Cardinal Functions on Ditopological Texture Spaces

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Abstract: In this paper we define the concept of dicardinal function, and then (co)weight, (co)densification, (co)net weight, (co)pseudo character which are able to be used in classifying of ditopological texture spaces. It is natural to ask how there are relationships between the set S ($\mathcal{P}$ or $\mathcal{Q}$) and dicardinal functions that we defined in ditopological texture spaces. Based on the question, our aim in the paper is to investigate dicardinal functions above for ditopological texture spaces. We obtain useful some results on bounds of S, the set $\mathcal{P}$ of all p-sets and the set $\mathcal{Q}$ of all q-sets by choosing the subclasses satisfying axiom T_0 or Co-T_1 the class of all ditopological texture spaces. Furthermore, we show that (co)weight and (co)densification restrict each others.

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1. Introduction

Being closest to set theory, cardinal invariants play a major role in general topology of which set theory forms the basis. They are used as a most useful tools in classifying topological spaces. Some important classes of topological spaces distinguished by them are separable spaces, compact spaces, topological spaces which has a countable basis. Also, by using cardinal invariants, it is possible to compare quantitively topological properties, and generalize the present results of them. Many researchers have contributed to development in theory of cardinal functions since 1920. In the 1920's, Alexandroff and Urysohn (1929) show that every compact, perfectly normal space has cardinality $\leq 2^{\omega}$. One of results that Čech and Pospíšil (1938) obtained states that every compact, first countable space has cardinality $\leq \omega$ or $\geq 2^{\omega}$. In 1940's, it is shown by Pondiczery (1944), Hewitt (1946) and Marczewski (1947) that a product of at most 2^{ω} separable space is separable. In 1965, one of Groot's results which generalizes Alexandroff and Urysohn's result above states that a Hausdorff space in which every subspace is Lindelöf has cardinality $\leq 2^{\omega}$. Arhangel'skii (1969) show that every Lindelöf, first countable, Hausdorff space has cardinality $\leq 2^{\omega}$.

The notion of fuzzy structure introduced by L. M. Brown (1993) in [1-2] is redenominated in texture space developed one of this structure by L. M. Brown and R. Ertürk (2000) in [4-5]. The structure makes it possible to be investigated mathematical concepts without any complement in account of the fact that, in a texture space$(S,\mathcal{S})$, $\mathcal{S}$ doesn't need to be a closed set under set-theoretical complement by the definition above. Based on the structure of texture space, it is obvius intiutively that a convenient topology on a texture space doesn't need to hold the existence of the duality of interior and closure and so not need to hold both axioms of open sets and ones of closed sets. In a series of three papers, L. M. Brown et al. present first two ones in 2004 and last one in 2006. In the first of them subtitled 'Basic concepts', the authors introduce a systematic form of the concepts of direlation, difunction, the category *dfTex* ditopological texture space in a categorical setting. In second paper, the category *dfDitop* of ditopological texture spaces and bicontinuous difunctions is defined. The subject of third paper is on separation axioms in general ditopological texture space. In a ditopological setting , L. M. Brown and M. M. Gohar study compactness in 2009, and strong compactness one year later [9-10].

In this study, after given concept of dicardinal function, (co)weight, (co) net weight, (co) densification, (co) pseudo character which are ones of most useful tools in classifying ditopological texture spaces are defined. It is natural to ask how there are relationships between the sets S ($\mathcal{P}$ or $\mathcal{Q}$) and dicardinal functions which will be defined on ditopological texture spaces. Based on the question, our aim in the paper is to investigate (co) weight, (co) net weight, (co) pseudo character, co (densification) for ditopological texture spaces. In the section 2 titled 'Texture Spaces', we recall the basic definitions of texture space, ditopology on the texture space and then, some definitions and theorems regarding the subjects. The concepts of ordinals, cardinals, cardinality of a set and cardinal functions and some theorems which are related to cardinals are given in the section 3. Finally, in the last section, we give the definition of dicardinal function in ditopological texture spaces. Then, we represent dicardinal functions and some important theorems for (a particular subclass of) the class of all ditopological texture spaces. For every ditopological texture space $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa)$, $co\text{-}r(\mathfrak{D}) \leq w(\mathfrak{D})$ and $r(\mathfrak{D}) \leq co\text{-}w(\mathfrak{D})$; if $\mathfrak{D}$ is T_0, then $|\mathcal{Q}| \leq \max\{o(\mathfrak{D}), c(\mathfrak{D})\}$ and $|S| \leq 2^{\max\{nw(\mathfrak{D}), co\text{-}nw(\mathfrak{D})\}}$. In particular, for every Kolmogorov ditopological coseparated texture space, $|S| \leq \max\{o(\mathfrak{D}), c(\mathfrak{D})\}$. If $\mathfrak{D}$ is $co\text{-}T_1$, then $|S| \leq nw(\mathfrak{D})^{\Psi(\mathfrak{D})}$.

2. Texture Spaces

The following definitions and propositions were introduced in [1-11].

A *texturing* on a non-empty set S is a set $\mathcal{S}$ containing $S, \emptyset$ of subsets of S with respect to inclusion satisfying the conditions: (i) $(\mathcal{S}, \subseteq)$ is a complete lattice (ii) $\mathcal{S}$ is completely distributive, (iii) $\mathcal{S}$ separates the points of S, (iv) Meets $\bigwedge$ and finite joins $\bigvee$ coincide with intersections $\bigcap$ and unions $\bigcup$, respectively, for $\mathcal{S}$. $(S, \mathcal{S})$ is then called *texture space*.

For each $s \in S$, the *p-set* P_s is defined by
$$P_s = \bigcap\{A \in \mathcal{S} \mid s \in A\},$$
and *q-set*
$$Q_s = \bigvee\{A \in \mathcal{S} \mid s \notin A\} = \bigvee\{P_r \mid r \in S, s \notin P_r\}.$$
We recall that a texture $(S, \mathcal{S})$ is said to be *coseparated* if, for all
$$s, t \in S, \quad Q_s \subseteq Q_t \Rightarrow P_s \subseteq P_t.$$
We define $\mathcal{P} = \{P_s \mid s \in S\}$ and $\mathcal{Q} = \{Q_s \mid s \in S\}$.

Texture space: Let $(S, \mathcal{S})$ a texture space, and τ, κ subsets of $\mathcal{S}$. $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa)$ is called a ditopological texture space, and the pair (τ, κ) a ditopology on $(S, \mathcal{S})$ if (τ, κ) satisfies

$(G1)$ $S, \emptyset \in \tau$,
$(G2)$ $G_1, G_2 \in \tau \Rightarrow G_1 \cap G_2 \in \tau$,
$(G3)$ $\mathcal{G} \subseteq \tau \Rightarrow \bigvee\mathcal{G} \in \tau$,
$(F1)$ $S, \emptyset \in \kappa$,
$(F2)$ $F_1, F_2 \in \kappa \Rightarrow F_1 \cup F_2 \in \kappa$,
$(F3)$ $\mathcal{F} \subseteq \kappa \Rightarrow \bigcap\mathcal{F} \in \kappa$.

Base and Cobase: Let $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa)$ a ditopological texture space, and $\mathcal{B}$ a subset of τ (κ). $\mathcal{B}$ is a base (cobase) for (τ, κ) if, for all $C \in \tau$ (κ), there exists a subset $\mathcal{B}_C$ of $\mathcal{B}$ such that $C = \bigvee\mathcal{B}_C$ $(\bigcap\mathcal{B}_C)$.

Interior and Closure: Let $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa)$ be a ditopological texture space. The interior and the closure of the set $A \in \mathcal{S}$ is defined, respectively:
$$]A[= \bigvee\{G \mid G \subseteq A\},$$
$$[A] = \bigcap\{K \mid A \subseteq K\}.$$

Dense and Codense: A set $A \in \mathcal{S}$ is said to be dense (codense) in (τ, κ) if $[A] = S$ $(]A[= \emptyset)$.

T_0 Axiom: Let $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa)$ be a ditopological space, $(\tau \cup \kappa)^{\vee}$ denote the set of arbitrary joins of sets in $(\tau \cup \kappa)$ and $(\tau \cup \kappa)^{\cap}$ the set of arbitrary intersections of sets in $\tau \cup \kappa$. $\mathfrak{D}$ is said to be T_0 if
$$(\forall s, t \in S)(\exists C \in (\tau \cup \kappa)^{\vee})\ (Q_s \nsubseteq Q_t \Rightarrow P_s \nsubseteq C \nsubseteq Q_t),$$
or equivalently,
$$(\forall s, t \in S)(\exists C \in (\tau \cup \kappa)^{\cap})\ (Q_s \nsubseteq Q_t \Rightarrow P_s \nsubseteq C \nsubseteq Q_t).$$

Proposition 2.1. The following are characteristic properties of T_0 ditopological texture space:
$$(\forall s, t \in S)(\exists C \in \tau \cup \kappa)\ (Q_s \nsubseteq Q_t \Rightarrow P_s \nsubseteq C \nsubseteq Q_t),$$
$$(\forall s, t \in S)([P_s] \subseteq [P_t] \text{ and }]Q_s[\subseteq]Q_t[\Rightarrow Q_s \subseteq Q_t).$$

R_0 and $Co\text{-}R_0$ Axioms Let $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa)$ be a ditopological space. $\mathfrak{D}$ is said to be R_0 if
$$(\forall s \in S)(\forall G \in \tau)\ (G \nsubseteq Q_s \Rightarrow [P_s] \nsubseteq G).$$
$\mathfrak{D}$ is said to be $Co\text{-}R_0$ if
$$(\forall s \in S)(\forall F \in \kappa)\ (P_s \nsubseteq F \Rightarrow F \subseteq]Q_s[.$$

T_1 and $Co\text{-}T_1$ Axioms Let $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa)$ be a ditopological space. $\mathfrak{D}$ is said to be T_1 $(Co\text{-}T_1)$ if it is T_0 and R_0 $(Co\text{-}R_0)$.

Proposition 2.2. The following are characteristic properties of T_1 ditopological texture space:
$$(\forall A \in \mathcal{S})(\exists \mathcal{F} \subseteq \kappa)\ (A = \bigvee\mathcal{F}),$$
$$(\forall s, t \in S)(\exists F \in \kappa)\ (Q_s \nsubseteq Q_t \Rightarrow P_s \nsubseteq F \nsubseteq Q_t).$$

Proposition 2.3. The following are characteristic properties of $Co\text{-}T_1$ ditopological texture space:
$$(\forall A \in \mathcal{S})(\exists \mathcal{G} \subseteq \tau)\ (A = \bigcap\mathcal{G}),$$
$$(\forall s, t \in S)(\exists G \in \tau)\ (Q_s \nsubseteq Q_t \Rightarrow P_s \nsubseteq G \nsubseteq Q_t).$$

3. Cardinal Functions

The following definitions and propositions were introduced in [12-24].

Cardinal and Ordinal: A set A is called *transitive* iff $\forall y \forall z(z \in y \wedge y \in A \Rightarrow z \in A)$. An *ordinal* α is a transitive set such that all $\beta \in \alpha$ are transitive. Then, we recall that a *cardinal* is an ordinal that there is no bijection from itself to a smaller ordinal.

Proposition 3.1. If $(X, \prec)$ is a well-ordering, then there is unique ordinal α and a unique isomorphism $\pi : (X, \prec) \rightarrow (\alpha, \in)$.

Proposition 3.2. Assuming axiom of choice, for every set X, there exists a relation $\prec$ on X such that $(X,\prec)$ is a well-ordering.

Cardinality of a Set: Let X be a set. From the facts above, there exists an ordinal that can be mapped one-to-one onto X. The smallest one of the ordinals is called the *cardinality* or *cardinal number* of X, written as $|X|$.

Sum and Product of Cardinals: Let κ and λ be cardinal numbers. The cardinal number $\kappa+\lambda := |\kappa\times\{0\}\cup\lambda\times\{1\}|$ is called the cardinal sum of κ and λ, and $\kappa.\lambda := |\kappa\times\lambda|$ cardinal product of κ and λ.

Proposition 3.3. If one of cardinal numbers κ and λ is nonzero and the other infinite, then $\kappa+\lambda=\kappa.\lambda=\max\{\kappa,\lambda\}$.
Let X be a set. The set of all subsets of X which have cardinality λ is denoted by $[X]^\lambda$, that is, $[X]^\lambda=\{A\subseteq X \mid |A|=\lambda\}$. The sets $[X]^{<\lambda}$ and $[X]^{\le\lambda}$ are defined analogously, that is, $[X]^{<\lambda}=\{A\subseteq X \mid |A|<\lambda\}$ and $[X]^{\le\lambda}=\{A\subseteq X \mid |A|\le\lambda\}$.

Proposition 3.4. Let X an infinite set of cardinality κ, and $\lambda\le\kappa$ a cardinal. Then $\left|[X]^{\le\lambda}\right|=\kappa^\lambda$.
We recall that a cardinal function is a function ϕ from the class of all topological spaces into the class of all infinite cardinals such that, if X and Y are homeomorphic, then $\phi(X)=\phi(Y)$.

4. Main Results

Since $\mathcal{S}$ separates the points of S, for every $s,t\in S, s\neq t\Rightarrow P_s\neq P_t$. From the definition p-set, for every $s,t\in S$, $P_s\neq P_t\Rightarrow s\neq t$. Thus $|S|=|\mathcal{P}|$. It need to be true that $|S|=|Q|$. To prove the equality, we need to use the definition of coseparated texture space.

Theorem 4.1. If a texture space $(S,\mathcal{S})$ is coseparated, then $|S|=|Q|$.

Proof. Consider a pair of distinct points $s,t\in S$. Then $P_s\neq P_t$, and so $P_s\not\subseteq P_t$ or $P_t\not\subseteq P_s$. Since $(S,\mathcal{S})$ is coseparated, $Q_s\not\subseteq Q_t$ or $Q_t\not\subseteq Q_s$; therefore $Q_s\neq Q_t$. Thusly, we have an one-one map from S to Q. Now, take $s,t\in S$ with $Q_s\neq Q_t$. Then $Q_s\not\subseteq Q_t$ or $Q_t\not\subseteq Q_s$. From the definition of q-set, it is immadiate that $P_s\not\subseteq P_t$ or $P_t\not\subseteq P_s$, and so $P_s\neq P_t$; therefore $s\neq t$. Thereby, we have an one-one map from Q to S.

Dicardinal Function: A function ϕ from the class of all ditopological texture spaces (or a particular subclass) into the class of all infinite cardinal numbers is called a *dicardinal function* if, for every pair $\mathfrak{D}_i=(S_i,\mathcal{S}_i,\tau_i,\kappa_i)_{i\in\{1,2\}}$ of ditopological texture spaces,
'$\mathfrak{D}_1,\mathfrak{D}_2$ are dihomeomorphic' $\Rightarrow \phi(\mathfrak{D}_1)=\phi(\mathfrak{D}_2)$.

Number of Open and Closed Sets: Let $\mathfrak{D}=(S,\mathcal{S},\tau,\kappa)$ be a ditopological texture space. $o(\mathfrak{D})$ and $c(\mathfrak{D})$ are defined as the number of open sets in $\mathfrak{D}$ plus ω and the number of closed sets in $\mathfrak{D}$ plus ω, respectively. $oc(\mathfrak{D})$ is defined as the number of sets in $\tau\cup\kappa$ plus ω.

Remark 4.1. Clearly, $\max\{o(\mathfrak{D}),c(\mathfrak{D})\}\le oc(\mathfrak{D})\le|\mathcal{S}|\le 2^{|S|}$. Also, neither of $o(\mathfrak{D})$ and $c(\mathfrak{D})$ dominates the other according to [?, Section 2]. If $\mathfrak{D}$ is a complemented ditopological texture space, then $o(\mathfrak{D})=c(\mathfrak{D})$.

Theorem 4.2. If a ditopological texture space $\mathfrak{D}=(S,\mathcal{S},\tau,\kappa)$ is T_0 (Kolmogorov), then $|Q|\le\max\{o(\mathfrak{D}),c(\mathfrak{D})\}$.

Proof. Define $\varphi: Q\to\tau\times\kappa$ by $\varphi(Q_s)=(]Q_s[,[P_s])$. Since $\mathfrak{D}$ is T_0, φ is one-one (See Proposition 2.1.(2)). Thus $|Q|\le|\tau\times\kappa|=\max\{o(\mathfrak{D}),c(\mathfrak{D})\}$.
Conclusion 4.1. For every Kolmogorov ditopological coseparated texture space, $|S|\le\max\{o(\mathfrak{D}),c(\mathfrak{D})\}$.

Weight and Coweight: Let $\mathfrak{D}=(S,\mathcal{S},\tau,\kappa)$ be a ditopological texture space. The *weight* and *coweight* of $\mathfrak{D}$ are defined as follows,

$$w(\mathfrak{D})=\min\{|\mathcal{B}| \mid \mathcal{B}\text{ a base for }(\tau,\kappa)\},$$
$$co\text{-}w(\mathfrak{D})=\min\{|\mathcal{B}| \mid \mathcal{B}\text{ a cobase for }(\tau,\kappa)\},$$

respectively.

Densifier and Codensifier: Let $\mathfrak{D}=(S,\mathcal{S},\tau,\kappa)$ be a ditopological texture space. A subset $\mathcal{A}$ of $\mathcal{S}$ is said to be *densifier* (*codensifier*) in $\mathfrak{D}$ if $\bigvee\mathcal{A}$ ($\bigcap\mathcal{A}$) is dense (codense) in (τ,κ).

Densification and Codensification: The densification and codensification of $\mathfrak{D}$ are defined as follows,

$$r(\mathfrak{D})=\min\{|\mathcal{A}| \mid \mathcal{A}\text{ densifier in }(\tau,\kappa)\},$$
$$co\text{-}r(\mathfrak{D})=\min\{|\mathcal{A}| \mid \mathcal{A}\text{ codensifier in }(\tau,\kappa)\},$$

respectively.

Now, we show that, in a ditopological texture space $\mathfrak{D}$, how there are relationships between (co)weight and (co)densification.

Theorem 4.3. For every ditopological texture space $\mathfrak{D}=(S,\mathcal{S},\tau,\kappa)$, we have

$$co\text{-}r(\mathfrak{D})\le w(\mathfrak{D}),$$
$$r(\mathfrak{D})\le co\text{-}w(\mathfrak{D}).$$

Proof. 1. Let $\mathcal{B} = \{B_p\}_{p\in P}$ be a base for the ditopological texture space $\mathfrak{D}$ not consisting of $\emptyset$ such that $|\mathcal{B}| = w(\mathfrak{D})$. Let us set $\mathcal{M} = \{Q_{a_p}\}_{p\in P}$ by choosing a certain q-set Q_{a_p} such that $B_p \not\subseteq Q_{a_p}$ for each $p \in P$. Since $\mathcal{B}$ is a base for $\mathfrak{D}$,

$$\forall G \in \tau, G \neq \emptyset \Rightarrow G \not\subseteq \bigcap\mathcal{M};$$

so $\mathcal{B}$ is codensifier in $\mathfrak{D}$. Let us define $\varphi : \mathcal{B} \to \mathcal{M}$ by $\varphi(\mathcal{B}_p) = Q_{a_p}$. Obviously, φ is onto; therefore $|\mathcal{M}| \leq |\mathcal{B}|$. Since $co\text{-}r(\mathfrak{D}) \leq |\mathcal{M}|$, $co\text{-}r(\mathfrak{D}) \leq w(\mathfrak{D})$.

2. Let $\mathcal{B} = \{B_r\}_{r\in R}$ be a cobase for the ditopological texture space $\mathfrak{D}$ not consisting of S such that $|\mathcal{B}| = co\text{-}w(\mathfrak{D})$. Let us set $\mathcal{N} = \{P_{b_r}\}_{r\in R}$ by choosing a certain p-set P_{b_r} such that $P_{b_r} \not\subseteq B_r$ for each $r \in R$. Since $\mathcal{B}$ is a cobase for $\mathfrak{D}$,

$$\forall K \in \kappa, K \neq S \Rightarrow \bigvee\mathcal{N} \not\subseteq K;$$

so $\mathcal{M}$ is densifier in $\mathfrak{D}$. Let us define $\phi : \mathcal{B} \to \mathcal{N}$ by $\phi(B_r) = P_{b_r}$. Obviously, ϕ is onto; therefore $|\mathcal{N}| \leq |\mathcal{B}|$. Since $r(\mathfrak{D}) \leq |\mathcal{N}|$, $r(\mathfrak{D}) \leq co\text{-}w(\mathfrak{D})$.

Net and Conet: Let $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa)$ be a ditopological texture space. A subset $\mathcal{N}$ of $\mathcal{S}$ is called a *net* for (τ, κ) if there exists a subset $\mathcal{N}^*$ of $\mathcal{N}$ such that $G = \bigvee\mathcal{N}^*$ for all $G \in \tau$. A subset $\mathcal{M}$ of $\mathcal{S}$ is called a *conet* for (τ, κ) if there exists a subset $\mathcal{M}^*$ of $\mathcal{M}$ such that $K = \bigcup\mathcal{M}^*$ for all $K \in \kappa$.

Net Weight and Conet Weight: The net *weight* and *conet weight* of $\mathfrak{D}$ are defined as follows,

$$nw(\mathfrak{D}) = \min\{|\mathcal{N}| \mid \mathcal{N} \text{ a net for } (\tau,\kappa)\},$$
$$co\text{-}nw(\mathfrak{D}) = \min\{|\mathcal{N}| \mid \mathcal{N} \text{ a conet for } (\tau,\kappa)\},$$

respectively.

Then, we show that net weight and conet weight have bounds on S in $\mathfrak{D}$.

Theorem 4.4. If a ditopological texture space $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa)$ is T_0, then $|S| \leq 2^{\max\{nw(\mathfrak{D}), co\text{-}nw(\mathfrak{D})\}}$.

Proof. Let $\mathcal{N} \subseteq \mathcal{S}$ be a net for (τ, κ) with $|\mathcal{N}| \leq nw(\mathfrak{D})$, let $\mathcal{M} \subseteq \mathcal{S}$ be a conet for (τ, κ) with $|\mathcal{M}| \leq co\text{-}nw(\mathfrak{D})$. Since $\mathfrak{D}$ is T_0 (See Proposition 2.1.(1)) and $\mathcal{N}, \mathcal{M}$ a net and conet for (τ, κ), respectively, for all $s, t \in S$, there exist a member N of $\mathcal{N}$ and a member of M of $\mathcal{M}$ satisfying

$$Q_s \not\subseteq Q_t \Rightarrow (P_s \not\subseteq N \not\subseteq Q_t \vee P_s \not\subseteq \mathcal{M} \not\subseteq Q_t).$$

Set $\mathcal{L}_{Q_s} = \{(N, M) \in \mathcal{N} \times \mathcal{M} \mid N, M \not\subseteq Q_s\}$ for each $Q_s \in \mathcal{Q}$. Define $\varphi : \mathcal{Q} \to \mathcal{P}(\mathcal{N} \times \mathcal{M})$ by $\varphi(Q_s) = \mathcal{L}_{Q_s}$. Then, clearly, φ is one-one. Thus $|\mathcal{Q}| \leq 2^{|\mathcal{N}\times\mathcal{M}|} \leq 2^{\max\{nw(\mathfrak{D}), co\text{-}nw(\mathfrak{D})\}}$.

Pseudo Base and Copseudo Base: Let $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa)$ be a ditopological texture space. A subset $\mathcal{V}$ of τ is called a *pseudo base* of A for (τ, κ) if $A = \bigcap\mathcal{V}$ for some $A \in \mathcal{S}$. A subset $\mathcal{K}$ of κ is called a *copseudo base* of A for (τ, κ) if $A = \bigcap\mathcal{K}$ for some $A \in \mathcal{S}$.

Pseudo Character and Copseudo Character: The *pseudo character* and *copseudo character* of $\mathfrak{D}$ are defined as follows,

$$\Psi(\mathfrak{D}) = \sup\{\Psi(s, \mathfrak{D}) \mid s \in S\},$$
$$co\text{-}\Psi(\mathfrak{D}) = \sup\{co\text{-}\Psi(s, \mathfrak{D}) \mid s \in S\},$$

respectively, where

$$\Psi(s, \mathfrak{D}) = \min\{|\mathcal{V}| \mid \mathcal{V} \text{ a pseudo base of } P_s \text{ for } (\tau,\kappa)\},$$
$$co\text{-}\Psi(s, \mathfrak{D}) = \min\{|\mathcal{K}| \mid \mathcal{K} \text{ a copseudo base of } P_s \text{ for } (\tau,\kappa)\}.$$

Theorem 4.5. If a ditopological texture space $\mathfrak{D} = (S, \mathcal{S}, \tau, \kappa)$ is $co\text{-}T_1$, then $|S| \leq nw(\mathfrak{D})^{\Psi(\mathfrak{D})}$.

Proof. Let $\mathcal{N} \subseteq \mathcal{S}$ be a net for (τ, κ) with $|\mathcal{N}| \leq nw(\mathfrak{D})$, let $\mathcal{V}_{P_s}$ be a pseudo base for (τ, κ) with $|\mathcal{V}_{P_s}| \leq \Psi(\mathfrak{D})$. Set $\mathcal{N}_{P_s} = \{N_V \mid V \in \mathcal{V}_{P_s}\}$ by choosing a certain element $N_V \in \mathcal{N}$ such that $P_s \subseteq N_V \subseteq V$ for each $V \in V_{P_s}$. Then, it is clear that $|N_{P_s}| \leq \Psi(\mathfrak{D})$. Moreover, since $\mathfrak{D}$ is $co\text{-}T_1$, by Preposition 2.3.(1), $P_s \subseteq \bigcap\mathcal{N}_{P_s} \subseteq \bigcap\mathcal{V}_{P_s} = P_s$; thus $\bigcap\mathcal{N}_{P_s} = P_s$. Then, the function $\Psi : \mathcal{P} \to [\mathcal{N}]^{\leq\Psi(\mathfrak{D})}$ defined by the equation $\Psi(P_s) = \mathcal{N}_{P_s}$ is one-one. Thus $|S| = |\mathcal{P}| \leq nw(\mathfrak{D})^{\Psi(\mathfrak{D})}$.

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