Mathematical modeling of radon subdiffusion into the cylindrical layer in ground

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Abstract. This paper describes a mathematical model the radon (^{222}Rn) transport in a cylindrical layer of porous ground, which has fractal properties. With the help of the mathematical apparatus of integral transforms and fractional calculus have been obtained their analytical solutions. These decisions depend on the fractional exponent included in the original model equation and related to the fractal dimension of the ground. It is shown that solutions are generalizations of previously known classical model theory of emanation method.

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Introduction

Emanation method is one of the main methods of radiometric prospecting uranium ores [1, 2]. It is the study of the distributions of emanations - radioactive substances (for example, radon) in porous ground or surface layer of the atmosphere with the help of mathematical models of stationary or non-stationary diffusion-advection. Mathematical models such processes are recorded by means of differential equations with initial and boundary conditions. Kind of differential equations can be varied depending on the specific task or area in which it sought a solution. In emanation method theory, this method represents artificial recesses (generation) of a porous ground, which can be of various geometric shapes: cylindrical, spherical or horizontal layers. In this study, we are interested in the distribution of radon in a cylindrical layer of porous ground.

Unlike classical mathematical models by Grammakov A.G and Bulashevich U.P. [1,3], we consider the porous ground as a fractal structure [4]. One of the basic properties of fractal media - is the memory effects in time (subdiffusion) and spatial coordinate (superdiffusion). Subdiffusion due to "pore-trapped" in the ground, which can be regarded as quasi-isolated from the other pores. Emanations, getting into such pores accumulate in them. Superdiffusion characterized shares wire channels between the pores, which emanations freely and openly transferred to the surface by diffusion, advection or effusion [5]. The part quasi-isolated pores and then wired channels depend on the fractal dimension of the ground [6], which varies depending on the deformation disturbances in the Earth's crust. Therefore, radon emanations are learning in order to predict earthquakes [7].

Statement of the problem

Consider the distribution of radon emanation in a cylindrical layer of porous soil due subdiffusion process. By this assumption, the process of diffusion of radon in a cylindrical layer should slow down.

Subdiffusion can be well described using the mathematical apparatus of fractional calculus [8]. Under this approach, the time derivative is a limiting case of a more general fractional order derivative. For example, one-dimensional equation subdiffusion volumetric radon activity concentration in the cylindrical elementary layer dr of unit length when emanation propagates along the axis z , can be written as :

$$\frac{\partial^{\beta} A(r,t)}{\partial t^{\beta}} = D\left(\frac{\partial^2 A(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial A(r,t)}{\partial r}\right) - \lambda A(r,t) + \lambda A_{\infty}, \quad (1)$$

Here $A(r,t)$ - volumetric activity of radon, A_{∞} - equilibrium volumetric activity of radon, D - diffusion coefficient of radon, λ - radon decay constant, $2.1\cdot 10^{-6}$ c^{-1}, r - the circle radius cylindrical layer, t - the time coordinate. Operator on the left hand side of equation (1) is understood in the sense of Gerasimov- Caputo [8].

Note, in the case $\beta = 1$ equation (1) becomes the equation of nonstationary diffusion of radon emanation theory radiometric prospecting method [5]:

$$\frac{\partial A(r,t)}{\partial t} = D\left(\frac{\partial^2 A(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial A(r,t)}{\partial r}\right) - \lambda A(r,t) + \lambda A_\infty \quad (2)$$

Note, the diffusion coefficient D corresponds to a single gradient pore emanations and calculated according to the relation $D = D^* \varepsilon$, where D^* - the diffusion coefficient corresponding to the gradient of the volume of emanation - ε ground porosity [5].

Equation (1) does not account for convective (advective) component, which would lead to a significant complication of the mathematical solution of the original equation.

Selection Gerasimov Caputo fractional differentiation operator of time due to the preservation of the initial conditions in the local setting , which is more familiar and natural for mathematical modeling of classical diffusion.

Decision

We apply the finite integral Hankel transform of zero order in cylindrical coordinates $r \in [0, r_0]$ for the equation (1) by the formula [9]:

$$H_0\{A(r,t)\}(\eta_n) = \tilde{A}(\eta_n,t) = \int_0^{r_0} rA(r,t)J_0(\eta_n r)dr, \quad (3)$$

Here $J_0(\eta_n r)$ – Bessel function of zero order and η_n - roots of the equation $J_0(r) = 0$.

Using the transformation (3) and Laplace transform in the variable *t* we obtain at the following equation:

$$\hat{\tilde{A}}(\eta_n,p) = \frac{p^{\beta-1}\tilde{A}(\eta_n,0)}{p^\beta + (D\eta_n^2 + \lambda)} + \frac{p^{-1}[\lambda A_\infty/\eta_n + D\eta_n A(r_0,t)]r_0 J_1(\eta_n r_0)}{p^\beta + (D\eta_n^2 + \lambda)}. \quad (4)$$

Inverse Laplace transform of (4) to the complex variable p leads us to the special functions of Mittag-Leffler [10]:

$$\tilde{A}(\eta_n,t) = \tilde{A}(\eta_n,0)E_{\beta,1}\left(-(D\eta_n^2+\lambda)t^\beta\right) + [\lambda A_\infty/\eta_n + D\eta_n A(r_0,t)]r_0 J_1(\eta_n r_0)t^\beta E_{\beta,\beta+1}\left(-(D\eta_n^2+\lambda)t^\beta\right) \quad (5)$$

где $E_{\alpha,\beta}(z)$ – function of Mittag-Leffler [10]. Inverse Hankel transform to equation (5) can be carried out using the formula [11]:

$$f(r) = \frac{2}{r_0^2}\sum_{n=1}^{\infty}\tilde{f}(\eta_n)\frac{J_0(\eta_n r)}{J_1^2(\eta_n r_0)}.$$

Therefore, the general solution of equation (1) can be written as:

$$A(r,t) = \frac{2}{r_0^2}\sum_{n=1}^{\infty}\frac{J_0(\eta_n r)\tilde{A}(\eta_n,0)E_{\beta,1}\left(-(D\eta_n^2+\lambda)t^\beta\right)}{J_1^2(\eta_n r_0)} + \frac{2}{r_0}\sum_{n=1}^{\infty}\frac{J_0(\eta_n r)[\lambda A_\infty/\eta_n + D\eta_n A(r_0,t)]\left(1-E_{\beta,1}\left(-(D\eta_n^2+\lambda)t^\beta\right)\right)}{(D\eta_n^2+\lambda)J_1(\eta_n r_0)} \quad (7)$$

In the limiting case where $\beta = 1$ solution of (7) is a solution of equation (2) and can be written as follows:

$$A(r,t) = \frac{2}{r_0^2}\sum_{n=1}^{\infty}\frac{J_0(\eta_n r)\tilde{A}(\eta_n,0)\exp\left(-(D\eta_n^2+\lambda)t\right)}{J_1^2(\eta_n r_0)} + \frac{2}{r_0}\sum_{n=1}^{\infty}\frac{J_0(\eta_n r)[\lambda A_\infty/\eta_n + D\eta_n A(r_0,t)]\left(1-\exp\left(-(D\eta_n^2+\lambda)t\right)\right)}{(D\eta_n^2+\lambda)J_1(\eta_n r_0)}$$

Solution (7) can be used in studying the distribution of emanation in a cylindrical borehole penetrated into porous ground emanates, as well as in the borehole environment.

Some special cases of the distribution of emanation in the workings of a cylindrical shape. Depending on the specific problem for the equation (1) sets the initial and boundary conditions. Consider some of the tasks of distribution of radon in a cylindrical layer of porous ground, proposed in [5], but for the case of non-stationary subdiffusion.

Problem 1. *Borehole with radius r_0 emanating reveals porous ground. Radon emanation is removed from the borehole well during drilling. Required to determine the distribution of radon in the borehole environment.*

For this problem, characterized by the following boundary conditions:

1) $A(r_0,t) = 0, r = r_0$; 2) $A(r,0) = A_\infty, t = 0, \forall r : r > r_0$;

3) $r \to \infty, A = A_\infty$ – finite value.

The solution to this problem, according to (8) can be written as:

$$A(r,t) = \frac{2A_\infty}{r_0}\sum_{n=1}^{\infty}\frac{J_0(\eta_n r)E_{\beta,1}\left(-(D\eta_n^2+\lambda)t^\beta\right)}{\eta_n J_1(\eta_n r_0)} + \frac{2\lambda A_\infty}{r_0}\sum_{n=1}^{\infty}\frac{J_0(\eta_n r)\left(1-E_{\beta,1}\left(-(D\eta_n^2+\lambda)t^\beta\right)\right)}{\eta_n(D\eta_n^2+\lambda)J_1(\eta_n r_0)}$$

Problem 2. *From the dry cylindrical production, revealing the body emanate, emanation is removed, after which it comes back as a result of subdiffusion environment. Find the distribution of radon inside production.*

This problem is solved with the following boundary conditions:

2) $A(r_0,t)=A_\infty, r=r_0$; 2)

$A(r,0)=0,\ t=0, \forall r: r\le r_0$;

3) $r=0, A(0,t)$ – finite value.

Taking into account boundary conditions, the solution (7) takes the following form:

$$A(r,t)=\frac{2A_\infty}{r_0}\sum_{n=1}^{\infty}\frac{J_0(\eta_n r)\left(1-E_{\beta,1}\left(-\left(D\eta_n^2+\lambda\right)t^\beta\right)\right)}{\eta_n J_1(\eta_n r_0)}$$

Conclusion

In this paper, as a first approximation proposed generalizations of known mathematical models Grammakov A.G. and Bulashevich U.P. radon diffusion in a cylindrical layer of porous ground when it has fractal properties. We obtained the family of solutions in the case when the distribution is due to the emanation of subdiffusion in a cylindrical porous soil layer according to the initial and boundary conditions.

In the future, some interest is to consider the model of radon migration, taking into account such transfer mechanisms as advection and superdiffusion and transport model of radon in ground- atmosphere system or many-layered media [12].

Development of mathematical modeling of radon transport in ground- atmosphere system may allow the development of a methodology to identify and interpretation anomalous effects in the time series of radon fields obtained on a network of stations Petropavlovsk-Kamchatsky geodynamic polygon.

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