Constructing two-state "on-ramp" traffic flow mathematical model

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Abstract: The purpose of this research is to find analytical functions for obtaining useful information of traffic flows properties such as capacity, average speed, queue and state (free or congested) for inhomogeneity "off-ramp" in the context of discrete dynamics. This work based on real empirical data collected by traffic detectors for long study period.

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Introduction

Having deal with large road networks in the field of transport planning, analysis, and engineering based on macroscopic traffic flow characteristics such as rate, average speed, density, occupancy, queue lengths, etc. [1].

The process of obtaining such information based on matching traffic demand and supply in dynamic. Traffic demand can be described by the information about the distribution of residence, employment and points of attraction (cinemas, theaters and shopping centers), etc. and can be obtained by various mathematical models or evaluated analytically. Traffic supply on the straightforward road bounded by maximum velocity and number of lanes and is less only in places where it changes the behavior - traffic light, on-ramp, off-ramp, lanes merge, crosswalk and etc. (hereinafter inhomogeneities/elements)

Nowadays for some inhomogeneities (traffic light, lanes merge) local analytical (see paper [2]) or rather empirical (including computational experiments results in work [3]) dependencies exists that can evaluate capacity and calculate traffic flow characteristics such as rate, average speed, queue length (in number of vehicles), density and their derivatives (for example, the average travel time through the element). The aforementioned inhomogeneities in case of traffic flows interactions are simplest because the major impact on traffic flow made by the element itself – periodic lane closing for traffic light and geometrical carriageway narrowing for lanes merge. However, for more complex elements (off-ramp, on-ramp) where traffic flow behavior on one lane depends on dynamic characteristics of other traffic flows currently no models that fully cover the important properties of the physical processes, occurring during the motion of traffic flow, exists. Further in this paper we will consider the behavior of traffic flows that passing through the most difficult (in the author opinion) inhomogeneity "off-ramp" and we will show the process of constructing its mathematical model based on empirical data collected by traffic detectors in the long study period (several months).

The Method

The empirical basis of this research were traffic detectors data aggregated over a time interval 300 second (hereinafter denoted by τ_n , where n indicates the number of time interval):

Rate denoted by q – number of vehicles passed through traffic detector loop in τ_n ;

Average speed denoted by V – arithmetic average speed of all vehicles from q ;

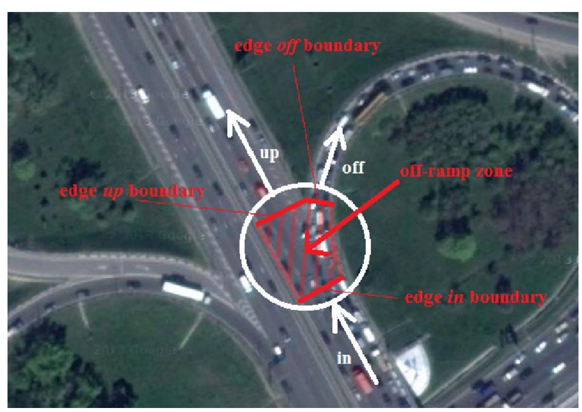

Figure 1. Directed graph off-ramp inhomogeneity representation

In case of discrete nature of data (aggregation by τ_n) we will consider vehicles motion on the road network in the context of discrete dynamics. Also let us consider the road network as a directed graph [4], where the roads are edges and nodes - inhomogeneities of the road network. In such case the inhomogeneity "off-ramp" will be presented as a directed graph with one vertex and three edges (two are outgoing and one is incoming). Let us denote the incoming edge as *in*, continuation of the edge *in* outgoing edge as *up*, and edge which is corresponding to the off-ramp road as *off* (see white color at Figure 1).

Let us introduce the following notation:

- $d_{in,up}^{\tau_n}$, $d_{in,off}^{\tau_n}$ – quantity of vehicles which want to move from edge in to edge up, off respectively in τ_n;
- $p_{in,up}^{\tau_n}, p_{in,off}^{\tau_n}$ – element “off-ramp” capacity from moves *in-up* and *in-off* respectively in τ_n;
- $a_{in,up}^{\tau_n}, a_{in,off}^{\tau_n}$ – quantity of vehicles which moved from edge in to edge up, off respectively in τ_n;
- $q_{in,up}^{\tau_n}, q_{in,off}^{\tau_n}$ – quantity of vehicles which did not move from edge in to edge up, off respectively in τ_n;
- V_{in}, V_{up}, V_{off} – traffic flows average speeds at edges *in,up* and *off* boundaries in τ_n (see red color at Figure 1);
- $R_{in,up}, R_{in,off}$ – vehicles trajectory curvature radiuses (included for accounting the direct proportional relationship between the radius and speed).

The discrete dynamics of traffic flow motion describes the process of vehicles movement on the road network in the context of classical deterministic theory of traffic flows [5] where flow can exist in one of two states: free or congested. Traffic flow is in free state when $d_{in,up}^{\tau_n} \le p_{in,up}^{\tau_n}$, and in congested state when $d_{in,up}^{\tau_n} > p_{in,up}^{\tau_n}$. In fact, this statement odds with the latest research in the theory of traffic flow (see for example the book [6]) where there are three phases of traffic flow - free, synchronized and wide moving jam. However, here we operates in the approximation of discrete dynamics that generally simplifies most of the physical processes of continuous dynamics, highlighting the only significant properties of dynamic systems, such as hysteresis [7].

Data source - traffic detectors were installed directly before and after inhomogeneity “off-ramp” near edges *in*, *up* и *off* boundaries (see Figure 1). That was done for downstream traffic flows states control (transition from free state to congested) on edges *up* and *off*. In fact the occurrence of congestion before inhomogeneity “off-ramp” can be affected by cascade proliferation from primary edge (*up*), and secondary edge (*off*)* (author's note: terminology relevant to the traffic rules in the right-side roads).

Let us consider the example of traffic detectors data collected a few months presented in two-dimensional histogram of the frequency in dependence of the average velocity from rate (see Figure 2).

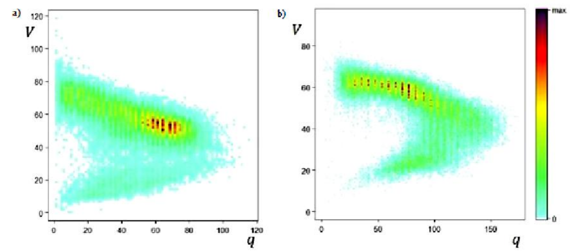

Figure 2. Traffic data examples

Visual analysis of graphs in Figure 2 indicates that the nature of the data dependence coincides with the behavior of the function which is the fundamental diagram of traffic flow (see Greenshields work [8]).

The analysis of dependencies in Figure 2 suggests the following conclusions:

- A separation of the set of values V, q for two subsets is observed: first of it corresponds to free state of traffic flow – the reduction of V where $q \to q_{max}$ from $q =$; second of it corresponds to congested state of traffic flow – the reduction of V where $q \to 0$ from $q = q_{max}$;
- The subsets corresponding to the free and congested states are limited above and below by V at a certain value of q. At the same time, the closer we are to the q to q_{max}, the less clearly visible upper limit for the congested state and the lower limit for the free state.

Typical range of values V at a certain value of q can be explained as following - every pair V, q has a certain hidden information:

- Composition of the traffic flows in the dynamic characteristics of vehicles;
- Behavioral strategies (properties) of the drivers (as an example of the phenomenological coefficients in microscopic models [9]);
- Weather conditions;
- The distribution of vehicles in space and time - the most essential information, which

is in fact forming a sequence, quantity and characteristics of waves of compression and decompression [10] traffic flows) defining delay on the movement;

- Other random factors: the influence of pedestrians, driveways, entrances to the yard area, etc.

The deterministic nature of developing mathematical model does not imply any assumption of spread of values V for certain values of q for free and congested traffic flow states. For further work in the context of discr ete dynamics the appreciation of transition to a deterministic mean dependencies was done. The main idea was to examine the frequency of the set of values V for one value of q and the allocation of their local maxima for free and congested traffic flow states. In more detail the methodology described in paper [11].

Let us introduce the additional notation: η – the number of lanes on the edge, l – lane ordinal number. As input parameters of the model we will use the static characteristics of inhomogeneity: $R_{in,up}, R_{in,off}$ $\eta_{in}, \eta_{off}, \eta_{up}$ and dynamic parameters of traffic flow $V_{off,l_{off,i}}, V_{up,l_{up,j}}, i \in \eta_{off}, j \in \eta_{up}, d^{\tau_n}_{in,up}, d^{\tau_n}_{in,off}$. Output parameters of the model will be:

- $V_{in,l_{in,k}}, k \in \eta_{in}$;
- $a^{\tau_n}_{in,up}$, $a^{\tau_n}_{in,off}$;
- $q^{\tau_n}_{in,up}, q^{\tau_n}_{in,off}$;
- $p^{\tau_n}_{in,up}$? $p^{\tau_n}_{in,off}$.

Three sets of traffic detectors data (on edges *in*, *up* and *off)* was decompose onto four variants (the Cartesian product of the possible states of traffic flow):

- Traffic flow state on edge *up* is free, on edge *off* is free;
- Traffic flow state on edge *up* – is free, on edge *off* – is congested;
- Traffic flow state on edge *up* – is congested, on edge *off* – is free;
- Traffic flow state on edge *up* – is congested, on edge *off* – is congested.

The process of decomposition was made by the author method (see work [11]) as follows. Each data set of traffic detectors data presented as the array of set [speed, frequency] for every flow value. Further, for each set with the help of author method local speed maximums and minimums founds. Then for each speed value its comparison with the values of central local speed minimum done - if the average speed of traffic flow is greater of central local minimum than the state is free, if less minimum than the state is congested. An example of such decomposition shown in Figure 3.

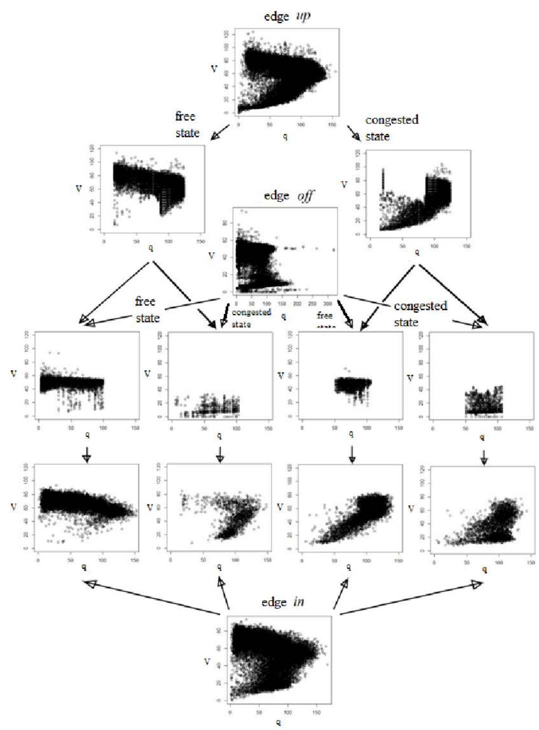

Figure 3. Traffic data decomposition example

Let us explain diagram in Figure 3. Originally traffic detectors which installed on the edges *in*, *up* и *off*, datasets joined by the date and time of measurement, thus obtaining, a single set. Further, it was divided into two subsets of free and congested traffic flow states on edge *up* and then each was divided into two subsets for free and congested traffic flow states of edges *off*.

Main Part

Let us consider four combinations of traffic flow states of edges *up* и *off*.

Traffic flow state on edge *up* is free, on edge *off* is free

The dependence of average speed from rate of traffic flow on Figure 3 (bottom left) shows that the most of values corresponds to free state of traffic flow. Formation of small region of pairs V , q in the area of the congested traffic flow state is due to the consideration lack of random factors (accidents, repairs, etc.) that may have been occurred on the roadway between traffic detectors installed on the edges *in*, *up* and *off*.

The analytical form of the element off-ramp" mathematical model was built by iterating various types of functions (selected on the basis of general considerations) to achieve the best (in terms of the minimum amount of standard deviations [12] theoretical values from empirical) approximation:

For lane 1 where traffic flows divided for two parts, i.e. vehicles makes movement to the edges *up* or *off*

$$V_{in,1} = V_{up}\left(R_{in,up}\right)\bullet\left(1-\frac{d_{in,off}^{\tau_n}}{\frac{d_{in,up}^{\tau_n}}{\eta_{in}-1}+d_{in,off}^{\tau_n}}\right)+V_{off}\left(R_{in,off}\right)\bullet\left(\frac{d_{in,off}^{\tau_n}}{\frac{d_{in,up}^{\tau_n}}{\eta_{in}-1}+d_{in,off}^{\tau_n}}\right) \quad (1)$$

for lane 2 (adjacent lane 1)

$$V_{in,2} = \min(V_{up}\left(R_{in,up}\right), V_{up}\left(R_{in,up}\right)\bullet\left(1-\frac{d_{in,off}^{\tau_n}}{\frac{d_{in,up}^{\tau_n}}{\eta_{in}-1}+d_{in,off}^{\tau_n}}\bullet\frac{1}{\eta_{in}}\right)+V_{off}\left(R_{in,off}\right)\bullet\left(\frac{d_{in,off}^{\tau_n}}{\frac{d_{in,up}^{\tau_n}}{\eta_{in}-1}+d_{in,off}^{\tau_n}}\bullet\frac{1}{\eta_{in}}\right)) \quad (2)$$

for lanes 3,4,5 and so on

$$V_{in,3(4,\ldots)} = V_{up}\left(R_{in,up}\right) \quad (3)$$

The meaning of (1) - (3) is the following. The more vehicles on edge *in* lane 1 want to make movement to the edge *off* the more influence has traffic flow average speed of edge *off* start bound (which generally can not be greater than the speed of vehicles moving on the road, having a radius of curvature $R_{in,off}$) and similarly for the movements on edge *up*. In fact, the expression $\frac{d_{in,off}^{\tau_n}}{\frac{d_{in,up}^{\tau_n}}{\eta_{in}}+d_{in,off}^{\tau_n}}$ reflects the proportion of the traffic flow (percent) which wants to make movement on edge *off*. Research has shown that for lane 2 the influence of vehicles movement desceases in the number of times proportional to the number of edhe *in* lanes and for the more "distant" lanes from off-ramp lane the influence is absent.

Traffic flow state on edge *up* is free, on edge *off* is congested

The dependence of average speed from rate of traffic flow on Figure 3 (the first right of the bottom left), shows that one part of values corresponds to free state of traffic flow and another part to the congested state. If there is a queue on the edge *off* and respectively it potential extending it to the edge *in* the formation of dynamic narrowing of the edge *in* roadway occurs. Hense the inhomogeneity capacity decreases both for the traffic flow that wants to make move from the edges *in* to the edge *off* and for the traffic flow that wants to make move from the *in* на to the edge *up*. Thus, when $d_{in,up}^{\tau_n} \le p_{in,up}^{\tau_n}$ that traffic flow on the edge *in* will be in free state and when $d_{in,up}^{\tau_n} > p_{in,up}^{\tau_n}$ that transition from free to congested state occurs. It should be noted that in reality in the period of the population activity (when most trips occurs) demand on inhomogeneity "off-ramp" primary and secondary edges is proportional. In case of this the most number of values observed for congested traffic from state.

The research showed that the analytical form of the mathematical model for various lanes of edge *in* can be estimated as following:

$$V_{in,l} = V_{up}\left(R_{in,up}\right)\bullet\left(1-\frac{d_{in,off}^{\tau_n}}{\frac{d_{in,up}^{\tau_n}}{\eta_{in}-1}+d_{in,off}^{\tau_n}}\bullet\frac{\eta_{in}-l+1}{\eta_{in}}\right)+V_{off}\left(R_{in,off}\right)\bullet\left(\frac{d_{in,off}^{\tau_n}}{\frac{d_{in,up}^{\tau_n}}{\eta_{in}-1}+d_{in,off}^{\tau_n}}\bullet\frac{\eta_{in}-l+1}{\eta_{in}}\right) \quad (4)$$

The meaning of function (4) is the following: in our notation (where number of lane 1 corresponds to the off-ramp lane, lane 2 – adjacent to the lane 1 and so on.) level of influence on the edge *off* inversely proportional to the number edge *in* lane: – the farther the lane of edge *in* (larger its number) from off-ramp lane to the edge *off* the less influence traffic flow movements has on considered traffic flow.

Traffic flow state on edge *up* is congested, on edge *off* – is free or congested

The dependence of average speed from rate of traffic flow on Figure 3 (the first left of the bottom right) shows that almost all values corresponds to the congested state of traffic flow. From this we can conclude that the the primary edge makes more influenc on the cascade jam propagation from the edge *up* to the edge *in*. However, the research showed that the behavior of the traffic flow on the edge *in* is equivalent to the previous part (see above "traffic flow state on edge up is free, on edge off is free"), and can be evaluated in a similar way to the relations (1) - (3). From this it follows that for these cases, the priority rules dictated by the traffic rules, play a key role in the behavior of traffic flows on the road network.

To determine the capacity of the element "off-ramp" were used fundamental relationship [13]:

$$q = V\rho \quad (5)$$

where [rho] – density of traffic flow. Assuming that the density is inversely proportional to the distance of security $D = \frac{1}{\rho}$, and safety distance and estimated as [14] $D = VT + s_0$, where T – driver's reaction time, s_0 – vehicles jam distance (bumper to bumper). We obtain that the capacity can be expressed in terms of the average speed:

$$q = \frac{V}{VT + s_0} \qquad (6)$$

Values of T, s_0 have been calibrated by the method of least squares. The final form of the function (6) shown in Figure 4 in red.

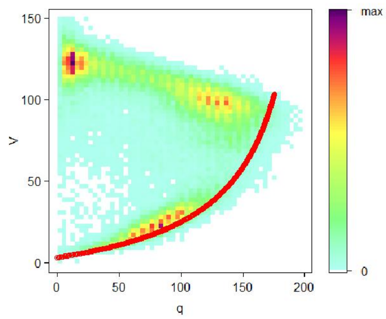

Figure 4. Calibrated function (6) and traffic data

Accordingly, for obtaining element “off-ramp” capacity we need to calculate the function by substituting the averaged across all lanes traffic flow average speeds (see the function (2-4)).

Conclusion

Founded analytical dependences in this research for the definition of the average speed of traffic flow and the capacity of the road network element "off-ramp" previously has no direct analogues. Such relationships could be explored during the computational experiments (microsimulation), but here we worked with real data, suggesting the greatest significance of the applied results. Constructed models not overloaded by many phenomenological constants and coefficients, and based on transport properties of the road network cleared for traffic engineers. Nevertheless, the approach (discrete dynamics) and small parametrizable of mathematical models does not allow us to work with small discrete steps and, for example, with strongly heterogeneous traffic flows that introduces some restrictions on their use. However, the linearity of computations as case of the linearity of mathematical model allows us to make quickly calculations at big road network and opens the possibility to rely on the obtained models for solving complex problems such as help finding a dynamic equilibrium distribution of traffic flows.

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