{0\ l}{a_q}(\Omega)$, dense multiplicity which is total of all infinitely differentiable finitary in Ω functions (Meirmanov, A.M., 1986). $\overset{2l,l}{a_q}(Q_T)$ with ℓ integer $(q \geq 1)$ is Banach space, which consists of elements $L_q(Q_T)$, having generalized derivatives of type $D^r_t D^s_x$ with any r and s, meeting inequality $2r + s \leq 2l$, where Q_T – cylinder $\Omega \times (0, T)$,which is a plurality of points (x,t) of (m+1)-dimensional Euclidean space R^{m+1} with x ∈ L-area in R^m, which is undefined open coherent multiple points R^m, that is an undefined open related multiplicity of points R^m, $t \in (0,T)$ (Konovalov A.N., 1988). Norm is determined by the equation:

$$\|u\|^{(2l)}_{q,Q_T} = \sum_{j-0}^{2l} << u >>^{(j)}_{q,Q_T} \qquad (1.3)$$

$$<< u >>^{(j)}_{q,Q_T} = \sum_{(2r+s=j)} \left\| D^r_t D^s_x u \right\|_{q,Q_T} . \qquad (1.4)$$

Summarization $\sum_{(2r+s+=j)}$ is conducted for all non-negative integers r and s meeting a following condition: $2r + s = j$. $H^l(\bar{\Omega})$ – Banach space, which elements are continuous in $\bar{\Omega}$ functions $u(x)$ (Bocharov O.B., Telegin I.G., 2005), having in $\bar{\Omega}$ continuous derivatives up to order of $[l]$, where $[l]$ is an integer part of l inclusively and ending value.

$$|u|^l_\Omega = < u >^{(l)}_\Omega + \sum_{j=0}^{[l]} < u >^{(j)}_\Omega \qquad (1.5)$$

where $< u >^{(0)}_\Omega = |u|^{(0)}_\Omega = \max_\Omega |u|$,

$$< u >^{(j)}_\Omega = \sum_{(j)} \left| D^j_x u \right|^{(0)}_\Omega ,$$

$$< u >^{(l)}_\Omega = \sum_{([l])} < D^{[l]}_x u >^{(l-[l])}_\Omega .$$

Equation (1.5) determines a rate $|u|^{(l)}_\Omega$ in $\chi^l(\bar{\Omega})$ (Peyret, R. and T. D. Taylor, 1983). Similarly $\chi^{l,l/2}(\bar{Q}_T)$ Banach space of functions $u(x.t)$ is defined, which are continuous in Q_T together with all derivatives of type $D^r_t D^s_x$ with $2r + s < l$.

$C^l(\bar{\Omega})$ $(C^l(\Omega))$ - a plurality of continuous in $\bar{\Omega}(\Omega)$ functions (Zhumagulov B.T, Zubov N.V., Monahov V.N., Smagulov Sh.S., 1996), having continuous in $\bar{\Omega}(\Omega)$ derivatives up to order of ℓ inclusive.

Materials and methods

In the presented study we confine out topic to a discussion of areas, piecewise-smooth boundaries. As such areas Ω are understood, shorting of which can be presented in a following form:

$\bar{\Omega} = \bar{\Omega}_1 U ... U \bar{\Omega}_N , \Omega_j \cap \Omega_i = ǿ$, at that each of $\bar{\Omega}_k$ can be in a homeomorphic way reflected on a single ball or cube using functions $z^k_i(x)(i = 1.2.....m; k = 1.....N)$, which are satisfying Lipschitz criteria in $\bar{\Omega}_k$, and those that are characterized by Jacopians of transformation $\left| \frac{\partial z^k}{\partial x} \right|$, which are restricted from below by a constant.

Designating $x^0 = (x^0_1 ...., x^0_m)$ as a certain point of border S of area Ω . Designating $(y_1,..., y_m)$ as a local Cartesian coordinate system with the beginning at the point x^0, if y and x are related by equations $y_i = a_{ik}(x_k - x^0_k)$, $i = 1,..., m$, where a_{ik} is orthogonal numeric matrix, and axis y_m is directed by external to Ω normal line to S at the point x^0.

Designating that the surface S belongs to class $C^l, l>1$, if there is a number $\rho>0$ such, that intersection - S with ball K_ρ of radius ρ with a center in undefined point $x^0 \in S$ is a connected surface, which equation in the local coordinate system $(y_1,...,y_m)$ with beginning at x^0 is in the form $y_m = a(y_1,..,y_{m-1})$, and $a(y_1,..,y_{m-1})$, is a function of class C^l in area $\bar{D}$, which is a projection $\bar{K}_\rho \cap S$ on a plane $y_m = 0$.

On the surface S of class C^{l_1}, $l>1$ settings function $\varphi(s)$. Designating that $\varphi(s)$ is a function class $C^l(s)$, $l>l_1$, if it is as a function $y_1,..,y_{m-1}$ is an element $C^l(\bar{D})$. The largest of norms $|\varphi(y)|_D^{(l)}$, calculated for all points x° of surface S, designating as norm $|\varphi(y)|_s^{(l)}$.

If φ is set to the entire $\bar{\Omega}$ and $\varphi(x) \in C^l(\bar{\Omega})$, on the border S of area, that was occupied by class C^{l_1} to $l_1 \geq \max\{1.l\}$, it defines the function $\varphi(s) = \varphi(x)_{x=s\in S}$ of class $C^l(s)$ .Opposite is also true: If $\varphi(s) \in C^l(S)$, $s \in C^l$, $l>1$, then $\varphi(s)$ can be extended to the entire area Ω so that continued function $\varphi(x)$ belongs to $C^l(\bar{\Omega})$. Moreover, that extension can be made for all the functions $\varphi(x)$ from $C^l(s)$ using only one design, so that norms $|\varphi(s)|_s^l$ and $|\varphi(x)|_\Omega^{(l)}$ will be equivalent. Exactly a continuation of $\varphi(s)$ to Ω will imply, formulating boundary conditions using function $\varphi(x)$.

Theorem 1. Designating $u(x) \in a_q^l(\Omega)$ where l is a positive number and $q>1$. If, $s \in C^l$, then $u/_s \in a_q^{l-1/q}(S)$ and there is inequality.

$$\|u\|_{q,s}^{(l-1/q)} \leq C\|u\|_{q,\Omega}^{(l)}.$$

And as opposite, any designated on S function $u(x) \in a_q^{l-1/q}(S)$ can be continued in area Ω so that $u(x) \in a_q^l(\Omega)$ and $\|u\|_{q,\Omega}^{(l)} \leq C\|u\|_{q,s}^{(l-1/q)}$.

Constants C in both inequalities do not depend on u.

Results

Changing focus of our discussion on certain auxiliary subjects. Designating $u(x)$ as function of $L_q(\Omega)$, $q \geq 1$. Designating $u^{(k)}(x)$ as function $u^{(k)}(x) = \max\{u(x)-k,0\}$. It is clear, that in a case of any k it also belongs to $L_q(\Omega)$, it can be easily seen that, if u and v belong to $L_q(\Omega)$, for all x from Ω following inequality exists:

$$\left|u^{(k)}(x) - v^{(k)}(x)\right| \leq |u(x)-v(x)|, \tag{3.1}$$

One of the consequences of that is well-known.

Lemma 1. If a sequence of functions $u_p(x)$ of $L_q(\Omega)$, $p=1,2...$, converges in function $u(x)$ in norm $L_q(\Omega)$, than sequence $u_p^{(k)}(x)$ converges to $U^{(k)}(x)$ in norm $L_q(\Omega)$.

The following statements are also true:

Lemma 2. If $u(x) \in L_q(\Omega)$ and has derivative $u_{x_i} \in L_q(\Omega)$, than $u^{(k)}$ has derivative $u_{x_i}^{(k)} \in L_q(\Omega)$ particularity, if $u(x) \in a_q^1(\Omega)$, then $u^{(K)}(x) \in a_q^1(\Omega)$. If, in addition, $vrei\max_s u \leq k_0$ then, in a case of $k \geq k_0$ of function $u^{(k)}(x)$ belong to $\overset{0}{a}{}_q^1(\Omega)$.

Lemma 3. Presuming that functions $u(x), u_1(x), u_2(x),...$ belongs to $L_q(\Omega)$ and have generalized derivatives u_{x_i} and $u_{p_{x_i}}$ $p=1,2...$, $L_q(\Omega)$ from for any of i. If $\|u_p - u\|_{q,\Omega} \to 0$ and $\left\|u_{p_{x_i}} - u_{x_i}\right\|_{q,\Omega} \to 0$ with $p \to \infty$, then

$u_p^{(k)}$ and $u_{p_{x_i}}^{(k)}$ converge in $L_q(\Omega)$ for $u^{(k)}$ and $u_{x_i}^{(k)}$ respectively.

Based on a well-known theorem or F.Rellich about compactness of attachment $a_2^1(\Omega)$ in $L_2(\Omega)$ following S.L.Sobolev theorem can be easily proven:

Theorem 2. For an undefined limited area Ω limited multiplicity of functions from $U(x)$ from $\overset{0}{a}{}^1_m(\Omega)$ are compact in $L_p(\Omega)$ from $p < mn/(n-m)$ with $m \le n$ and are compact in $C^\alpha(\bar{\Omega})$ with $\tau < 1 - n/m$ in a case $m > n$. For strictly Lipschitz are and limited sums of those areas restricted multiplicities of functions $U(x)$ from $a^1_m(\Omega)$ are compact in $L_p(\Omega)$ from $p < nm/(n-m)$ with $m \le n$ and are compact in $C^\alpha(\bar{\Omega})$ with $\tau < 1 - n/m$ in a case $m > n$.

Designating X as normalized space. Reflection of multiplicity R of real numbers, which possess properties of linearity and continuity is referred to as linear limited function of function. Plurality of all linear limited functionals in X is referred to as adjoint space and is indicate by symbol X^* . The result of functional v from X^* on element u from X defining as (v, u).

Using the adjoint space X^* weak convergence term is defined. Sequence $\{U_n\}$ of X weakly converges to $u \in X$, if for any $v \in X^*$ $\underset{n\to\infty}{Lim}(v, u_n) = (v, u)$. Multiplicity K in Banach space X is called compact, if from any sequence of its elements subconsequence can be separated out, which converges to element from K. Concept of weakly compactness is defined in a same manner: Multiplicity K of Banach space X is called weakly compact, if from any sequence of its elements subconsequence can be separated out, which weakly converges to element from K.

Discussion

After replacement of independent variables and desired functions: $t' = t/\tau,\ x' = x\sqrt{c/\tau k}$, $a' = a;\ u' = uc/\chi$, $H(u) = H'(uc/\chi)$ following system of equations is obtained (Mukhambetzhanov, S. T. and Baishemirov, Zh. D., 2012):

$$u_t - \nabla u + w_t = 0 \qquad (4.1)$$

$$a_t = \frac{1}{\tau}(H(u) - a) \qquad (4.2)$$

with initial

$$u(x,0) = u_0(x),\ a(x,0) = a_0(x),\ x \in \Omega \quad (4.3)$$

and boundary conditions

$$u(x,t) = u_s(x,t),\ (x,t) \in S_T \qquad (4.4)$$

Function $H(u) = 1$ in a case of $u(x,t) > 0$, $H(u) = 0$ in $u(x,t) < 0$ and $H(0) = a(x,t)$ (Barenblatt, G. I., T.W. Patzek and D.B. Silin, 2002).

Theorem 4. Presuming that boundary $S \in O^2$ and function $v \in a_q^{2,1}(Q_T)$ satisfies conditions (4.3), (4.4), $a_0(x)$ is measurable and $0 \le a_0(x) \le 1,\ x \in \Omega$. Then there is the only solution of problem (4.1) - (4.4). In that context following evaluations are true:

$$\|u\|^{(2)}_{q,Q_T} \le c_1(1 + \|v\|^{(2)}_{q,Q_T}) \qquad (4.5)$$

$$0 \le a(x,t) \le 1, |a_t| \le 1 \ . \qquad (4.6)$$

Positive constant c_1 depends on only q, Ω and T 1 (Mukhambetzhanov, S. T. and Baishemirov, Zh. D. , 2013):

Note. In a case of $q > (m+2)/2$ solution $u \in \chi^\alpha(Q_T)$ for some α >0. For $q > m+2$. ∇u also becomes of Hölder type (Schavelev, N. P. , A. V. Karpov and V.S. Sysoev, 1982).

A proof of a uniqueness of the task's solution is of a certain interest. For the proof of uniqueness a solubility of adjoint task is shown. Presuming that u_i, a_i ($i = 1,2$) are two solutions of the problem (4.1) - (4.4). Supposing, that $u = u_1 - u_2$, $a = a_1 - a_2$, $\bar{\chi} = \chi(u_1) - \chi(u_2)$. Then functions $u, a, \bar{\chi}$ are meeting equations (4.1) - (4.2) and conditions

$$u|_{t=0} = a|_{t=0},\ u|_{S_T} = 0 \ . \qquad (4.7)$$

Auxiliary functions F^δ and $F^{\varepsilon,\delta}$ are introduced: $F^\delta = \bar{H}/u$ in multiplicity $B = \{(x,t) \in Q_T |\ |u| > \delta\}$, and $F^\delta = 0$ in Q_T / B , and function $F^{\varepsilon.\delta}$ is selected considering conditions $F^{\varepsilon,\delta} \in \overset{0}{C}{}^{\infty}(Q_T)$, $0 \le F^{\varepsilon,\delta} \le F^{\delta}$, $\lim\limits_{\varepsilon \leftarrow 0} \|F^{\varepsilon,\delta} - F^{\delta}\|_{1,Q_T} = 0$.

In Q_T functions φ, ψ are considered, which are sufficiently smooth and satisfy conditions:

$$\varphi,\psi\big|_{t\in[T_0,T]} = 0,\ \varphi\big|_{S_T},\ 0 < T_0 \le T \qquad (4.8)$$

From (4.1), (4.2), (4.7), and (4.8) equality is developed:

$$\iint_{Q_T} \left\langle u \cdot M_1(\varphi,\psi) + a \cdot M_2(\varphi,\psi) + (\bar{\chi} - F^{\varepsilon,\delta} u)\psi \right\rangle dxdt = 0, (4.9)$$

where $M_1(\varphi,\psi) = \varphi_t + \nabla\varphi + F^{\varepsilon,\delta}\psi$, $M_2(\varphi,\psi) = \varphi_t + \psi_t + \psi$. Presuming, that $G_1, G_2 \in \overset{0}{C}{}^{\infty}(Q_{T_0})$. In area Q_{T_0} equations are considered

$$M_i(\varphi,\psi) = G_i(x,t),\ i = 1,2..... \qquad (4.10)$$

Solubility of the problem (4.8), (4.10) is shown in a standard manner, based on local theorem of existence and a priori estimates. From (4.2) representation for $\psi(x,t)$ is developed:

$$\psi(x,t) = \int_t^{T_0} \{\psi(x,s) + G_2(x,s)\} ds - \varphi(x,t) \quad (4.11)$$

Function $\Phi(x,t)$ is determined by equation

$$\Phi(x,t) = e^t \cdot \varphi(x,t) \qquad (4.12)$$

Following equation is a consequence of (4.2) - (4.12) (Baishemirov, Zh.D., B.E. Bekbauov and A. Kaltayev, 2012):

$$\Phi(\Phi_t + \nabla\Phi) - (1 + F^{\varepsilon,\delta}) \cdot \Phi^2 = \left\{ G_1 - F^{\varepsilon,\delta} \cdot \int_t^{T_0} (\psi + G_2) ds \right\} e^t \Phi \quad (4.13)$$

Discussing (4.13) at the point of inner maximum of function (Pen'kovksy V.I., 1996). The first two members at that point are not positive and, considering (4.12) and from (4.11), (4.14) following evaluations are obtained:

$$|\varphi| \le |\Phi| \le \left\{ \int_0^{T_0} (|\psi| + |G_2|) dt + |G_1| \right\} e^{T_0} \quad (4.14)$$

$$\|\psi\|_{\infty,Q_{T_0}} \le e^{T_0} \cdot \|G_1\|_{\infty,Q_{T_0}} + C_2 T_0, \qquad (4.15)$$

Where constant C_2 only depends on T and norm $\|G_i\|_{\infty,Q_{T_0}}$ $i = 1,2......$ After limiting transition in $\varepsilon \to 0$ (Zhumagulov, B. T. and V. N. Monakhov, 2003), and then in $\delta \to 0$, because of evaluation (4.15) and conditions (4.7), equation (4.9) becomes:

$$\iint_{Q_{T_0}} \{U \cdot G_1 + wG_2\} dxdt + \iint_E \psi \cdot wdxdt = 0, \quad (4.16)$$

where $E = \{(x,t) \in Q_T | u = 0\}$. It can be considered that $G_i(x,t),\ i = 1,2.....$ are undefined functions $\overset{0}{C}{}^{\infty}(Q_{T_0})$. Presuming, that $T_0 = \min\{T, 1/2G_2\}$.

Selecting $G_1 = 0, G_2 = signw(x,t)$, from (4.15), (4.16) obtaining $w = 0$ in Q_{T_0}. For $G_1 = signu(x,t)$ $G_2(x,t) = 0$ from (4.16) equality $u = 0$ in Q_{T_0} is derived. Similarly uniqueness in $\Omega \times [T_0, 2T_0]$, $\Omega \times [2T_0, 3T_0]$ and so on is shown. For the final number of steps uniqueness Q_T is obtained.

Conclusion

1.Generalized solution for Stefan problem

Presuming that Ω_T -cylinder $\Omega \times (0,T)$, where $\Omega \subset R^n$, $\partial\Omega \equiv S$ -boundary of area Ω, $S_T = S \times (0,T)$. Stefan problem is a problem about a definition in Ω_T function $c(x,t)$ satisfying equation

$$\frac{\partial U}{\partial t} = \Delta c, \qquad (5.1)$$

and conditions

$$c\big|_{S_T} = c^0(s,t), (s,t) \in S_T \qquad (5.2)$$

$$U\big|_{t=0} = u_0(x),\ x \in \Omega \qquad (5.3)$$

Theorem 5. Presuming that designated temperature $c^0 \in a_2^2(\Omega_T)$ primary distribution $u_0(x)$ unit of internal energy is measurable limited function that function is $c_0(x) = \chi(u_0) \in a_2^1(\Omega)$ .Then there is the only generalized solution for Stefan problem (5.1) - (5.3): $u \in L_\infty(\Omega_T)$ and $c = \chi(u) \in L_\infty(0,T; a_2^1(\Omega)) \cap a_2^{1,1}(\Omega_T)$

2.Periodic in time generalized solution for Stefan problem

Presuming $c^0(s,t)$ -periodic in time with period T function. The task is to obtainment in $Q_\infty = \Omega \times (-\infty, +\infty)$ function $c(x,t)$ meeting equation, conditions (5.2) and

$$U(x,t) = U(x, t+T),\ (x,t) \in Q_\infty \quad (5.4)$$

The solution for Stefan problem is called periodic in time with period T.

Theorem 6. If $c^0(x,t)$ -periodic in time with period T function of class $a_2^2(\Omega\times(-T,T))$, , then there is the only periodic in time with period T generalized solution for Stefan problem (5.1), (5.2), (5.4): $U \in L_\infty(Q_\infty)$ $c = \chi(U) \in L_\infty(-\infty,+\infty; a_2^1(\Omega))$.

Obtaining a new system of equations after a replacement of independent variables, desired functions, we were able to obtain evaluations and prove that the solution to that problem exists and it is unique. A solubility of adjoin solutions had proved uniqueness. On the basis of local theorem of existence and a priori evaluations it was proved that a solubility of the problem is standard. We presume, that in the future it is necessary to describe the work in more detail and more acceptable for a creation of mathematical models.

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