Recurrence Formula to Determine the Number of New Elements for Fuzzy Topographic Topological Mapping

Mohamed Sayed[1,2] and Tahir Ahmad[1,3]

[1]Department of Mathematical Science, Universiti Teknologi Malaysia, 81310 Skudai, Johor, Malaysia
[2]Department of Mathematical Science, International University of Africa, Sudan
[3]Ibnusina Institute, Universiti Teknologi Malaysia, 81310 Skudai, Johor, Malaysia
mohamedelsafi@yahoo.com and tahir@ibnusina.utm.my

Abstract: Fuzzy Topographic topological Mapping (FTTM) is a mathematical model for solving neuromagnetic inverse problem. FTTM consists of four topological spaces which are connected by three different algorithms. In 2006, Yun showed that FTTM version 1 and FTTM version 2 are homeomorphic, and this homeomorphism generated 14 new elements of FTTM. This paper presents a recurrence formula to determine the number of the new elements from a combination of k versions of FTTM with respect to n components.

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Keywords: FTTM; Sequence of FTTM; Number of the new elements; Element of order k.

1. Introduction:

In 1999 Fuzzy Topographic Topological Mapping (FTTM) was introduced to solve neuromagnetic inverse problem to determine the location of epileptic foci in epilepsy disorder patient (Ahmad, *et al.*, 2000). FTTM version 1 was developed to present a 3-D view of unbounded single current source in one angle observation (upper of a head model) (Yun, 2001; Zakaria, 2002). It consists of three algorithms, which link between the four components of the model as shown in Figure1 (For more detail, see Tan, *et al*., 2014).

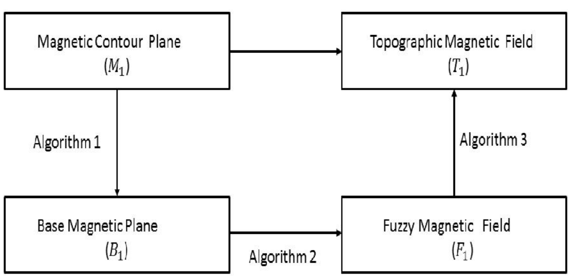

Figure 1: FTTM 1

The four components are Magnetic Contour Plane M_1, Base Magnetic Plane B_1, Fuzzy Magnetic Field F_1, and Topographic Magnetic Field T_1. M_1 is is a magnetic field in a plane above a current source with $z = 0$. The plane is lowered down to B_1, which is a plane with $z = h$. Then the entire B_1 is fuzzified into a fuzzy environment (F_1), where all the current fields reading are fuzzified. Finally, a three dimensional presentation of F_1 is plotted on B_1. The final process is defuzzification of the fuzzified data to obtain a 3-D view of the current source (T_1), (Yun, 2001, Yun, 2006). *FTTM* Version 2 was developed to present a 3-D view of a bounded multi current source in four angles of observation (upper, left, right and back of the head model) (Rahman, 2002). It consists of three algorithms, which link between the four components of the model. The four components are Magnetic Image Plane M_2, Base Magnetic Image Plane B_2, Fuzzy Magnetic Image Field F_2 and Topographic Magnetic Image Field T_2, as shown in Figure 2.

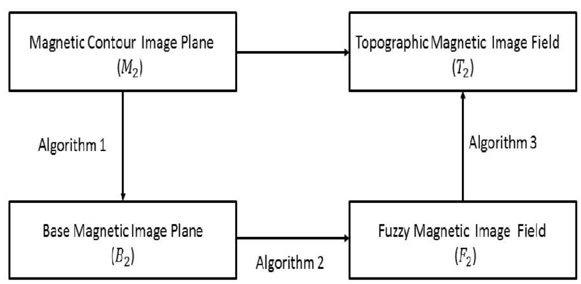

Figure 2: FTTM 2

2. Literature Review:

FTTM Version 1 as well as FTTM Version 2 were designed to have equivalent topological structures between their components. Ahmad *et al.* (2005) shown that the components of FTTM Version 1 are homeomorphic. On the other hand, Yun (2006) proved that the components of FTTM version 2 are homeomorphic. Yun (2006) also showed that FTTM version 1 and FTTM version 2 are homeomorphic component wise (see Figure 3), and this homeomorphism generated 14 new elements, namely:

$$(M_1,B_1,F_1,T_2),(M_1,B_1,F_2,T_1),(M_1,B_2,F_1,T_1),\\(M_2,B_1,F_1,T_1),(M_1,B_1,F_2,T_2),(M_1,B_2,F_1,T_2),\\(M_2,B_1,F_1,T_2),(M_1,B_2,F_2,T_1),(M_2,B_1,F_2,T_1),\\(M_2,B_2,F_1,T_1),(M_1,B_2,F_2,T_2),(M_2,B_2,F_1,T_2),\\(M_2,B_2,F_2,T_1),(M_2,B_1,F_2,T_2)$$

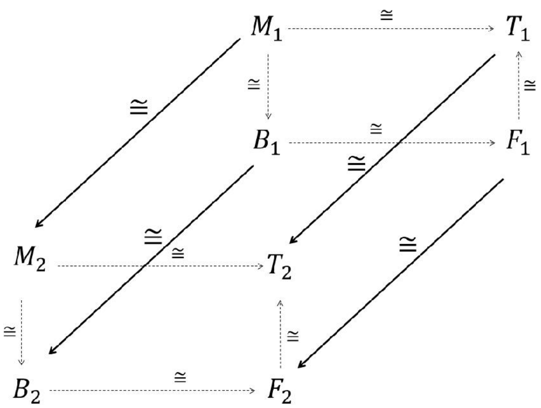

Figure 3: Homeomorphism between *FTTM 1* and *FTTM 2*

In general, Yun (2006) proposed a conjecture as follows:

If there exist k elements of *FTTM*, then the numbers of new elements are: k^4-k.
The above conjecture was finally proved in Ahmad *et al.* (2010).

3. Material and Methods:

Definition 1: Let $F_{n,i}=\{A_{1,i},A_{2,i},A_{3,i},\dots,A_{n,i}\}$ be *FTTM* version *i,* where $A_{1,i},A_{2,i},A_{3,i},\dots,A_{n,i}$ are topological spaces and $A_{1,i}\cong A_{2,i}\cong A_{3,i}\cong\cdots\cong$ *An,i*. The sequence of k *FTTM* is

$$F_n^k=\{F_{n,1},F_{n,2},F_{n,3},\dots,F_{n,k}\}$$

such that n represents the component number in each version.

Definition 2 : (Sayed *et al.*, 2013) [8].
The new element is said to be an element of order k if its components appear in exactly k versions of *FTTM*.

Remark 1: From a combination of k versions of *FTTM* we can generate new elements of order up to n. In other words, the maximum order of the new element is n.

Definition 3 : (Sayed *et al.*, 2013) [9].
Let $F_n^k=\{F_{n,1},F_{n,2},F_{n,3},\dots,F_{n,k}\}$ be a sequence of k FTTM. Then,

i. $F\,(n,k)$ is the set of all new elements of order k that could be generated from F_n^k for $k\le n$.
ii. $\#F(n,k)$ is the cardinality of $F(n,k)$.

Theorem 1:
$$\#F(n,k)=k(\#F(n-1,k-1)+\#F(n-1,k))$$
$$where\ \#F(n,1)=1\ and\ \#F(n,k)=0\ \ for\ \ k>n$$

Proof: Let n and k be integers such that $1\le k\le n$ and

$$F_n^k=\{F_{n,1},F_{n,2},F_{n,3},\dots,F_{n,k}\} \qquad (1)$$

be a sequence of k *FTTM* such that

$$F_{n,1}=\{A_{1,1},A_{2,1},A_{3,1},\dots,A_{n,1}\}$$
$$F_{n,2}=\{A_{1,2},A_{2,2},A_{3,2},\dots,A_{n,2}\}$$
$$F_{n,3}=\{A_{1,3},A_{2,3},A_{3,3},\dots,A_{n,3}\}$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$F_{n,k}=\{A_{1,k},A_{2,k},A_{3,k},\dots,A_{n,k}\}$$

Suppose $F(n,k)$ be the set of all new elements which are generated from (1). Partition $F(n,k)$ in k subsets.
Let the first subset contain only the new elements that end with the component $A_{n,1}$. Call it $F(n,k)_{A_{n,1}}$.
Let the second subset contain only the new elements that end with the component $A_{n,2}$. Call it $F(n,k)_{A_{n,2}}$.
Let the third subset contain only the new elements that end with the component $A_{n,3}$. Call it $F(n,k)_{A_{n,3}}$.

$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$

Let the k^{th} subset contain only the new elements that end with the component $A_{n,k}$. Call it $F(n,k)_{A_{n,k}}$.
Consequently,

$$F(n,k)=F(n,k)_{A_{n,1}}\cup F(n,k)_{A_{n,2}}\cup\cdots\cup F(n,k)_{A_{n,k}}$$
$$=\bigcup_{j=1}^{k}F(n,k)_{A_{n,j}}$$

Since $F(n,k)_{A_{n,j}}$ are mutually disjoint $\forall\, j=1,2,3,\cdots,k$, therefore

$$\#F(n,k)=\#F(n,k)_{A_{n,1}}+\#F(n,k)_{A_{n,2}}+\cdots+\#F(n,k)_{A_{n,k}}$$
$$=\sum_{j=1}^{k}\#F(n,k)_{A_{n,j}}$$

But

$$\#F(n,k)_{A_{n,1}}=\#F(n,k)_{A_{n,2}}=\cdots=\#F(n,k)_{A_{n,k}}$$

Thus

$$\#F(n,k)=k(\#F(n,k)_{A_{n,k}}) \qquad (2)$$

Now consider the set $F(n,k)_{A_{n,k}}$ and eliminates the component $A_{n,k}$ from all elements of $F(n,k)_{A_{n,k}}$ (this reduction does not change the cardinality of the set). So each element contains $(n-1)$ components. Partition the set $F(n,k)_{A_{n,k}}$ (after elimination) into two subsets. In the first subset list all of these new elements which do not contain any component from $F_{n,k}$. In the second subset, list all of the remaining elements which contain some components of $F_{n,k}$. The first subset contains $\#F(n-1,k-1)$ elements, and the second subset contains $\#F(n-1,k-1)$ elements. Hence

$$\#F(n,k)_{A_{n,k}} = \#F(n-1,k-1) + \#F(n-1,k)$$

From (2) the unknown number $\#F(n,k)$ satisfies the recurrence.

Hence,

$$\#F(n,k) = k(\#F(n-1,k-1) + \#F(n-1,k))$$

Such that $\#F(n,1) = 1$ and $\#F(n,k) = 0$ for $k > n$.

This completes the proof. □

Corollary 1: If $n = k$ then $\#F(n,k) = k!$

Proof: From Theorem (1) the recurrence is given as

$$\#F(n,k) = k(\#F(n-1,k-1) + \#F(n-1,k))$$

When $n = k$, it will yield

$$\#F(k,k) = k\left(\#F(k-1,k-1) + \underbrace{\#F(k-1,k)}_{=0}\right)$$

$$= k(\#F(k-1,k-1))$$

$$= k(k-1)\left(\#F(k-2,k-2) + \underbrace{\#F(k-1,k)}_{=0}\right)$$

$$= k(k-1)(\#F(k-2,k-2))$$

$$= k(k-1)(k-2)\left(\#F(k-2,k-2) + \right.$$

#F(k−1,k)

=0

$$\vdots \qquad \vdots \qquad \vdots$$

$$= k(k-1)(k-2)(k-3)\cdots(3)(2)(\underbrace{\#F(1,1)}_{=1})$$

$$= k(k-1)(k-2)(k-3)\cdots(3)(2)(1)$$

$$= k\,!$$ □

Corollary 2:

$$\#F(k+1,k) = \sum_{j=1}^{k} \frac{k!\,(k-j)!}{(k-j)!}$$

Proof:

Notice,

$$\#F(k+1,k) = k(\#F(k,k-1) + \#F(k,k))$$

$$= k\big(\#F(k,k-1)\big) + k\left(\underbrace{\#F(k,k)}_{=k!}\right)$$

$$= k\big(\#F(k,k-1)\big) + kk!$$

$$= k(k-1)\big(\#F(k-1,k-2) + \#F(k-1,k-1)\big) + kk!$$

$$= k(k-1)\#F(k-1,k-2) + k(k-1)\left(\underbrace{\#F(k-1,k-1)}_{=(k-1)!}\right) + kk!$$

$$= k(k-1)\#F(k-1,k-2) + k(k-1)(k-1)! + kk!$$

$$= k(k-1)(k-2)\#F(k-2,k-3) + k(k-1)(k-2)\left(\underbrace{\#F(k-2,k-2)}_{=(k-2)!}\right) + k(k-1)(k1)! + kk!$$

$$= k(k-1)(k-2)\#F(k-2,k-3) + k(k-1)(k-2)(k-2)! + k(k-1)(k1)! + kk!$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$= k(k-1)(k-2)\cdots(3)(2)\underbrace{\#F(2,1)}_{=1} + k(k-1)\cdots(3)(2)(2)! + k(k-1)(k-2)\cdots(4)(3)(3)! + \cdots + k(k-1)(k-1)! + kk!$$

$$= k(k-1)(k-2)\cdots(3)(2)(1)! + k(k-1)(k-2)\cdots(3)(2)(2)! + \cdots + k(k-1)(k-1)! + kk!$$

$$= k(k-1)(k-2)\cdots(3)(2)(1) + (k(k-1)(k-2)\cdots(3)(2)!)(2) + (k(k-1)(k-2)\cdots(4)(3)!)3 + (k(k-1)!)(k-1) + kk!$$

$$= \frac{k!(1)!}{(0)!} + \frac{k!(2)!}{(1)!} + \frac{k!(3)!}{(2)!} + \cdots + \frac{k!(k)!}{(k-1)!}$$

$$= \frac{k!(k)!}{(k-1)!} + \frac{k!(k-1)!}{(k-2)!} + \frac{k!(k-2)!}{(k-3)!} + \cdots + \frac{k!(1)!}{(0)!}$$

$$= \sum_{j=1}^{k} \frac{k!\,(k-j+1)!}{(k-j)!}$$ □

Corollary 3:

$$\#F(n,2) = \sum_{j=1}^{n-1} 2^j \qquad , n \geq 2$$

Proof: By induction on n and using Theorem (1) we have

$$\#F(n,k) = k(\#F(n-1,k-1) + \#F(n-1,k))$$

Therefor

$$\#F(2,2) = 2\left(\underbrace{\#F(1,1)}_{=1} + \underbrace{\#F(1,2)}_{=0}\right)$$

$= 2^1$, so the result holds when $n = 2$

$$\#F(3,2) = 2\left(\underbrace{\#F(2,1)}_{=1} + \underbrace{\#F(2,2)}_{=2}\right)$$

$= 2^1 + 2^2$, the result holds when $n = 3$

Assume the formula holds when $n = m$.

$$\#F(m,2) = 2^1 + 2^2 + 2^3 + \cdots + 2^{m-1}$$

Now

$$\#F(m+1,2) = 2\left(\underbrace{\#F(m,1)}_{=1} + \#F(m,2)\right)$$
$$= 2(1 + 2^1 + 2^2 + \cdots + 2^{m-1})$$
$$= 2^1 + 2^2 + 2^3 + \cdots + 2^m$$
$$= \sum_{j=1}^{m} 2^j$$

Consequently, if the formula holds when $n = m$, it is also true when $n = m + 1$. Thus, by induction, the formula holds for every positive integer $n \geq 2$. □

4. Results:

This paper presented a new formula to determine the number of the new elements of order *k*. In general, the result of this paper extends the sequence of FTTM to accommodate any number of components with a condition that the number of versions must be less than or equal to the number of components.

5. Discussion and Conclusion:

The sequence of *FTTM* was mentioned firstly by Yun (2006) when she conjectured that if there exist k versions of *FTTM*, then they will generate another $k^4 - k$ new *FTTM* and finally proven in Ahmad *et. al* (2010) . However, in this paper we generalized the sequence of *FTTM* to accommodate any number of its components.

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Corresponding Author:
Prof. Dr. Tahir Aahma
Ibnusina Institute,
Universiti Teknologi Malaysia
81310 Skudai, Johor, Malaysia
Email: tahir@ibnusina.utm.my

References:

1. Ahmad, T., Shah, R., Zakaria, F. and Yun, L. L., 2000. Development Of Detection Model For Neuromagnetic Field. Proceedings of BIOMED. Kuala Lumpur: Universiti Malaysia. 119–121.
2. Yun, L. L., 2001. Homeomorfisma Antara Melalui Struktur Permukaan Riemann serta Deduksi Teknik Pembuktiannya bagi Homeomorfisma pemetaan Topologi Topografi kabur (FTTM). M.sc. thesis. University Teknologi Malaysia.
3. Zakaria, F., 2002. Algorithma Penyelesaian Masalah Songsang Arus Tunggal Tak Terbatas MEG. M.sc. thesis. University Technology Malaysia.
4. Tan, L. K., and Ahmad, T., 2014. Structural Stability of Flat Electroencephalography. Life Science Journal. 2014. 11(8).
5. Yun, L. L., 2006. Algebraic Structures of Fuzzy Topographic Topological Mapping for Solving Neuromagnetic Inverse Problem. PhD. thesis. University Teknologi Malaysia.
6. Rahman, W. E. Z. W. A., Ahmad, T. and Shah, R., 2002. Simulating The Neuronal Current Sources In The Brain. Proceedings of BIOMED. Kuala Lumpur: Universiti Malaysia. 19–22.
7. Ahmad, T., Shah, R., Yun, L. L., Zakaria, F. and Zarina, W. E. , 2005. Homeomorphism of Fuzzy Topographic Topological Mapping (FTTM). Matematika. 1(21): 35– 42.
8. Ahmad, T., Jamian, S. S. and Talib, J., 2010. Generalized Finite Sequence of Fuzzy Topographic Topological Mapping. Journal of Mathematics and Statistics. 6(2): 151– 156
9. Sayed, M., and Ahmad,T., 2013. Graph of Finite Sequence of fuzzy Topographic topological Mapping of Order Two. Journal of Mathematics and Statistics. 9(1): 18– 23.
10. Sayed, M. and Ahmad, T., 2013. On Properties of the Graph of Fuzzy Topographic Topological Mapping. Malaysian Journal of Fundamental and Applied Sciences. 9(3):139-142.