Formation at students of abilities of mathematical thinking by complication of trigonometrical expressions

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Abstract. Formation process theoretically locates in article abilities of mathematical thinking at students by complication of trigonometrical expressions. Ways of transformation of trigonometrical expressions by means of complication process are considered. The purpose of transformation process is development of an intellectual outlook of students. This process decides on the help of complication. Actuating of cogitative activity by means of expressions with small information before expression with bigger information is called as complication. Complication and simplification - mutually the return processes. In comparison with simplification in the course of complication the share of formation and development of knowledge is bigger. The formation Model at students of abilities of mathematical thinking by complication of trigonometrical expressions is developed.

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Introduction

Analyzing researches Cong, I.M. [1, p.554], Orlov, A.N., Timbalist, O.V. [2, p.279], Karl Wesley Kosko and Anderson Norton [3, p.340], Abylkasymova A.E. [4, p.272], Sakenov, D. Zh. [5, p.1431], Esmukhan M. E. [6, p.300], David K. Pugalee [7, p.236], George L. Trigg [8, p.33], Kellah Edens and Ellen Potter [9, p.184], Page Starr and Vladimir Rokhlin [10, p.1117], Kai Velten [11, p.40], Cheryl, A. Lubinski and Albert D. Otto [12, p.336], Peter, D. Miller, Nicholas M. Ercolani, Igor M. Krichever and C. David Levermore [13, p.1369], Bidosov, B.E. [14, p.224], Alpysov, A.K., [15, p.17], we consider that the mathematical thinking — is one of types of the thinking, directed on the solution of mathematical problems and the tasks, being characterized use of mathematical concepts and symbols. In the higher education the mathematical thinking occupies one of leading places, as during studying fundamental mathematical sciences, and special disciplines, owing to that role which it plays in formation of intellectual potential of the personality.

However now the problem of development of mathematical thinking is solved along with assimilation by students of a program material and, as a rule, isn't allocated as an independent task. Objectively in the course of knowledge acquisition at students in a certain degree the mathematical thinking develops, but such spontaneous way is obviously insufficient: students have to realize clearly cogitative tasks, know the main ways of their decision, be able plan independently the activity, in particular at the solution of trigonometrical expressions.

Relevance of research of formation of abilities of mathematical thinking by complication of trigonometrical expressions is caused by need of permission of the following contradictions between: need of educational practice for development of mathematical thinking and insufficient readiness of this problem for pedagogical science; the importance of use of mathematical thinking in connection with the increased requirements to future experts, an increasing flow of information on the one hand, and lack of exhaustive researches of the pedagogical conditions providing efficiency of its development with another; need of purposeful management of mathematical cognitive activity in the course of which there is a development of mathematical thinking and degree of theoretical judgment of its essence and structure.

There is a scientific problem, under what conditions achievement by students of a level of development of mathematical thinking, sufficient is possible to provide requirement of educational practice and aspiration of the personality to self-development. For formation of creative abilities value of mathematics is special. Mathematical tasks help with development of laws and properties, and also with improvement of process of mathematical thinking. Basis of process of mathematical thinking are mathematical expressions. Without obtaining information on expression we can't think, and also solve a problem. Information as a part of expression at the solution of a task sets cogitative activity in motion. Usually this process is carried out at expression movement with bigger information before expression with smaller information. During performance of this process in the opposite direction and when transforming task, qualification of readers increases. It is complication process. If when transforming one or some members of initial expression are replaced with only one member, then it will be simplification process. And if when transforming one member of initial expression is replaced by several members, then this process is called as complication. On the basis of the analysis of works of Karl Wesley Kosko and Anderson Norton [3, p.340], Sakenov, D. Zh. [5, p.1431], Esmukhan M. E. [6, p.300], David K. Pugalee [7, p.236], George L. Trigg [8, p.33], Kai Velten [11, p.40], Cheryl, A. Lubinski and Albert D. Otto [12, p.336], Peter, D. Miller, Nicholas M. Ercolani, Igor M. Krichever and C. David Levermore [13, p.1369] in work we define the research direction that process of complication is of great importance in formation of knowledge and mathematical thinking. Research objective is justification of formation of abilities of mathematical thinking of students by complication of trigonometrical expressions. Considered complications of trigonometrical expressions help with development of system of mathematical thinking of students, to deep assimilation of theoretical materials. The tasks solved on the basis of this direction, increase knowledge and efficiency, and also qualification of students.

Methods

The logic of research is caused by use of system of the methods complementing each other: the theoretical analysis philosophical, psychology and pedagogical pedagogical, and natural-science literature on a problem; modeling; studying of pedagogical experience; diagnostics, questioning, purposeful pedagogical supervision, interviewing; conversations, oral and written polls, testing; pedagogical experiment; introspection and self-assessment students of the activity; statistical processing of materials of research; analysis of results of skilled and experimental work.

Main part

The mathematical thinking is the cerebration of the personality subordinated to mathematical laws, directed on studying of world around and establishment of regularities between various subjects and the reality phenomena. Development of mathematical thinking happens by means of inclusion of students in mathematical cognitive activity which us is understood as a form of active knowledge the person of spatial representations and the quantitative relations of world around, for the purpose of their transformation and change. The structure of mathematical cognitive activity is based on the general structure of activity, and has own specifics expressed by mathematical subject actions and mathematical abilities which center of association the operational component of mathematical thinking is - nature of interrelation of structure of mathematical thinking consists in it with structure of mathematical cognitive activity.

Pedagogical conditions of formation of abilities of mathematical thinking by complication of trigonometrical expressions:

- formation of positive motivation of development of mathematical thinking;
- readiness of teachers for activity on development of mathematical thinking of students, ensuring variability of mathematical cognitive activity taking into account conceptual provisions of the content of education;
- establishment of the subject and subject relations between the teacher and students;
- development of informative activity and independence of students by means of implementation of problem training.

Example 1. To prove identity

$$1=\frac{1+\sin 2\alpha}{cos2\alpha ctg\left(\frac{\pi}{4}-\alpha\right)}$$

Decision. We will prove this identity a complication method, i.e. we will transform by means of the law of replacement of expression with small information (the simplest structure of expression) expression with bigger information (difficult structure of expression). For initial object we take number 1 in the left part, and for a reference point we take expression in the right part of equality. We will write down a statement of the problem in mathematical language.

$$B: B_1(a);\quad O: B_2(A).$$

$$B:\ 1;\qquad O:\frac{1+\sin 2\alpha}{cos2\alpha ctg\left(\frac{\pi}{4}-\alpha\right)}.$$

According to a reference point at first it is necessary to transform 1 in the form of fraction, i.e. we will write down 1 in a look of $\frac{a}{a}$. Certainly, at once it is visible that for an a it is impossible to receive expression $1+\sin 2\alpha$. As this structure isn't connected directly with a cotangent. So, for initial object it is necessary to take or $cos2\alpha$, or $ctg\left(\frac{\pi}{4}-\alpha\right)$. This information is available in argument of a cotangent (a difference of two corners) and if to consider that the cotangent will be transformed through a cosine $ctg\left(\frac{\pi}{4}-\alpha\right)$, is independent expression, and is dependent expression for it $cos2\alpha$. During complication the thought moves from dependent to independent expression. So, it is necessary to take $cos2\alpha$ for *a*. We will write down results of system of thinking in mathematical language

$$B:{}_1 = B_1:\frac{\cos 2\alpha}{\cos 2\alpha}.$$

If to compare dependent and independent expressions ($cos2\alpha$ and $ctg\left(\frac{\pi}{4}-\alpha\right)$) as as a part of a cotangent there is one α, thus there is a need to transform $cos2\alpha$ through one α. Thus,

$$B_1 \Rightarrow B_2:\frac{\cos^2\alpha-\sin^2\alpha}{\cos^2\alpha-\sin^2\alpha}.$$

we will compare B_2 with $ctg\left(\frac{\pi}{4}-\alpha\right)$.

Difference here in degrees. Using a formula of a difference of squares of two expressions, we eliminate this difference:

$$B_2 \Rightarrow B_3:$$

$$\frac{(\cos\alpha-\sin\alpha)(\cos\alpha+\sin\alpha)}{(\cos\alpha-\sin\alpha)(\cos\alpha+\sin\alpha)}$$

we will compare B_3 with $ctg\left(\frac{\pi}{4}-\alpha\right)$.

Here a difference in absence in argument of function of $\pi/4$. To enter this corner it is necessary numerator and a denominator of fraction to increase on $\sqrt{2}/2$. Then

$$B_3 \Rightarrow B_4:$$

$$\frac{\frac{\sqrt{2}}{2}(\cos\alpha-\sin\alpha)(\cos\alpha+\sin\alpha)}{\frac{\sqrt{2}}{2}(\cos\alpha-\sin\alpha)(\cos\alpha+\sin\alpha)}=$$

$$B_5:\frac{\left(\frac{\sqrt{2}}{2}\cos\alpha-\frac{\sqrt{2}}{2}\sin\alpha\right)(\cos\alpha+\sin\alpha)}{\left(\frac{\sqrt{2}}{2}\cos\alpha+\frac{\sqrt{2}}{2}\sin\alpha\right)(\cos\alpha-\sin\alpha)}=B_6:\frac{\left(\sin\frac{\pi}{4}\cos\alpha-\cos\frac{\pi}{4}\sin\alpha\right)(\cos\alpha+\sin\alpha)}{\left(\cos\frac{\pi}{4}\cos\alpha+\sin\frac{\pi}{4}\sin\alpha\right)(\cos\alpha-\sin\alpha)}=$$

$$=B_7:\frac{\sin\left(\frac{\pi}{4}-\alpha\right)(\cos\alpha+\sin\alpha)}{\cos\left(\frac{\pi}{4}-\alpha\right)(\cos\alpha-\sin\alpha)}=B_8:\frac{tg\left(\frac{\pi}{4}-\alpha\right)(\cos\alpha+\sin\alpha)}{(\cos\alpha-\sin\alpha)}$$

Transformations from B_4 to B_8 are carried out through internal information. It quickly is defined when transforming function of a cotangent by a cosine and a sine. We will compare structure B_8 to structure O. Difference in lack of expression $cos2\alpha$ and a cotangent. For elimination of this difference we will increase fraction on $\cos\alpha+\sin\alpha$ and we will replace a tangent with a cotangent. Then

$$B_8 \Rightarrow B_9:$$

$$\frac{(\cos\alpha+\sin\alpha)(\cos\alpha+\sin\alpha)}{ctg\left(\frac{\pi}{4}-\alpha\right)(\cos\alpha-\sin\alpha)(\cos\alpha+\sin\alpha)}=$$

$$=B_{10}:\frac{(\cos\alpha+\sin\alpha)^2}{ctg\left(\frac{\pi}{4}-\alpha\right)(\cos^2\alpha-\sin^2\alpha)}$$

$$=B_{11}:\frac{1+\sin 2\alpha}{ctg\left(\frac{\pi}{4}-\alpha\right)\cos 2\alpha}.$$

In the course of transformation we took development of thinking of pupils for the basic. As noticed from practice, at pupils the thinking when they carry out on one operation develops.

Now in a considered task we will change a way of the proof. We will define similarity and a difference in thinking system.

Example 2. To prove identity

$$\frac{1+\sin 2\alpha}{\cos 2\alpha\, ctg\left(\frac{\pi}{4}-\alpha\right)}=1$$

Decision. We will obtain the evidence of identity by means of regularity of transition of expression with bigger information before expression with smaller information (simplification process). For initial object we take expression in the left part of equality, and for a reference point we take expression in the right part of equality. We will write down a statement of the problem in mathematical language

$$B:B_1(A);\qquad O:B_2(a).$$

$$B: \quad \frac{1+\sin 2\alpha}{\cos 2\alpha\, ctg\left(\frac{\pi}{4}-\alpha\right)}; \qquad O: \; 1$$

Process of transformation of number 1 in the right part of equality in connection with incomplete information is carried out through internal information. For the purpose of elimination $1+\sin 2\alpha$, $\cos 2\alpha$, $\frac{\pi}{4}$, transformation can be begun with a cotangent. If information (an example 1) serves in a reference point for management of thought, the requirement of the second example such service can't fulfill. In this regard the thinking system for an example 2 remained disorder. This is the first feature. As transformation is carried out through internal information them it is possible to write down one behind another. Thus,

$$B: \frac{1+\sin 2\alpha}{\cos 2\alpha\, ctg\left(\frac{\pi}{4}-\alpha\right)} = B_1: \frac{1+\sin 2\alpha}{\cos 2\alpha \dfrac{\cos\left(\frac{\pi}{4}-\alpha\right)}{\sin\left(\frac{\pi}{4}-\alpha\right)}} =$$

$$B_2: \frac{(1+\sin 2\alpha)\sin\left(\frac{\pi}{4}-\alpha\right)}{\cos 2\alpha \cos\left(\frac{\pi}{4}-\alpha\right)} =$$

$$B_3: \frac{(1+\sin 2\alpha)\left(\sin\frac{\pi}{4}\cos\alpha-\cos\frac{\pi}{4}\sin\alpha\right)}{\cos 2\alpha\left(\cos\frac{\pi}{4}\cos\alpha+\sin\frac{\pi}{4}\sin\alpha\right)} = B_4: \frac{(1+\sin 2\alpha)\frac{\sqrt{2}}{2}(\cos\alpha-\sin\alpha)}{\cos 2\alpha\frac{\sqrt{2}}{2}(\cos\alpha+\sin\alpha)} =$$

$$B_5: \frac{(1+2\sin\alpha\cos\alpha)(\cos\alpha-\sin\alpha)}{(\cos^2\alpha-\sin^2\alpha)(\cos\alpha+\sin\alpha)} = B_6: \frac{(\sin^2\alpha+\cos^2\alpha+2\sin\alpha\cos\alpha)(\cos\alpha-\sin\alpha)}{(\cos\alpha+\sin\alpha)^2(\cos\alpha-\sin\alpha)} =$$

$$B_7: \frac{(\cos\alpha+\sin\alpha)^2}{(\cos\alpha+\sin\alpha)^2} = 1.$$

Now we will define regularity between complication and simplification for formation of knowledge. At complication dependence between requirements of functions is defined and thus carried-out transformations will be ordered. In a different way, the system of thinking of pupils is ordered, their thoughts are put in action.

As we noticed, during process of complication eleven are executed, and at simplification seven operations. The set of operations defines thinking system, thus the continuity is defined. Thus, it is possible to claim that the share of the complicated tasks for process of formation of knowledge is more powerful, in comparison with simplification process. In a different way, by drawing up tasks through complication process the thinking of pupils in connection with uniform motion, will gradually develop. And for thinking development the method of simplification gives smaller effect. It can be seen in calculations below

$$\frac{1+\sin 2\alpha}{\cos 2\alpha\, ctg\left(\frac{\pi}{4}-\alpha\right)} = 1 \Rightarrow$$

The fraction is equal to unit in only case when the numerator and a denominator are equal, i.e.

$$1+\sin 2\alpha = \cos 2\alpha\, ctg\left(\frac{\pi}{4}-\alpha\right),$$

$$\cos 2\alpha\, ctg\left(\frac{\pi}{4}-\alpha\right) = (\cos^2\alpha-\sin^2\alpha)\frac{\cos\alpha+\sin\alpha}{\cos\alpha-\sin\alpha},$$

$$\cos 2\alpha\, ctg\left(\frac{\pi}{4}-\alpha\right) = (\cos\alpha+\sin\alpha)^2 = 1+\sin 2\alpha.$$

Complication and simplification mutually return operations. If when transforming one or some members of initial expression are replaced with one member, this process is called as simplification. Because the number of members here decreases. During transformation if to increase numerator and a denominator of fraction (to divide) into the same number or expression, its value isn't changed and when using formulas of abridged multiplication the players of expression become simpler. If when transforming one member of initial expression is replaced by several members, then this process is called as complication.

Example 3. To prove identity

$$\frac{\sin 4\alpha - \sin 2\alpha + \sin\alpha}{\cos 4\alpha + \cos 2\alpha + \cos\alpha} = tg\alpha$$

Decision. Because process of complication is of great importance in formation of knowledge, we will prove identity by means of a complication method. We will write down the requirement of a task in mathematical language

$$B: \quad tg\alpha; \qquad O: \quad \frac{\sin 4\alpha - \sin 2\alpha + \sin\alpha}{\cos 4\alpha + \cos 2\alpha + \cos\alpha}$$

Structures B and O different. By consideration from the point of view of compliance B the structure has to be also fraction

$$B:\ tg\alpha = B_1:\ \frac{\sin\alpha}{\cos\alpha}$$

In fraction O the numerator and a denominator are the sums. That in structure B_1 there was a sum it is necessary to increase fraction by one function. On property of fraction B_1 from multiplication of numerator and a fraction denominator by any function its structure doesn't change. As are trigonometrical expressions B_1 and O, the fraction should be increased by any trigonometrical function. To receive the sum, it is necessary numerator and a denominator to increase on. Then

$$B_1 \Rightarrow B_2:\ \frac{\sin\alpha\cdot 2\cos\beta}{\cos\alpha\cdot 2\cos\beta} = B_3:\ \frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\cos(\alpha-\beta)+\cos(\alpha+\beta)}.$$

We will compare B_3 and O. From here we will come to a conclusion that β - is an unknown size and arguments of trigonometrical functions have to be equal, i.e. $\alpha+\beta=4\alpha,\ \beta-\alpha=2\alpha.$ From here $\beta=3\alpha.$ Between B_3 and O there is one more difference. This unequal quantity of the composed. To eliminate this difference it is necessary to increase B_1 on $2\cos 3\alpha+1$. Then

$$B_1 \Rightarrow B_4:\ \frac{\sin\alpha(2\cos 3\alpha+1)}{\cos\alpha(2\cos 3\alpha+1)}$$

$$= \frac{2\sin\alpha\cos 3\alpha+\sin\alpha}{2\cos\alpha\cos 3\alpha+\sin\alpha} = \frac{\sin 4\alpha-\sin 2\alpha+\sin\alpha}{\cos 4\alpha+\cos 2\alpha+\cos\alpha}.$$

So, the identity is proved.

Example 4. To prove identity

$$\frac{\sin^4\alpha+\cos^4\alpha-1}{\sin^6\alpha+\cos^6\alpha-1} = \frac{2}{3}$$

Decision. We will prove identity by means of regularity of transition of expression with small information to expression with bigger information. We will write down the requirement of a task in mathematical language:

$$B:\ \frac{2}{3};\quad O:\ \frac{\sin^4\alpha+\cos^4\alpha-1}{\sin^6\alpha+\cos^6\alpha-1}$$

According to a reference point the initial object needs to be transformed in the form of fraction. As process of complication is carried out gradually, necessary means for transformation are also gradually and from structure of difficult expression information gradually reveals. In this regard from each structure O information search is carried out. In a reviewed example O is fraction. To allocate an indicator of the third degree from an indicator of the sixth degree, we enter designations $a=\sin^2\alpha$, $b=\cos^2\alpha$, and we will transform a denominator. Then the denominator of fraction O will be transformed in a look a^3+b^3-1. If to spread out here the sum, we will receive expression $(a+b)(a^2-ab+b^2)-1$. From here we will come to a conclusion that information on dependence on a sign and expression structure in O is available. It ($-ab$), i.e. process of complication begins with expression ($-\sin^2\alpha\cos^2\alpha$). So:

$$B:\ \frac{2}{3} = B_1:\ \frac{-2\sin^2\alpha\cos^2\alpha}{-3\sin^2\alpha\cos^2\alpha}$$

We will add 1 and (-1) to numerator and a denominator B_1. To restore necessary indicators for a sine and the cosine, reduced 1 it is replaceable on trigonometrical 1, then we will build them respectively in the second and third degree. They too are necessary for process of complication of the transformations following from a reference point. Then

$$B_1 \Rightarrow B_2:\ \frac{1-2\sin^2\alpha\cos^2\alpha-1}{1-3\sin^2\alpha\cos^2\alpha-1} = B_3:\ \frac{(\sin^2\alpha+\cos^2\alpha)^2-2\sin^2\alpha\cos^2\alpha-1}{(\sin^2\alpha+\cos^2\alpha)^3-3\sin^2\alpha\cos^2\alpha-1} =$$

$$= B_4:\ \frac{\sin^4\alpha+\cos^4\alpha+2\sin^2\alpha\cos^2\alpha-2\sin^2\alpha\cos^2\alpha-1}{\sin^6\alpha+\cos^6\alpha+3\sin^4\alpha\cos^2\alpha+3\sin^2\alpha\cos^4\alpha-3\sin^2\alpha\cos^2\alpha-1} =$$

$$= B_5:\ \frac{\sin^4\alpha+\cos^4\alpha-1}{\sin^6\alpha+\cos^6\alpha+3\sin^2\alpha\cos^2\alpha(\sin^2\alpha+\cos^2\alpha)-3\sin^2\alpha\cos^2\alpha-1} =$$

$$= B_6:\ \frac{\sin^4\alpha+\cos^4\alpha-1}{\sin^6\alpha+\cos^6\alpha-1}.$$

So, the identity is proved. From this the conclusion that in the course of complication the knowledge is formed follows.

Thus, we see that for formation of knowledge and increase of ability of thinking the share of process of complication in comparison with simplification is bigger.

Example 5. To prove identity

$$\frac{tg\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)(1+\sin\alpha)\frac{1}{\cos\alpha}-2\cos 2\alpha}{tg\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)(1+\sin\alpha)\frac{1}{\cos\alpha}+2\cos 2\alpha}=tg\left(\frac{\pi}{6}+\alpha\right)tg\left(\frac{\pi}{6}-\alpha\right)$$

Decision. We will prove identity by means of complication process. We will write down a statement of the problem in mathematical language

$$B:\ tg\left(\frac{\pi}{6}+\alpha\right)tg\left(\frac{\pi}{6}-\alpha\right) \qquad O:$$

$$\frac{tg\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)(1+\sin\alpha)\frac{1}{\cos\alpha}-2\cos 2\alpha}{tg\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)(1+\sin\alpha)\frac{1}{\cos\alpha}+2\cos 2\alpha}.$$

We will bring the requirement of a task into accord with the set expression

$$B\Rightarrow B_1:$$

$$\frac{\sin\left(\frac{\pi}{6}+\alpha\right)\sin\left(\frac{\pi}{6}-\alpha\right)}{\cos\left(\frac{\pi}{6}+\alpha\right)\cos\left(\frac{\pi}{6}-\alpha\right)}=$$

$$=B_2:\frac{\frac{1}{2}\left(\cos\left(\frac{\pi}{6}+\alpha-\alpha+\frac{\pi}{6}\right)-\cos\left(\frac{\pi}{6}+\alpha+\alpha-\frac{\pi}{6}\right)\right)}{\frac{1}{2}\left(\cos\left(\frac{\pi}{6}+\alpha-\alpha+\frac{\pi}{6}\right)+\cos\left(\frac{\pi}{6}+\alpha+\alpha-\frac{\pi}{6}\right)\right)}=$$

$$=B_3:\frac{\cos\frac{\pi}{3}-\cos 2\alpha}{\cos\frac{\pi}{3}+\cos 2\alpha}=B_4:\frac{2\left(\frac{1}{2}-\cos 2\alpha\right)}{2\left(\frac{1}{2}+\cos 2\alpha\right)}=B_5:\frac{1-2\cos 2\alpha}{1+2\cos 2\alpha}.$$

Thus, presence at structure of O a cosine of a double corner and as a result of decomposition of work of trigonometrical functions B_1 in the sum, turns out expression B_5. We will assume that in expressions B_5 and O unequal elements have to be equal. Then

$$tg\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)(1+\sin\alpha)\frac{1}{\cos\alpha}=1$$

We will transform structures of multipliers in the left part of expression.

$$tg\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)(1+\sin\alpha)\frac{1}{\cos\alpha}=1 \qquad \Rightarrow$$

$$\frac{tg\frac{\pi}{4}-tg\frac{\alpha}{2}}{1+tg\frac{\alpha}{2}}(1+\sin\alpha)\frac{1}{\cos\alpha}=1$$

$$\frac{1-tg\frac{\alpha}{2}}{1+tg\frac{\alpha}{2}}(1+\sin\alpha)\frac{1}{\cos\alpha}=1 \qquad \Rightarrow$$

$$\frac{\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}}(1+\sin\alpha)\frac{1}{\cos\alpha}=1$$

$$\frac{\left(\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}\right)\left(\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}\right)}{\left(\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}\right)\left(\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}\right)}(1+\sin\alpha)\frac{1}{\cos\alpha}=1,$$

$$\frac{\left(\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}\right)}{\left(\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}+2\cos\frac{\alpha}{2}\sin\frac{\alpha}{2}\right)}(1+\sin\alpha)\frac{1}{\cos\alpha}=1,$$

$$\frac{\cos\alpha}{(1+\sin\alpha)}(1+\sin\alpha)\frac{1}{\cos\alpha}=1,$$

$$\Rightarrow \quad 1=1.$$

Thus, one part of expression is defined from the point of view of an arrangement, and another is defined through proofs.

Example 6. To prove identity.

$$\frac{\cos^2\frac{\alpha}{4}\cos\frac{\alpha}{2}\left(1+ctg^2\frac{3\alpha}{4}\right)}{ctg^2\frac{\alpha}{4}-ctg^2\frac{3\alpha}{4}}=\frac{1}{8}$$

Decision. In comparison with the examples reviewed above this example has the feature. This division of one corner into some corners. As a result of it during complication process the thought comes to crisis. In this case by expression transformation with bigger information to expression with smaller information we look for recovery from the crisis ways. Thus, to process of complication simplification process increases.

$$B:\frac{\cos^2\frac{\alpha}{4}\cos\frac{\alpha}{2}\left(1+ctg^2\frac{3\alpha}{4}\right)}{ctg^2\frac{\alpha}{4}-ctg^2\frac{3\alpha}{4}}=B_1:\frac{\cos^2\frac{\alpha}{4}\cos\frac{\alpha}{2}\cdot\frac{1}{\sin^2\frac{3\alpha}{4}}}{\frac{\cos^2\frac{\alpha}{4}}{\sin^2\frac{\alpha}{4}}-\frac{\cos^2\frac{3\alpha}{4}}{\sin^2\frac{3\alpha}{4}}}=$$

$$=B_2:\frac{\cos^2\frac{\alpha}{4}\cos\frac{\alpha}{2}\cdot\sin^2\frac{\alpha}{4}\sin^2\frac{3\alpha}{4}}{\sin^2\frac{3\alpha}{4}\left(\cos^2\frac{\alpha}{4}\sin^2\frac{3\alpha}{4}-\cos^2\frac{3\alpha}{4}\sin^2\frac{\alpha}{4}\right)}=$$

$$=B_3:\frac{\cos^2\frac{\alpha}{4}\cos\frac{\alpha}{2}\cdot\sin^2\frac{\alpha}{4}}{\cos^2\frac{\alpha}{4}\sin^2\frac{3\alpha}{4}-\cos^2\frac{3\alpha}{4}\sin^2\frac{\alpha}{4}}=$$

$$=B_4:\frac{\sin^2\frac{\alpha}{2}\cos\frac{\alpha}{2}}{4\left[\left(\cos\frac{\alpha}{4}\sin\frac{3\alpha}{4}\right)^2-\left(\cos\frac{3\alpha}{4}\sin\frac{\alpha}{4}\right)^2\right]}=$$

$$=B_5:\frac{\sin^2\frac{\alpha}{2}\cos\frac{\alpha}{2}}{4\left(\cos\frac{\alpha}{4}\sin\frac{3\alpha}{4}-\cos\frac{3\alpha}{4}\sin\frac{\alpha}{4}\right)\left(\cos\frac{\alpha}{4}\sin\frac{3\alpha}{4}+\cos\frac{3\alpha}{4}\sin\frac{\alpha}{4}\right)}=$$

$$=B_6:\frac{\sin^2\frac{\alpha}{2}\cos\frac{\alpha}{2}}{4\sin\left(\frac{3\alpha}{4}-\frac{\alpha}{4}\right)\sin\left(\frac{3\alpha}{4}+\frac{\alpha}{4}\right)}=B_7:\frac{\sin^2\frac{\alpha}{2}\cos\frac{\alpha}{2}}{4\sin^2\frac{\alpha}{2}\sin\alpha}=B_8:\frac{\sin^2\frac{\alpha}{2}\cos\frac{\alpha}{2}}{8\sin^2\frac{\alpha}{2}\cos\frac{\alpha}{2}}.$$

Now to define a difference between processes of complication and simplification we will stop on complication process. We will write components of calculation of complication:

$$B:\frac{1}{8};$$

$$O:\frac{\cos^2\frac{\alpha}{4}\cos\frac{\alpha}{2}\left(1+ctg^2\frac{3\alpha}{4}\right)}{ctg^2\frac{\alpha}{4}-ctg^2\frac{3\alpha}{4}}.$$

$$B:\frac{1}{8}=B_8:\frac{\sin^2\frac{\alpha}{2}\cos\frac{\alpha}{2}}{8\sin^2\frac{\alpha}{2}\cos\frac{\alpha}{2}}=B_1:\frac{\sin^2\frac{\alpha}{2}\cos\frac{\alpha}{2}}{4\left(2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\right)\sin\frac{\alpha}{2}}=B_2:\frac{\sin^2\frac{\alpha}{2}\cos\frac{\alpha}{2}}{4\sin\alpha\sin\frac{\alpha}{2}}.$$

We select by grouping in a fraction denominator work (B_1) some multipliers, and transition to a sine of the angle α follows from reference point information. Really, to pass to corners $\frac{\alpha}{4}$ and $\frac{3\alpha}{4}$ at first it is necessary to integrate a corner, during simplification this thought wasn't told. If to compare B_2 and O, it is necessary to transform corners α and $\alpha/2$ necessary information for realization of this thought is available in structure B_2.

By means of work of sine the necessary means will register

$$\sin\beta\sin\gamma=\frac{1}{2}\left[\cos(\beta-\gamma)-\cos(\beta+\gamma)\right], \;\Rightarrow$$

$$\beta-\gamma=\frac{\alpha}{2};\;\beta+\gamma=\alpha\;\Rightarrow$$

$$\Rightarrow\beta=\frac{3\alpha}{4};\;\gamma=\alpha/4.$$

Then it will be transformed as follows:

$$B_2\Rightarrow B_3:\frac{\sin^2\frac{\alpha}{2}\cos\frac{\alpha}{2}}{4\sin\left(\frac{3\alpha}{4}-\frac{\alpha}{4}\right)\sin\left(\frac{3\alpha}{4}+\frac{\alpha}{4}\right)}=$$

$$=B_4:\frac{\sin^2\frac{\alpha}{2}\cos\frac{\alpha}{2}}{4\left(\sin\frac{3\alpha}{4}\cos\frac{\alpha}{4}-\sin\frac{\alpha}{4}\cos\frac{3\alpha}{4}\right)\left(\sin\frac{3\alpha}{4}\cos\frac{\alpha}{4}+\sin\frac{\alpha}{4}\cos\frac{3\alpha}{4}\right)}$$

Because between squares of cotangents there is a minus sign, and it follows from information in a reference point or in a different way to receive a difference it is necessary a sine of a difference and the sum of two corners to spread out in the sum. At simplification cotangents were transformed through a sine and a cosine. Then we used the rule of division of fraction on fraction. And at complication to pass to a cotangent even in the absence of the general multipliers, using the rule, we take out work of sine for a bracket. Then

$$B_5: \frac{\sin^2\frac{\alpha}{2}\cos\frac{\alpha}{2}}{4\sin\frac{\alpha}{4}\sin\frac{3\alpha}{4}\left(ctg\frac{\alpha}{4}-ctg\frac{3\alpha}{4}\right)\left(ctg\frac{\alpha}{4}+ctg\frac{3\alpha}{4}\right)}=$$

$$B_6: \frac{\sin^2\frac{\alpha}{2}\cos\frac{\alpha}{2}}{4\sin\frac{\alpha}{4}\sin\frac{3\alpha}{4}\left(ctg^2\frac{\alpha}{4}-ctg^2\frac{3\alpha}{4}\right)}$$

As noticed in the course of complication, at identical arguments of sine by simplification it is possible to approach B_6 to structure O. Then

$$B_7: \frac{4\sin^2\frac{\alpha}{4}\cos^2\frac{\alpha}{4}\cos\frac{\alpha}{2}}{4\sin^2\frac{\alpha}{4}\sin^2\frac{3\alpha}{4}\left(ctg^2\frac{\alpha}{4}-ctg^2\frac{3\alpha}{4}\right)}=$$

$$B_8: \frac{\cos^2\frac{\alpha}{4}\cos\frac{\alpha}{2}\cdot\frac{1}{\sin^2\frac{3\alpha}{4}}}{ctg^2\frac{\alpha}{4}-ctg^2\frac{3\alpha}{4}}=$$

$$=B_9: \frac{\cos^2\frac{\alpha}{4}\cos\frac{\alpha}{2}\left(1+ctg^2\frac{3\alpha}{4}\right)}{ctg^2\frac{\alpha}{4}-ctg^2\frac{3\alpha}{4}}.$$

The severity of an order of performance of actions in the course of complication isn't identical in comparison with order of performance of actions in the course of simplification.

We as a result developed formation Model at students of abilities of mathematical thinking by complication of trigonometrical expressions which is given in figure 1.

The explanation to figure 1. Formation model at students of abilities of mathematical thinking by complication of trigonometrical expressions:

I – Abilities of mathematical thinking of students.

II – Technology of complication of trigonometrical expressions.

III – Pedagogical conditions of formation of abilities of mathematical thinking by complication of trigonometrical expressions.

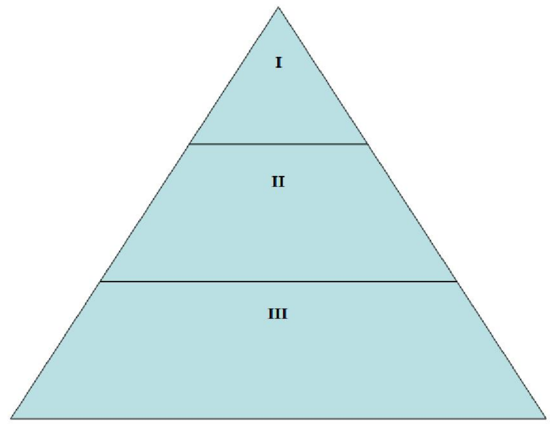

Figure 1. **Formation model at students of abilities of mathematical thinking by complication of trigonometrical expressions**

Conclusion

In work the technology of transformation of trigonometrical expressions by means of complication process is proved. The purpose of process of transformation is development of an intellectual outlook of pupils. This process decides on the help of complication. Actuating of cogitative activity by means of expressions with small information before expression with bigger information is called as complication. Complication and simplification - mutually the return processes. In comparison with simplification in the course of complication the share of formation and development of knowledge is bigger.

As a result of the conducted research it is expanded scientific ideas of process of formation of abilities of mathematical thinking by complication of trigonometrical expressions, developments of mathematical thinking of students. Pedagogical conditions of formation of abilities of mathematical thinking by complication of trigonometrical expressions are developed. The formation Model at students of abilities of mathematical thinking by complication of trigonometrical expressions is developed.

The recommendation of results of research is that the technology is developed, pedagogical conditions, model of formation of abilities of mathematical thinking by complication of the trigonometrical expressions, allowing the teacher purposefully to build management of mathematical cognitive activity of students which can be used in mass practice of work in higher educational institutions together with created and by practical consideration checked system of tasks for teachers, and also methodical recommendations.

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