An Investigation of the longitudinal fluctuations of viscoelastic cores

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Abstract: It is necessary to select a problem from dynamic problems of viscoelasticity about fluctuation of the viscoelastic systems which decisions are consolidated to Voltaire's integro-differential equation of the II type. The solution of this equation requires a problem of an analytical type of a kernel, or ths equation solves with various numerical methods. In this paper the approximate solution of this equation for any kernels at small viscosity is proposed. The decision in the form of a row is received, the original of which first member is the solution of this equation received by a known method averaging, and the accounting of the subsequent members of a row improves objective accuracy. Influence of the subsequent members of a row on the decision is calculated for Rzhanitsin's kernel and shown that it increases at increase in frequency.

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1. Introduction

It is known that at the solution of non-stationary dynamic problems for various designs the analytical type of kernels of a relaxation isn't given. Therefore decisions are under construction by means of some approximate methods which give a final solution of the integro-differential equations of fluctuations of the viscoelastic systems, averagings realized by a method, a freezing method, a method of continuations and a method of integral transformations of Laplace, Fourier, Mellin and their combinations [1,2,8,13].

However, methods of integral transformations are inseparably connected with complex problems of the invers transformation which in case of more real c between tension and deformation inevitably result in need of larger number of cuts on branches in the course of contour integration [4,5].

2. Material and Methods

In this article the approximate solution of the equation of longitudinal fluctuations of viscoelastic cores for any kernels at small viscosity is proposed. The image of the decision in the form of a row is prezented, the original of which first member is the solution of this equation received by a method of averaging, and the accounting of the subsequent members improves the accuracy of the solution of an objective.

It is know that the equation of the longitudinal fluctuations of elastic cores has the following form [1,4,14]:

$$\rho \frac{\partial^2 u(x,t)}{\partial t^2} = E \frac{\partial^2 u(x,t)}{\partial x^2}, \qquad (1)$$

where

$$E = 2\mu(1+\nu) = 2G(1+\nu),$$

ν - Poisson's number which change in small limits and therefore is considered further constant, ρ - material density, E - the instantaneous Young's modulus. $u(x,t)$ - the movement, μ - Lame's constant.

Replacing μ with $\mu-\mu^*$ in (1) can be obtained:

$$\rho \frac{\partial^2 u(x,t)}{\partial t^2} = 2\mu(1+\nu)\frac{\partial^2 u(x,t)}{\partial x^2} - 2(1+\nu)\mu^* \frac{\partial^2 u(x,t)}{\partial x^2}$$

Here

$$2(1+\nu)\mu^* = (1+\nu)\int_0^t \tilde{\Gamma}(t-\tau)\frac{\partial^2 u(x,\tau)}{\partial x^2} d\tau .$$

Taking into account this in the last equation can be obtain:

$$\tilde{\Gamma}(t) = 2GR(t)$$

and to assume that the material of a core possesses small viscosity, the integral $\int_0^t R(s)ds$ is positive and small rather to unit, therefore

$$\int_0^t R(s)ds = \varepsilon \int_0^t R(s)ds \le \varepsilon .$$

Therefore can be obtained following equation:

$$\rho\frac{\partial^2 u(x,t)}{\partial t^2}=E\left[\frac{\partial^2 u(x,t)}{\partial x^2}-\varepsilon\int_0^t R(t-\tau)\frac{\partial^2 u(x,\tau)}{\partial x^2}d\tau\right] \quad (2)$$

Accept initial conditions in the following form:

$$u(x,t)=\varphi_0(x) \quad as \quad t=0$$
$$\frac{\partial u(x,t)}{\partial t}=\varphi_1(x)\ as\, t=0 \quad (3)$$

And boundary conditions we accept in the following form:

$$u(x,t)=0 \quad as \quad x=0$$
$$u(x,t)=0\ as\ x=l \quad (4)$$

Where l - core length.

The particle solution of the equation (2), is identical not equal to zero we look for in following:

$u(x,t)=X(x)T(t)$

Taking into account is in the (2) for finding of functions and, we obtain two independent equations:

$$X''(x)+\left(\frac{\lambda}{c}\right)^2 X(x)=0$$
$$T''(t)+\lambda^2 T(t)=\varepsilon\lambda^2\int_0^t R(t-\tau)T(\tau) \quad (5)$$

Solving the first equation under boundary conditions (4), we find

$$X_k(x)=\cos\frac{\lambda_k x}{c}$$

where

$$\lambda_k=\pm\frac{k\pi c}{l};\ k=0,1,2,\ldots$$

$c=\sqrt{\frac{E}{\rho}}$ - the speed of waves distribute.

Thus, the common solution of the equation of fluctuation of a viscoelastic core has following form:

$$u(x,t)=\sum_{k=0}^{\infty}X_k(x)T_k(t) \quad (6)$$

where $X_k(x)$ - the coordinate functions found at the solution of the corresponding elastic problem also don't depend on the parameters characterizing viscous properties of a material of a core, and function $T_k(t)$ is the solution of the integro-differential equation (5).

Thus, the problem of determine of bias in a core is reduced to determine of functions $T_k(t)$ from the equation (5).

Applying Laplace's integrated transformation to the equation (5) respect to time parameter t and taking into account (3), can be obtained:

$$\overline{T}_k(p)=\frac{p\varphi_0+\varphi_1}{p^2+\lambda_k^2-\varepsilon\lambda_k^2\overline{R}} \quad (7)$$

Here at small values of time parameter p - is rather large. If we consider materials with instant elasticity, the image $\overline{R}(p)$ with increase p tends to zero therefore the inequality is executed:

$$\left|\frac{\varepsilon\ \lambda_k^2\ R(p)}{p^2+\lambda_k^2}\right|<1$$

Then the equation (7) can be represent in the following form:

$$\overline{T}_k(p)=\frac{p\varphi_0+\varphi_1}{p^2+\lambda_k^2}\cdot\frac{1}{1-\dfrac{\varepsilon\lambda_k^2\overline{R}(p)}{p^2+\lambda_k^2}}$$

or

$$\overline{T}_k(p)=\frac{p\varphi_0+\varphi_1}{p^2+\lambda_k^2}\sum_{n=0}^{\infty}\left(\frac{\varepsilon\lambda_k^2\overline{R}(p)}{p^2+\lambda_k^2}\right)^n. \quad (8)$$

Here

$$\frac{\varepsilon\lambda_k^2\overline{R}(p)}{p^2+\lambda_k^2}\ \dot{=}\ \varepsilon\lambda_k\int_0^t\sin\lambda_k(t-\tau)R(\tau)d\tau=$$

$$=\varepsilon\lambda_k\sin\lambda_k t\int_0^t R(\tau)\cos\lambda_k\tau d\tau-\varepsilon\lambda_k\cos\lambda_k t\int_0^t R(\tau)\sin\lambda_k\tau d\tau=$$

$$=\varepsilon\lambda_k\sin\lambda_k t\int_0^\infty R(\tau)\cos\lambda_k\tau d\tau-\varepsilon\lambda_k\cos\lambda_k t\int_0^\infty R(\tau)\sin\lambda_k\tau d\tau-$$

$$-\varepsilon\lambda_k\sin\lambda_k t\int_t^\infty R(\tau)\cos\lambda_k\tau d\tau+\varepsilon\lambda_k\cos\lambda_k t\int_t^\infty R(\tau)\sin\lambda_k\tau d\tau$$

If to accept the following designation:

$$R_s=\int_0^\infty R(\tau)\sin\lambda_k\tau d\tau,\ R_c=\int_0^\infty R(\tau)\cos\lambda_k\tau d\tau$$

$$A(t)=\sin\lambda_k t\int_t^\infty R(\tau)\cos\lambda_k\tau d\tau-\cos\lambda_k t\int_t^\infty R(\tau)\sin\lambda_k\tau d\tau$$

Then we are obtained:

$$\frac{\varepsilon\lambda_k^2\overline{R}(p)}{p^2+\lambda_k^2}\ \dot{=}\ \varepsilon\lambda_k R_c\sin\lambda_k t-\varepsilon\lambda_k R_s\cos\lambda_k t-\varepsilon\lambda_k A(t)$$

The last formula in Laplace's can be following form:

$$\frac{\varepsilon\lambda_k^2\overline{R}(p)}{p^2+\lambda_k^2}=\frac{\varepsilon\lambda_k^2R_c-\varepsilon\lambda_k pR_s-\varepsilon\lambda_k\left(p^2+\lambda_k^2\right)\overline{A}(p)}{p^2+\lambda_k^2}$$

Considering it in (8), we receive:

$$\overline{T}_k(p)=\frac{p\varphi_0+\varphi_1}{\overline{a}(p)-\varepsilon\lambda_k^2\overline{b}(p)} \quad (9)$$

where

$$\overline{a}(p)=\left(p+\frac{1}{2}\varepsilon R_s\lambda_k\right)^2+\lambda_k^2\left(1-\frac{1}{2}\varepsilon R_c\right)^2$$

$$\overline{b}(p)=\overline{R}(p)+R_s\frac{p}{\lambda_k}+R_c+\frac{\varepsilon}{4}\left(R_s^2+R_c^2\right)$$

Here $\left|\dfrac{\varepsilon\lambda_k^2\overline{b}(p)}{\overline{a}(p)}\right|<1$.

Therefore we can write a formula (9) in following form:

$$\overline{T}_k(p)=\frac{p\varphi_0+\varphi_1}{\overline{a}(p)}\left[1+\varepsilon\lambda_k^2\frac{\overline{b}(p)}{\overline{a}(p)}+\varepsilon^2\lambda_k^4\frac{\overline{b}^2(p)}{\overline{a}^2(p)}+\ldots\right] \quad (10)$$

From here for the first member after the inverse Laplace's transformations it is found:

$$T_1(t)=\exp\left(-\frac{1}{2}\varepsilon R_s\lambda_k t\right)\times$$
$$\times\left[\varphi_0\cos\lambda_k\left(1-\frac{1}{2}\varepsilon\Gamma_c\right)t+\frac{\varphi_1-\frac{1}{2}\varepsilon R_s\lambda_k}{\lambda_k\left(1-\frac{1}{2}\varepsilon R_c\right)}\sin\lambda_k\left(1-\frac{1}{2}\varepsilon R_c\right)t\right] \quad (11)$$

The last formula is the solution of the objective, received by an averaging method [1,2].

For finding of the following approach we will present it in a look:

$$T_2(t)=\varepsilon\lambda_k^2T_{k_0}(t)*L^{-1}\left[\frac{\overline{b}(p)}{\overline{a}(p)}\right] \quad (12)$$

where

$$f(t)*g(t)=\int_0^t f(t-\tau)g(\tau)d\tau\ .$$

L^{-1} - the operator of the return transformation of Laplace. Means, it is necessary to calculate the original of functions $\dfrac{\overline{b}(p)}{\overline{a}(p)}$. For this purpose we will present it in following:

$$\frac{\overline{b}(p)}{\overline{a}(p)}=\frac{\overline{R}(p)}{\overline{a}(p)}+\frac{R_s}{\lambda_k}\cdot\frac{p+d}{\overline{a}(p)};$$

where $d=\dfrac{R_c}{R_s}\lambda_k+\dfrac{\varepsilon\lambda_k}{4R_s}\left(R_s^2+R_c^2\right)$

Then

$$L^{-1}\left[\frac{\overline{b}(p)}{\overline{a}(p)}\right]=R(t)*\exp\left(-\frac{\varepsilon}{2}R_s\lambda_k t\right)\cdot\frac{\sin\lambda_k\left(1-\frac{1}{2}\varepsilon R_c\right)t}{\lambda_k\left(1-\frac{1}{2}\right)\varepsilon R_c}+$$
$$\frac{R_s}{\lambda_k}\exp\left(-\frac{1}{2}\varepsilon R_s\lambda_k t\right)\left[\cos\lambda_k\left(1-\frac{1}{2}\varepsilon R_c\right)t+\right.$$
$$\left.\frac{d-\frac{\varepsilon}{2}R_s\lambda_k}{\lambda_k\left(1-\frac{1}{2}\varepsilon R_c\right)}\sin\lambda_k\left(1-\frac{1}{2}\varepsilon R_c\right)t\right] \quad (13)$$

Regeneration of originals of the following approximations of a row (11) doesn't present difficulty.

From (11) and (12) receives that existence of viscosity of a material in (2) leads to attenuation of summary fluctuations of a core under the exponential law and phase shift is observed.

For calculation of influence of the member (12) on the decision, we will consider Rzhanitsin's kernel [4,12,14] $R(t)=\varepsilon t^{\alpha-1}\exp(-\beta t)$.

Where $0<\alpha<1$, β - a constant, ε - some small parameter.

For this kernel from a formula (12) can be obtained:

$$T_2(t)=\exp\left(-\frac{1}{2}\varepsilon\lambda R_s t\right)\times$$
$$\times\left\{\left[\frac{\varepsilon A_1}{2}\cos\lambda\left(1-\frac{1}{2}\varepsilon R_c\right)t+\frac{\varepsilon A_4}{2}\sin\lambda\left(1-\frac{1}{2}\varepsilon R_c\right)t\right]\times\right.$$
$$\times\int_0^t e^{-\beta\tau}\tau^{\alpha-1}\sin2\lambda\left(1-\frac{1}{2}\varepsilon R_c\right)\tau d\tau+$$
$$+\left[\frac{\varepsilon A_1}{2}\sin\lambda\left(1-\frac{1}{2}\varepsilon R_c\right)t-\frac{\varepsilon A_4}{2}\cos\lambda\left(1-\frac{1}{2}\varepsilon R_c\right)t\right]\times$$
$$\times\int_0^t e^{-\beta\tau}\tau^{\alpha-1}d\tau-\left[\frac{\varepsilon A_1}{2}\sin\lambda\left(1-\frac{1}{2}\varepsilon R_c\right)t-\right.$$
$$\left.-\frac{\varepsilon A_4}{2}\cos\lambda\left(1-\frac{1}{2}\varepsilon R_c\right)t\right]\times\int_0^t e^{-\beta\tau}\tau^{\alpha-1}\cos2\lambda\left(1-\frac{1}{2}\varepsilon R_c\right)\tau d\tau+$$
$$+\frac{t}{2}\left(A_2-A_6\right)\cos\lambda\left(1-\frac{1}{2}\varepsilon R_c\right)t+$$
$$\left.+\left[\frac{A_2+A_6}{2\lambda\left(1-\frac{1}{2}\varepsilon R_c\right)}+\frac{t}{2}\left(A_3+A_5\right)\right]\sin\lambda\left(1-\frac{1}{2}\varepsilon R_c\right)t\right\}$$

where $R_s=\varepsilon\Gamma(\alpha)\left(\beta^2+\lambda^2\right)^{-\frac{\alpha}{2}}\sin\left[\alpha arctg\left(\frac{\lambda}{\beta}\right)\right]$

$$R_c = \varepsilon\Gamma(\alpha)\left(\beta^2+\lambda^2\right)^{-\frac{\alpha}{2}}\cos\left[\alpha arctg\left(\frac{\lambda}{\beta}\right)\right]$$

$\Gamma(\alpha)$ - Euler's Gamma function.

$$A_1 = \frac{\varepsilon\lambda\varphi_0}{1-\frac{1}{2}\varepsilon R_c}; \qquad A_2 = \varepsilon\lambda\varphi_0 R_s;$$

$$A_3 = \frac{\varepsilon R_s\varphi_0\left(d-\frac{1}{2}\varepsilon R_s\lambda\right)}{1-\frac{1}{2}\varepsilon R_c};$$

$$A_4 = \frac{\varepsilon\left(\varphi_1-\frac{1}{2}\varepsilon R_s\varphi_0\right)}{\left(1-\frac{1}{2}\varepsilon R_c\right)^2};$$

$$A_5 = \frac{\varepsilon R_s\left(\varphi_1-\frac{1}{2}\varepsilon\lambda R_s\varphi_0\right)}{1-\frac{1}{2}\varepsilon R_c}$$

$$A_6 = \frac{\varepsilon R_s\left(\varphi_1-\frac{1}{2}\varepsilon R_s\varphi_0\lambda\right)\left(d-\frac{1}{2}\varepsilon R_s\lambda\right)}{\lambda\left(1-\frac{1}{2}\varepsilon R_c\right)^2}.$$

3. Results.

For polypropylene are constructed graphs functions $T_1(t)$ and $T_2(t)$ at the following values of parameters: $\alpha = 0.1$; $\beta = 0.05$; $\varepsilon = 0.09$.

Fig. 1. Graphs functions $T_1(t)$ and $T_2(t)$ for parameters: $\varphi_0 = 0$; $\varphi_1 = 1$.

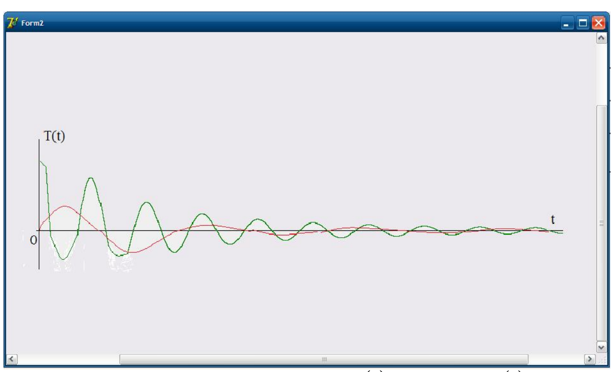
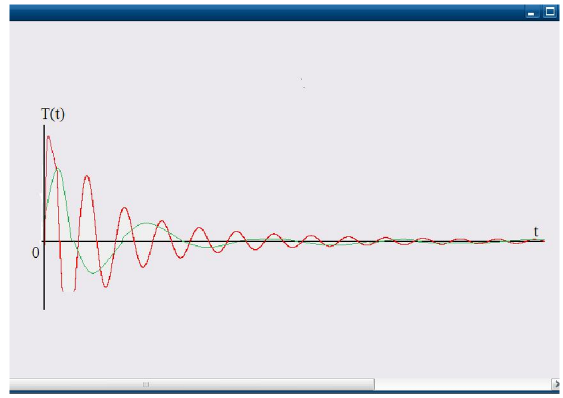


Fig. 2. Graphs functions $T_1(t)$ and $T_2(t)$ for parameters: $\varphi_0 = 1$; $\varphi_1 = 0$.



From figures it is visible that the accounting of the subsequent members of a row improves the decision accuracy as at the great values of frequency the error is small, and with increase in frequency it increases. At $\lambda = 100$ amplitude $T_2(t)$ at some values of time makes $20-25\%$ amplitudes $T_1(t)$.

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