

Analysis Of An Ac-Dc Full-Controlled Converter Supplying Dc Series Motor Parallel With An Inductive R-L Load

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Abstract: This paper is concerned with the detailed study of performance characteristics of an ac-dc full-controlled converter supplying dc series motor parallel with an inductive R-L load. The converter-loads combination is simulated on a digital computer. Different modes of operation (continuous and discontinuous converter currents) are considered. A formula is derived for the critical firing angle. Different performance parameters are studied for both constant motor horsepower level and constant firing angle. The effects of variation of the passive load phase angle and impedance have been studied. In order to verify the accuracy and validity of the analysis presented, the converter-loads combination has been assembled and investigated experimentally.

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1. Introduction

In recent years, the DC series motor drives are engaged with wide range of applications. The application of DC series motor in industrial environment has increased due to high performance and high starting torque as suitable drive system (Masilamani Muruganadam and Muthusamy Madheswaran, 2013) and (Muruganadam M and Madheswaran M, 2012). In phase controlled converters, thyristor commutation is easily achieved by the naturel or line commutation. No additional circuitry is required for the commutation process. Phase controlled converters are simple, less expensive and are extensively used in industries (Hart D H ,2011), (Rashed M H,2004) ,(Singh M D, 2003) and (Sen P C, 1981). Most of the work reported in the literature is related to AC-DC converter supplying a dc motor (Sen P C,1981) , (Sen P C and Dorodla S R, 1975) and (Sen P C, 1990) or two-parallel passive loads (Yung- Chung Li,1978) and (Al-Johani A H,1987) or two motor loads (Al-Subaie,1999) , (Al-Turki Y A, Al-Hindawi M M and Al Subaie O T,2001) and (Al-Hindawi M M, Al-Turki Y A and Al Subaie O T,2000). The analysis of full-converter supplying separately excited DC motor parallel with an inductive R-L load and the analysis of semi converter supplying series motor parallel with the same inductive R-L load is reported in (Al-Turki Y, Al-Hindawi M M and Al-Sobaie O T,2000) and (Al-Hindawi M M, Al-Turki Y A and Maghrabi M,2006). The presence of full-controlled converter supplying DC series motor connected in parallel with an inductive R-L load is commonly found in practice. The analysis and performance parameters reported in

the literature for an AC-DC converters cannot be applied in such a situation.

In this paper, analysis of a phase-controlled (AC-DC full-controlled) converter feeding a series dc motor parallel with an inductive R-L load will be investigated. The steady-state analysis of this system will be obtained for each of the two modes of operation, i.e. continuous and discontinuous converter current. An expression for the critical firing angle at which the converter current changes from one mode to the other will be deduced. Performance parameters such as, input power factor, supply current distortion factor, torque-speed characteristics, and motor current ripple factor have been derived and studied for both constant firing angle and constant motor horsepower level. The effects of variation of the passive load phase angle and impedance have been studied. The theoretical and experimental results are compared to check model effectiveness.

2. Steady state analysis

Figure 1 shows an AC-DC full-controlled converter supplying dc series motor parallel with an inductive R-L load. The sinusoidal voltage source is considered to have an rms value of V volts, frequency of $\omega/2\pi$ Hz and zero internal impedance. Changing the thyristors firing-angles controls the load voltage and currents. The converter current may be continuous or discontinuous depending on the value of the firing angles, the load parameters, and the motor speed. Thyristors S1 and S3 are fired at an angle α relative to the supply zero voltage whereas S2 and S4 are gated at $\pi+\alpha$. Each of the two modes of operation will be treated separately.

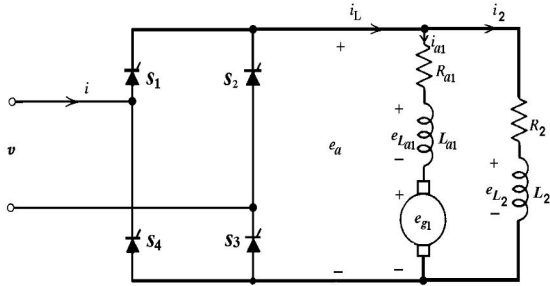


Figure 1. Full controlled converter supplying dc series motor parallel with an inductive R-L load

2.1 Continuous Mode:

The converter current i_L in this mode has a positive value throughout the period of $\alpha \leq \omega t \leq \pi + \alpha$. The load voltage across each branch is equal to the supply voltage assuming zero drop across each thyristor. This means that: for the dc series motor,

$$\sqrt{2}V \sin \omega t = i_{c1}R_{a1} + L_{a1} \frac{d}{dt}i_{c1} + K_{af}i_{c1}^N + K_{res}N, \quad (1)$$

and for the passive load,

$$\sqrt{2}V \sin \omega t = i_{c2}R_2 + L_2 \frac{d}{dt}i_{c2} \quad (2)$$

Where: i_{c1} is the armature current (i_{a1}) and i_{c2} is the passive load current during the continuous mode.

During this mode, the converter current i_{cL} equals the supply current i_{cS} and is given by:

$$i_{cS} = i_{cL} = i_{c1} + i_{c2}, \quad \alpha \leq \omega t \leq \pi + \alpha \quad (3)$$

If steady-state speed and magnetic linearity are assumed, the solution of equation (1) is of the form:

$$i_{c1} = \sqrt{2}(V/Z_1) \sin(\omega t - \phi_1) + K_{c1} e^{-(\omega t - \alpha)/Q_1} - (K_{res}N/R_1)(1 - e^{-(\omega t - \alpha)/Q_1}) \quad (4)$$

Where: $R_1 = R_{a1} + K_{af}N$, K_{af} and K_{res} are constants. Full expressions for the average and rms values of the motor current (I_{c1av} and I_{c1rms}) are given in the appendix and the ripple factor of i_{c1} is given

by:
$$K_{cr1} = \sqrt{I_{c1rms}^2 / I_{c1av}^2 - 1} \quad (6)$$

The solution of equation (2) is:

$$i_{c2} = \sqrt{2}(V/Z_2) \sin(\omega t - \phi_2) + K_{c2} e^{-(\omega t - \alpha)/Q_2} \quad (7)$$

Where: $Z_2 = \sqrt{R_2^2 + (\omega L_2)^2}$, $Q_2 = \omega L_2 / R_2$,

$$\phi_2 = \tan^{-1}(\omega L_2 / R_2),$$

$K_{c2} = I_{c2} - \sqrt{2}(V/Z_2) \sin(\alpha - \phi_2)$, and I_{c2} is the initial value of i_{c2} at $\omega t = \alpha$.

Under steady state operation, i_{c2} at $\omega t = \alpha$ equals that at $\omega t = \pi + \alpha$ and is equal to:

$$I_{c2} = -\sqrt{2}(V/Z_2) [(1 + e^{(-\pi/Q_2)}) / (1 - e^{(-\pi/Q_2)})] \sin(\alpha - \phi_2) \quad (8)$$

Full expressions for the average and rms value of the passive load current (I_{c2av} and I_{c2rms}) are given in the appendix and the rms value of the supply current is obtained as:

$$I_{csrms} = \sqrt{(1/\pi) \int_{\alpha}^{\pi+\alpha} i_{cS}^2 d\omega t = \sqrt{I_{c1rms}^2 + I_{c2rms}^2 + (1/\pi) \int_{\alpha}^{\pi+\alpha} 2i_{c1}i_{c2} d\omega t} \quad (9)$$

The rms value of the fundamental component of the

supply current is:
$$I_{cf} = \sqrt{(a_{c1}^2 + b_{c1}^2)/2} \quad (10)$$

Full expressions for the rms value of the supply

current (I_{csrms}) and the amplitudes of the cosine and sine fundamental components (a_{c1} and b_{c1}) of the same current are given in the appendix and the phase

angle of that component is given by:
$$\theta = \tan^{-1}(a_{c1}/b_{c1}) \quad (11)$$

The supply power factor is:

$$Pf = (I_{cf} / I_{csrms}) \cos \theta \quad (12)$$

Whereas the supply current distortion factor is:

$$Df = I_{cf} / I_{csrms} \quad (13)$$

The critical firing angle α_c , which is the highest firing angle for the continuous mode, satisfies the condition

$$of: I_{c1} + I_{c2} = 0 \quad (14)$$

Therefore, the critical firing angle α_c is given from:

$$\alpha_c = \sin^{-1} [(A_1 C_1 \pm \sqrt{A_1^2 C_1^2 + (A_1 + B_1)(B_1 - C_1)}) / (A_1^2 + B_1^2)] \quad (15)$$

Where: $A_1 = F_1 \cos \phi_1 + F_2 \cos \phi_2$,

$$F_1 = -(\sqrt{2}V/Z_1)(1 + e^{(-\pi/Q_1)}) / (1 - e^{(-\pi/Q_1)}),$$

$$F_2 = -(\sqrt{2}V/Z_2)(1 + e^{(-\pi/Q_2)}) / (1 - e^{(-\pi/Q_2)}),$$

$$B_1 = F_1 \sin \phi_1 + F_2 \sin \phi_2, \text{ and } C_1 = K_{res}N/R_1.$$

If K_{res} has a negligible value, α_c is independent of the supply voltage.

When $\phi_1 = \phi_2 = \phi$ then $\alpha_c = \phi$, ignoring the effect of K_{res} .

2.2 Discontinuous Mode

In this mode, $\alpha > \alpha_c$ and the current i_{d1} increases from zero at $\omega t = \alpha$ to a maximum value and then decreases to become zero again at $\omega t = \beta$ which is referred to as the extinction angle. The angle β is, of course, less than $\pi + \alpha$.

Throughout the period $\alpha \leq \omega t \leq \beta$, the load voltage equals the supply voltage. This means that: for the motor,

$$\sqrt{2}V \sin \omega t = i_{d1}R_{d1} + L_{d1} \frac{d}{dt} i_{d1} + K_{af} i_{d1}^N + K_{res} N \quad (16)$$

and for the passive load,

$$\sqrt{2}V \sin \omega t = i_{d2}R_2 + L_2 \frac{d}{dt} i_{d2} \quad (17)$$

During $\alpha \leq \omega t \leq \beta$, the converter current i_{dl} equals the supply current and is given by:

$$i_{ds} = i_{dl} = i_{d1} + i_{d2}$$

The solution of equation (16) is of the form:

$$i_{d1} = \sqrt{2}(V/Z_1) \sin(\omega t - \phi_1) + K_{d1} e^{-(\omega t - \alpha)/Q_1} - (K_{res} N / R_1) (1 - e^{-(\omega t - \alpha)/Q_1}) \quad (19)$$

$$\text{Where: } K_{d1} = I_{d1} - (\sqrt{2}V/Z_1) \sin(\alpha - \phi_1) \quad (20)$$

and I_{d1} is the initial value of i_{d1} at $\omega t = \alpha$.

The solution of equation (17) is of the form:

$$i_{d2} = \sqrt{2}(V/Z_2) \sin(\omega t - \phi_2) + K_{d2} e^{-(\omega t - \alpha)/Q_2} \quad (21)$$

$$\text{Where: } K_{d2} = I_{d2} - (\sqrt{2}V/Z_2) \sin(\alpha - \phi_2) \quad (22)$$

and I_{d2} is the initial value of i_{d2} at $\omega t = \alpha$.

During the period $\beta \leq \omega t \leq \pi + \alpha$, the converter current is

equal to zero, and $i_{d1} = -i_{d2}$.

During this period the motor and the passive load are connected in parallel. Thus:

$$R_1 i_{d1} + L_{d1} \frac{d}{dt} i_{d1} + K_{res} N = R_2 i_{d2} + L_2 \frac{d}{dt} i_{d2} \quad (23)$$

Solving (23), one obtains:

$$i_{d1} = (-K_{res} N) / (R_1 + R_2) + (I_{d1} + K_{res} N / (R_1 + R_2)) e^{-(\omega t - \beta)/Q} \quad (24)$$

Similarly:

$$i_{d2} = (K_{res} N) / (R_1 + R_2) + (I_{d2} - K_{res} N / (R_1 + R_2)) e^{-(\omega t - \beta)/Q} \quad (25)$$

Where: $Q = \omega(L_{d1} + L_2) / (R_1 + R_2)$,

I_{d1} is the initial value of i_{d1} at $\omega t = \beta$ and is given by:

$$I_{d1} = (\sqrt{2}V/Z_1) \sin(\beta - \phi_1) + K_{d1} e^{-(\beta - \alpha)/Q_1} - (K_{res} N / R_1) (1 - e^{-(\beta - \alpha)/Q_1}) \quad (26)$$

and I_{d2} is the initial value of i_{d2} at $\omega t = \beta$ and is given by:

$$I_{d2} = (\sqrt{2}V/Z_2) \sin(\beta - \phi_2) + K_{d2} e^{-(\beta - \alpha)/Q_2} \quad (27)$$

At $\omega t = \pi + \alpha$, i_{d1} equals I_{d1} , and i_{d2} equals I_{d2} . Therefore, from equations (20), (24), and (26) one gets:

$$I_{d1} = [(-K_{res} N) / (R_1 + R_2)] (1 - e^{-(\pi + \alpha - \beta)/Q}) + (\sqrt{2}V/Z_1) (\sin(\beta - \phi_1) - \sin(\alpha - \phi_1)) e^{-(\beta - \alpha)/Q_1} - (K_{res} N / R_1) (1 - e^{-(\beta - \alpha)/Q_1}) e^{-(\pi + \alpha - \beta)/Q} - (K_{res} N / R_1) (1 - e^{-(\beta - \alpha)/Q_1}) e^{-(\pi + \alpha - \beta)/Q} / (1 - e^{-(\beta - \alpha)/Q_1}) e^{-(\pi + \alpha - \beta)/Q} \quad (28)$$

Similarly: from equations (22), (25), and (27) one gets:

$$I_{d2} = [(K_{res} N) / (R_1 + R_2)] (1 - e^{-(\pi + \alpha - \beta)/Q}) + (\sqrt{2}V/Z_2) (\sin(\beta - \phi_2) - \sin(\alpha - \phi_2)) e^{-(\beta - \alpha)/Q_2} - (K_{res} N / R_1) (1 - e^{-(\beta - \alpha)/Q_1}) e^{-(\pi + \alpha - \beta)/Q} / (1 - e^{-(\beta - \alpha)/Q_1}) e^{-(\pi + \alpha - \beta)/Q} \quad (29)$$

The extinction angle β is determined by the solution of the equation $I_{d1} = -I_{d2}$. (30)

Full expressions for I_{d1av} , I_{d1rms} , I_{d2av} , I_{d2rms} , and I_{dsrms} are given in the appendix. Where: I_{d1av}

and I_{d1rms} are the average and rms values of the motor current respectively.

I_{d2av} and I_{d2rms} are the average and rms values of the passive load current respectively, and I_{dsrms} is the rms value of the supply current.

The ripple factor of i_{d1} will be given by:

$$K_{dr1} = \sqrt{(I_{d1rms} / I_{d1av})^2} - 1 \quad (31)$$

The rms value of the fundamental component of the

$$\text{supply current is: } I_{df} = \sqrt{(a_{d1}^2 + b_{d1}^2)} / 2 \quad (32)$$

Where: a_{d1} and b_{d1} are the amplitudes of the cosine and sine fundamental components of i_{ds} respectively and their full expressions are given in the appendix. phase angle of the fundamental component of the

$$\text{supply current is given by: } \theta = \tan^{-1}(a_{d1} / b_{d1}) \quad (33)$$

And the supply power factor is:

$$Pf = (I_{df} / I_{dsrms}) \cos \theta \quad (34)$$

Whereas the supply current distortion factor is:

$$Df = I_{df} / I_{dsrms} \quad (35)$$

The developed motor power is given by:

$$P_{motor} = K_{af} N I_{motorrms}^2 \quad (36)$$

and the motor torque is obtained from:

$$T_{motor} = K_{af} I_{motorrms}^2$$

3. System Performance

A computer program has been developed (based on the equations derived in section 2) to study the system performance. The following data of the load-combination parameters are used:

Motor: $R_{a1} = 0.15 \Omega$, $L_{a1} = 0.02 \text{ H}$, $K_{af} = 0.03 \text{ H}$, $K_{res} = 0.075 \text{ V/rad/s}$.
 Passive load: $R_2 = 1.0 \Omega$, $L_2 = 0.012 \text{ H}$.

and the rms value of the supply voltage is 120.0 volts.

3.1 Computer Results

3.1.1 Current Waveforms:

The waveforms of the converter, motor, and passive load currents are obtained using equations (3), (4) and (7) in the continuous mode and equations (18), (19), (21), (24), and (25) in the discontinuous mode. These waveforms are shown in figure 2 and figure 3 for continuous mode and in figure 4 for discontinuous mode.

3.1.2 Critical Firing Angle:

According to equation (15), the variation of the critical firing angle α_c , with respect to the motor speed N , is shown in figure 5. It is noted that the critical firing angle α_c , decreases in a non-linear fashion as the motor speed is increased.

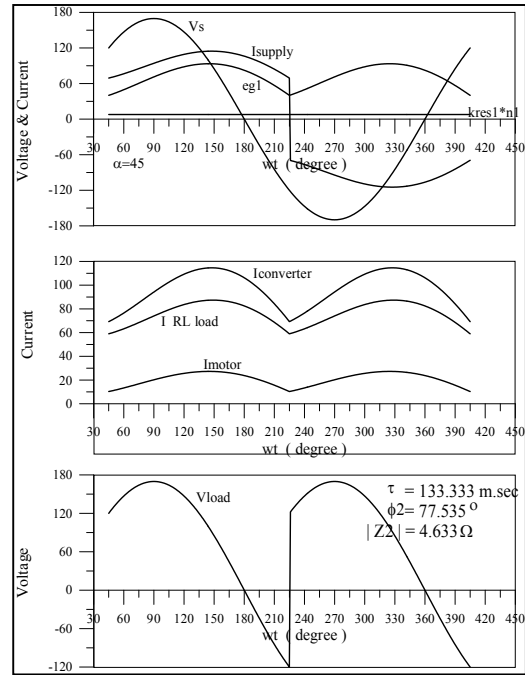


Figure 2. Current and voltage waveforms in the continuous converter current mode of operation for $N=1000 \text{ rpm}$ and $\alpha = 45^\circ$.

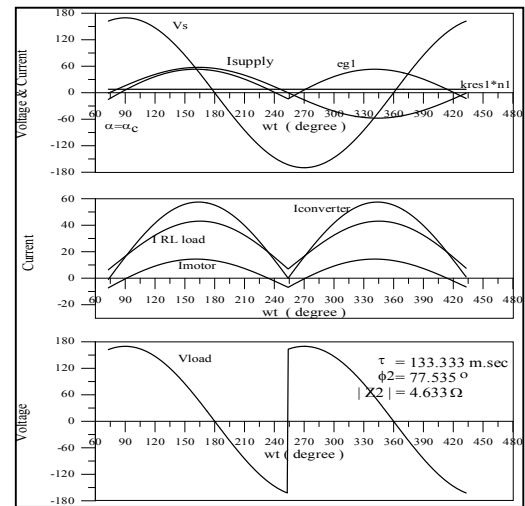


Figure 3. Current and voltage waveforms in the continuous converter current mode of operation for $N=1000 \text{ rpm}$ and $\alpha = \alpha_c = 73.922^\circ$.

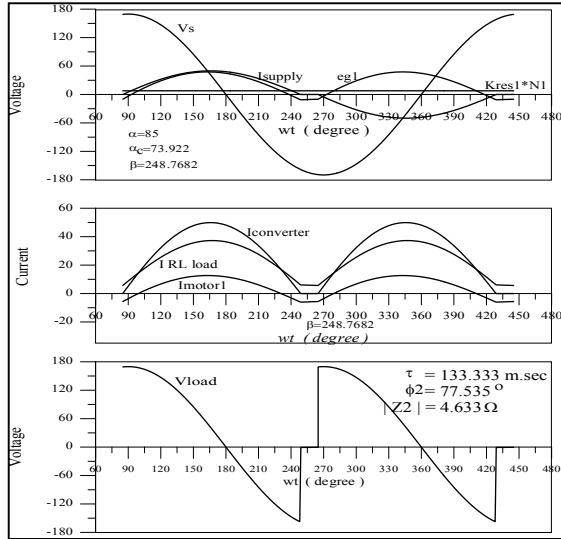


Figure 4. Current and voltage waveforms in the discontinuous converter current mode of operation for $N=1000\text{rpm}$ and $\alpha=85^\circ$.

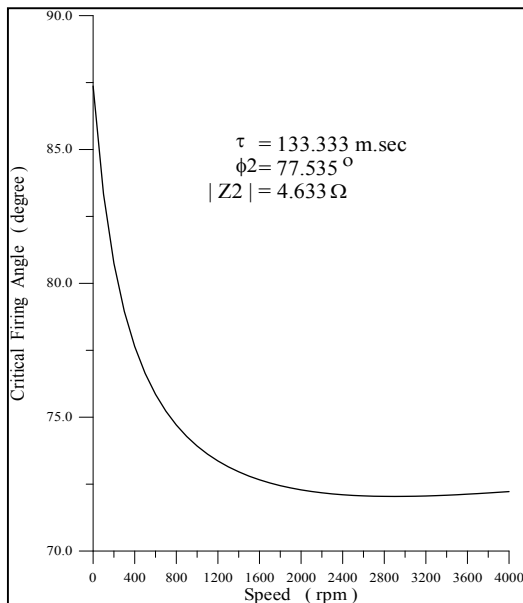


Figure 5. Critical firing angle versus motor speed

3.1.3 Motor Performance Parameters for Constant-HP Operation:

Series motors are normally used for constant-horsepower applications (Masilamani Muruganadam and Muthusamy Madheswaran,2013) and (Muruganadam M and Madheswaran M, 2012). However, the torque-speed curves for a particular firing angle do not conform to constant power characteristics. If the motor is required to operate at a constant power level, the firing angle has to be adjusted accordingly (Sen P C,1981).

Figure 6 shows the variation of firing angle α , with respect to the motor speed N , for different values of

horsepower level, H_p . It is seen that α decreases as N , or the H_p , is increased

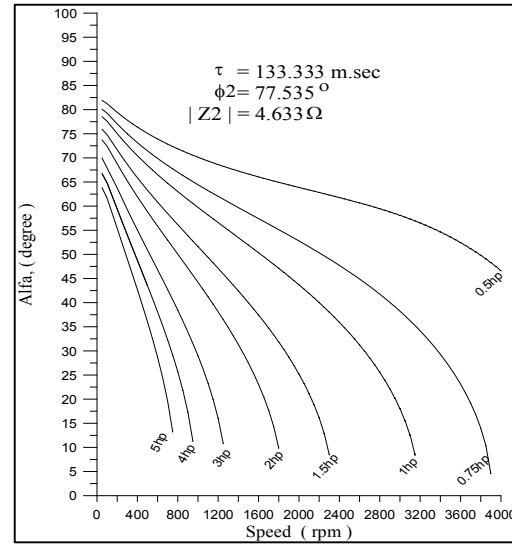


Figure 6. Firing angle versus motor speed at different values of horsepower level.

Different performance parameters are presented in figures 7-10. Figure 7 shows that the motor torque-speed, $T-N$, characteristics are drooping in nature. The system produces high torque at low speed and low torque at high speed which is typical for series dc motors (Stephen J Chapman,2012 and Fitzgerald, Kingsley A E C, Jr. and Umans S D,2003). Figure 8 shows that the supply power factor $p.f.$ deteriorates with a decrease in motor speed N . Increasing the motor speed leads to higher motor branch resistance $R1$; thus leading to a higher power factor. Keeping the motor speed constant, the increase in the value of horsepower level, leads to a lower α , i.e. higher power factor as shown in Figure. 8. It is evident from figures 9 and 10 that, each of the supply current distortion factor Df , and motor current ripple factor Rf increases as the load horsepower is decreased provided that N is constant. The best distortion factor is obtained at α which is the closer to the critical value α_c . For constant speed, as the value of H_p decreases, α is increased leading to a lower dc component of the motor current. Therefore Rf increases as shown in Figure 10 for constant N .

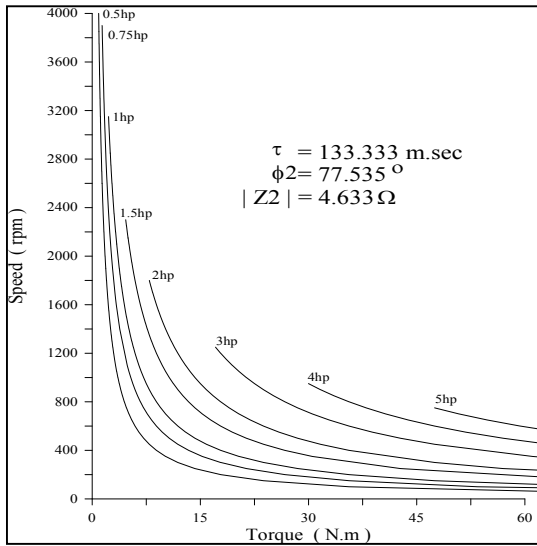


Figure 7. Torque-speed characteristics at different values of horsepower level.

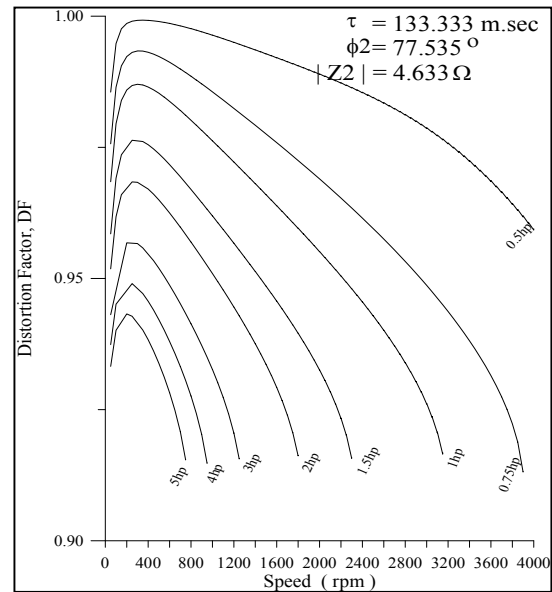


Figure 9. Supply current distortion factor versus motor speed at different values of horsepower level.

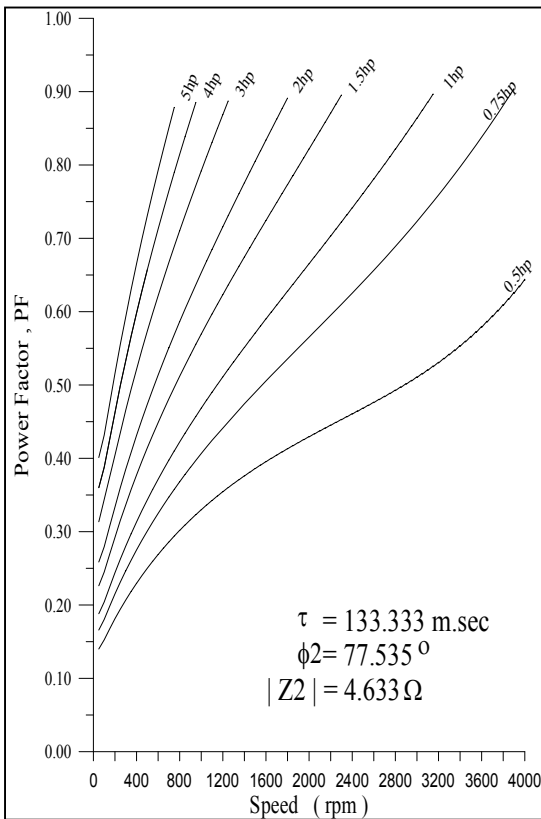


Figure 8. Supply power factor versus motor speed at different values of horsepower level.

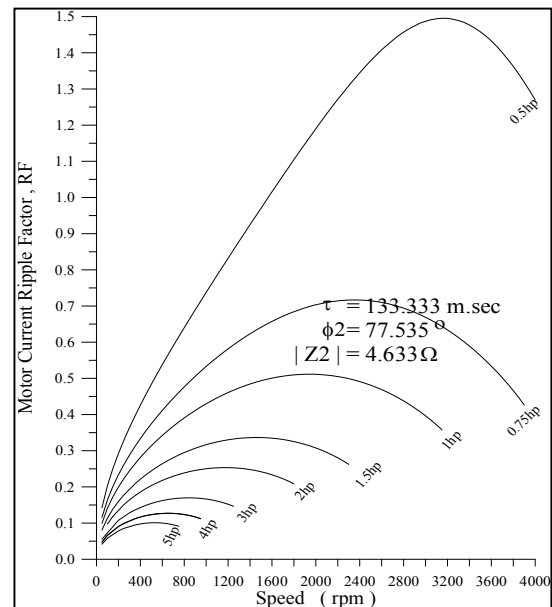


Figure 10. Motor current ripple factor versus motor speed at different values of horsepower level.

3.1.4 Motor Performance Parameters for Constant firing angle Operation:

Different performance parameters are shown in figures 11-14. Figure 11 shows the T-N characteristics for different values of α . Dotted lines in the figure represent the regions of discontinuous converter current. As expected, the converter current is discontinuous at high values of α , high speed and low values of torque. Figure 12 shows that p.f. decreases as α , is increased. Increase of α leads to increase of the displacement angle of the fundamental

component of the supply current which leads to a lower p.f. Figure 13 shows that Df increases as α is increased for continuous converter current mode. Opposite conclusion is obtained for the discontinuous converter current mode. From figures 5 and 13, it is observed that, the best Df is obtained at α which is closest to α_c . This was observed for constant Hp operation. Figure 14 shows that motor current Rf increases with increase either of α or N. Increasing α or N decreases the dc component of the motor current and this leads to high motor current Rf.

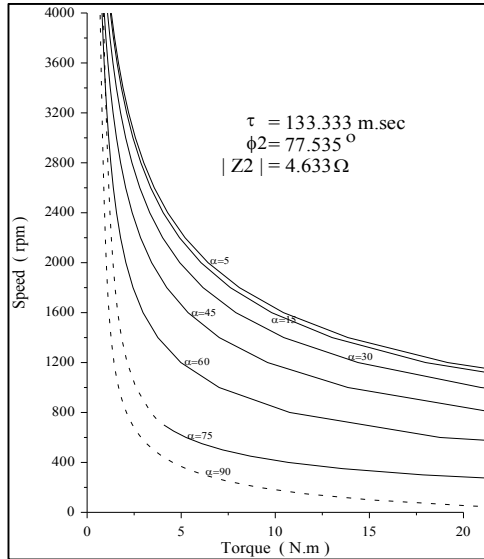


Figure 11. Torque-speed characteristics at different values of firing angle.

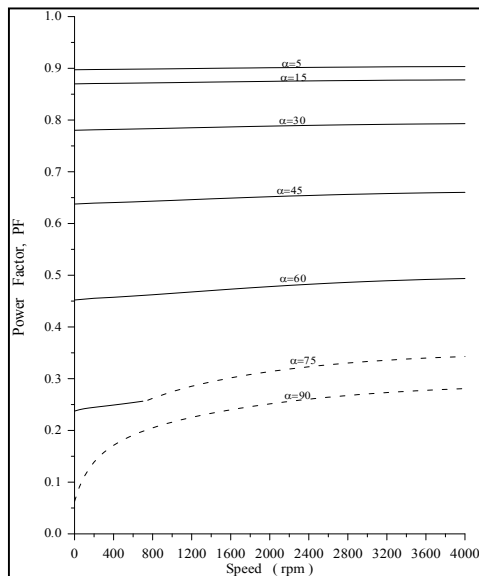


Figure 12. Supply power factor versus motor speed at different values of firing angle

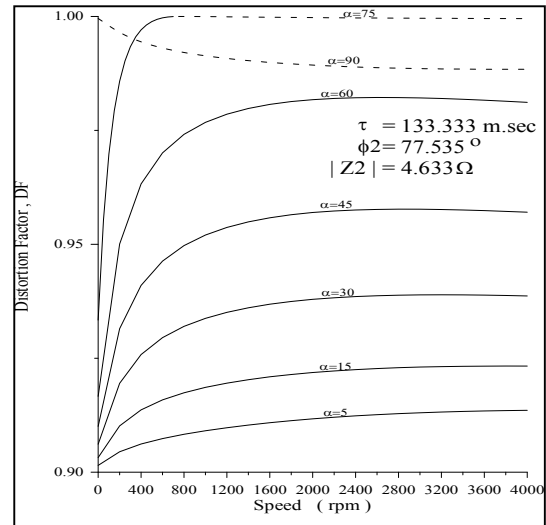


Figure 13. Supply current distortion factor versus motor speed at different values of firing angle

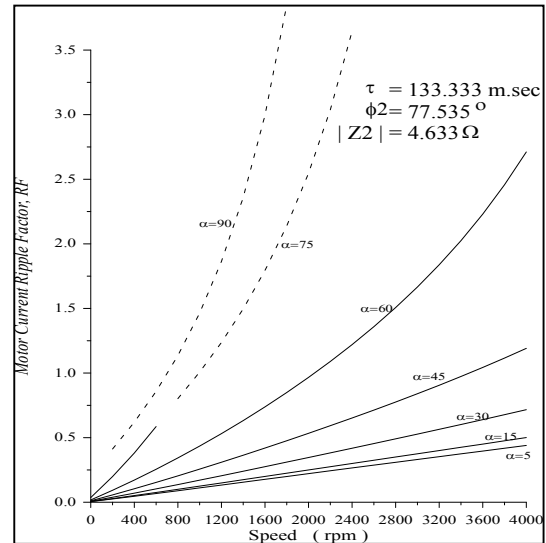


Figure 14. Motor current ripple factor versus motor speed at different values of firing angle.

3.1.5 Effect of load impedance.

3.1.5.1 ϕ_2 Variation.

The variation of α_c with respect to N for different values of passive load phase angle ϕ_2 is shown in Figure 15. As expected, when $|Z_2|$ and motor speed remain unaltered α_c increases as ϕ_2 is increased. T-N characteristics are shown in Figure 16. For the same speed high torque is obtained at low ϕ_2 , keeping α constant. P.f. versus N is shown in Figure 17. P.f. is high for smaller ϕ_2 , keeping α constant. Figure 18 shows the variation of motor current Rf

versus N . Rf is low for smaller ϕ_2 , keeping α and motor speed constants.

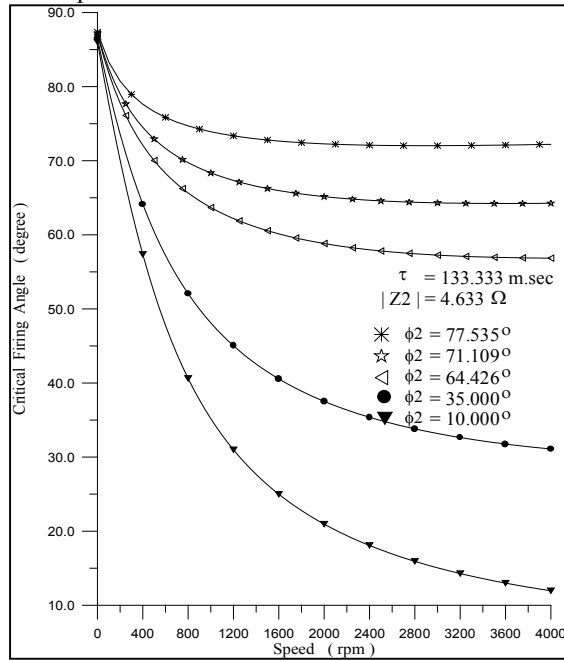


Figure 15. Critical firing angle versus motor speed at different values of passive load phase angle ϕ_2 , keeping $|Z_2|$ unchanged.

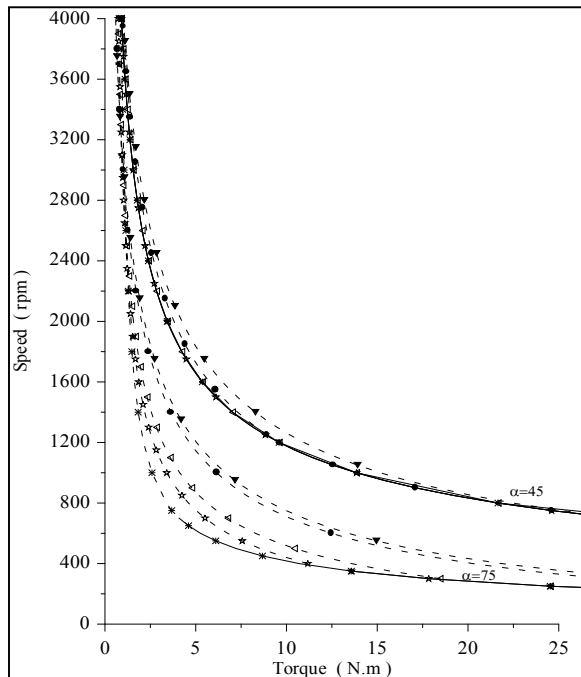


Figure 16. Torque-speed characteristics at different values of firing angle and for different values of passive load phase angle ϕ_2 .

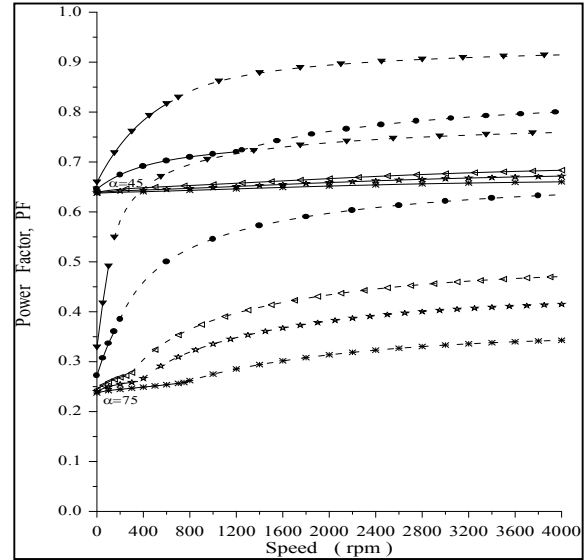


Figure 17. Supply power factor versus motor speed at different values of firing angle and for different values of passive load phase angle ϕ_2

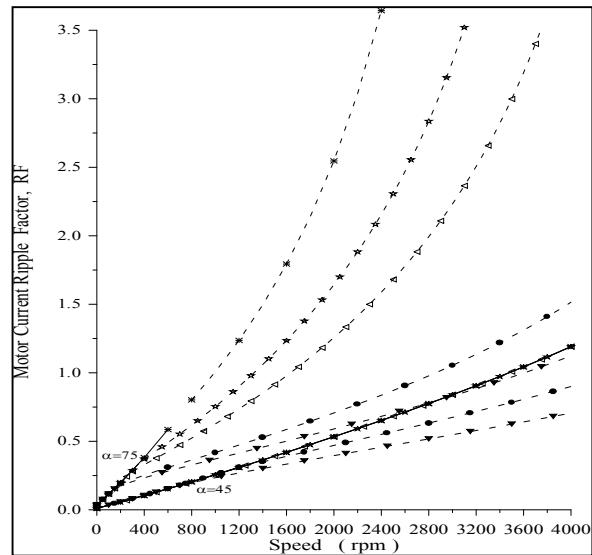


Figure 18. Motor current ripple factor versus motor speed at different values of firing angle and for different values of passive load phase angle ϕ_2 .

3.1.5.2 $|Z_2|$ Variation.

The variation of α_c with respect to N for different values of passive load impedance $|Z_2|$ are shown in figures 19 and 20 for $\phi_2=55^\circ$ and $\phi_2=77.535^\circ$, respectively. The impedance $|Z_{1b}|$ is calculated at rated speed. It is noted that the curves intersect at a common α_c where both loads have the same phase angle as α_c . This confirms the fact that when $\phi_1 = \phi_2$, then α_c equals both as could be shown from equation 15, assuming a negligible value of K_{res} is considered. At speeds lower than the intersection point speed N_i , the

value of α_c increases as $|Z2|$ is increased. However, the situation is reversed for speeds higher than N_i . This is an interesting conclusion which is applicable under any situation as long as it is possible to have same phase angle for both loads. Figure 21 shows T-N characteristics. It is evident that in the continuous converter current mode of operation, the variation of $|Z2|$ has no effect on the T-N characteristics. For the discontinuous converter current mode of operation and at speeds lower than N_i , the torque increases with the decrease of $|Z2|$ for the same speed. The opposite is true for speeds above N_i . P.f. versus N is shown in Figure 22. It is clear that for the speed smaller than N_i , p.f. increases as $|Z2|$ is decreased and the opposite is valid for speeds higher than N_i . The relationship between Df and N is presented in Figure 23. Again, the best distortion factor is obtained for α nearer to α_c . The variation of motor current Rf versus N is shown in Figure 24. It is noted that, higher motor current Rf is obtained for higher $|Z2|$ for speeds smaller than N_i . The opposite result is valid for speeds higher than N_i .

3.2 Experimental Results

In order to verify the accuracy and validity of the analysis presented an AC-DC full controlled converter has been assembled. A firing circuit has been designed and built to match the scheme of cosine control of the firing angle α .

The following load parameters are measured:

- | | |
|-------------------------|---------------------------|
| Motor: | Passive load: |
| Ra1=0.61Ω. | R2=3.83Ω. |
| La1=0.0053 H. | L2=0.043 H. (unsaturated) |
| Kaf=0.0099222 H. | |
| Kres=0.0247446 V/rad/s. | |

and the rms value of the supply voltage is 120.0 volts.

The experimental oscillograms of i_l , i_1 and i_2 are shown in Figure 25 and Figure 26 for continuous mode of operation at $\alpha=48^\circ$ and at $\alpha_c=55.64^\circ$ respectively. The experimental oscillograms of these currents are shown in Figure 27 for discontinuous mode of operation at $\alpha=78.55^\circ$. The same parameters of the loads connected to the assembled converter have been entered to the computer program developed to get the waveforms of the total load current and the branch currents at various firing angles. The theoretical waveforms of the branch currents i_1 and i_2 and the total load current i_l at firing angles of 48, 55.64 and 78.55 are shown in Figures 28, 29 and 30 respectively. Inspecting the theoretical and experimental results, it is evident that they are in good agreement. This confirms and verifies the validity and accuracy of the analysis developed in this paper.

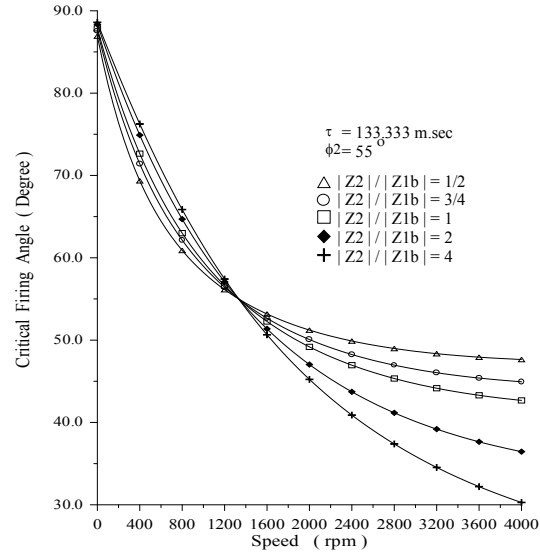


Figure 19. Critical firing angle versus motor speed for different values of passive load impedance $|Z2|$ keeping $\phi_2 = 55^\circ$.

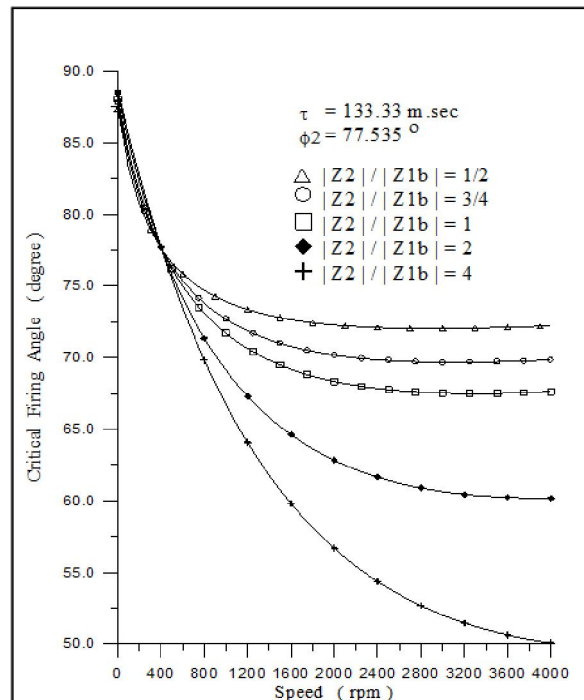


Figure 20. Critical firing angle versus motor speed for different values of passive load impedance $|Z2|$ keeping $\phi_2 = 77.535^\circ$.

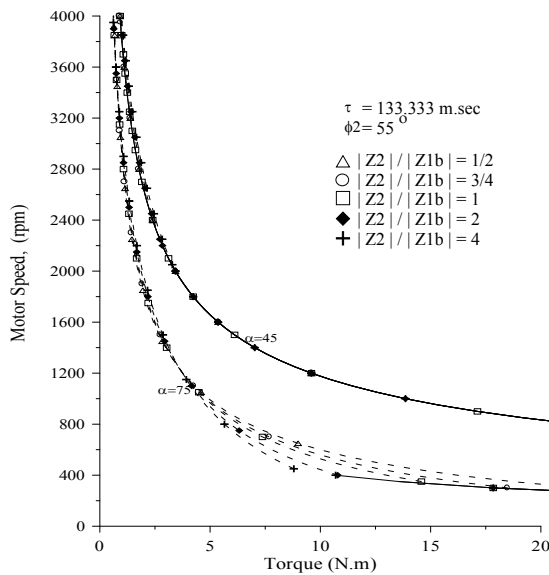


Figure 21. Torque-speed characteristics at different values of firing angle for different value of passive load impedance $|Z_2|$ keeping $\phi_2 = 55^\circ$.

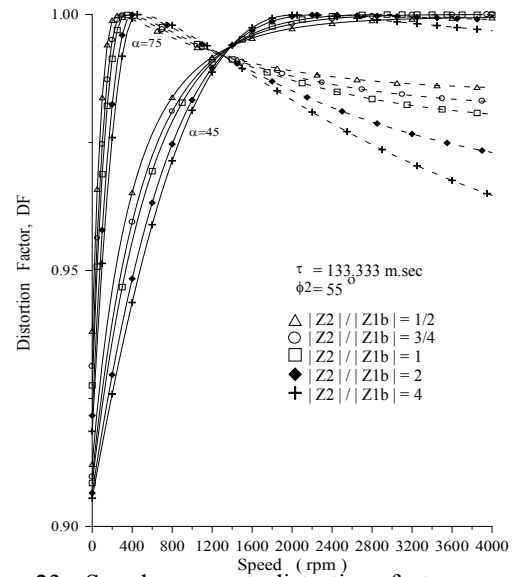


Figure 23. Supply current distortion factor versus motor speed at different values of firing angle for different value of passive load impedance $|Z_2|$ keeping $\phi_2 = 55^\circ$.

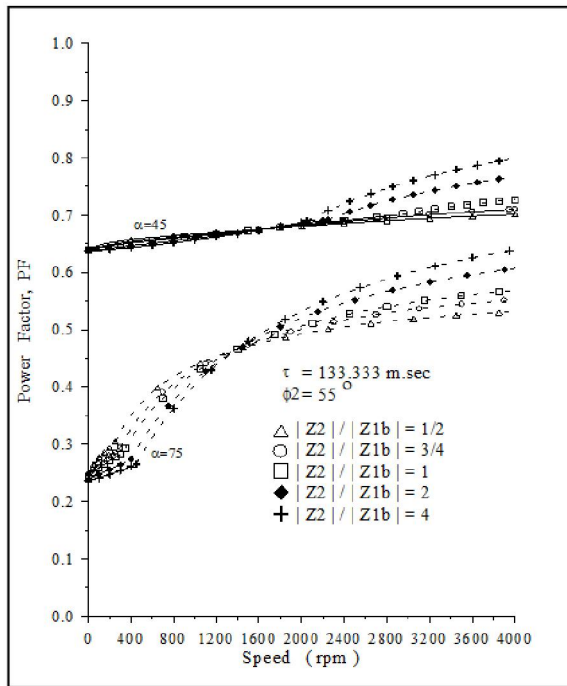


Figure 22. Supply power factor versus speed at different values of firing angle for different value of passive load impedance $|Z_2|$ keeping $\phi_2 = 55^\circ$.

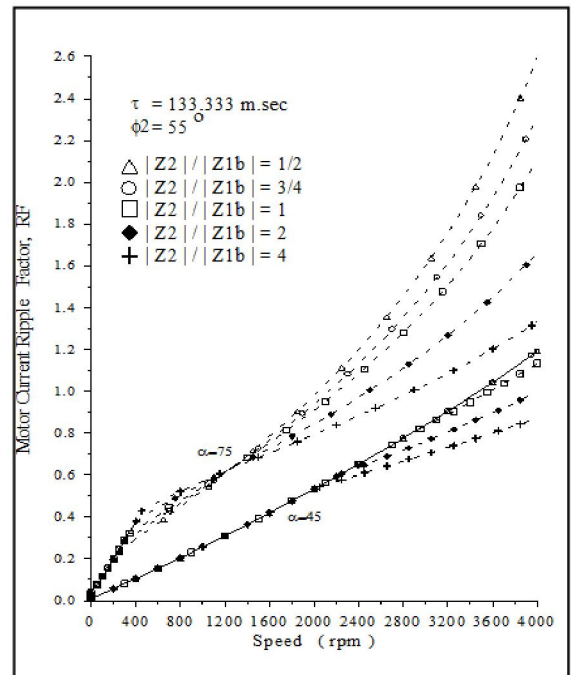
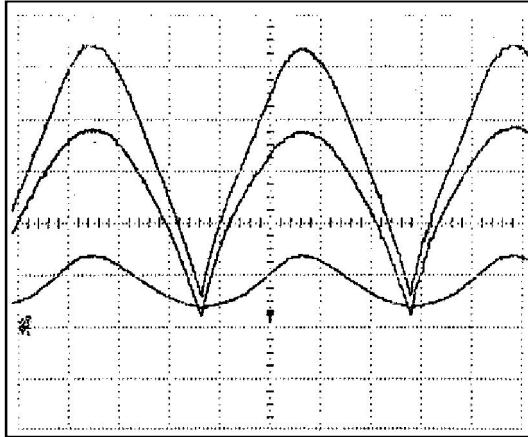


Figure 24. Motor current ripple factor versus motor speed at different values of firing angle for different value of passive load impedance $|Z_2|$ keeping $\phi_2 = 55^\circ$.



Continuous converter current mode
 Figure 25. Oscillograms of the converter current, i_{cl} , motor current, i_{c1} , and passive load current, i_{c2} , at $\alpha = 48^\circ$, $N=800$ rpm (time scale = 2 m.sec./div, current scale = 20 amp/div).

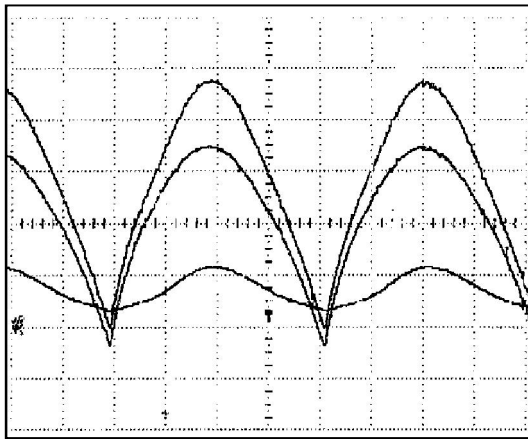
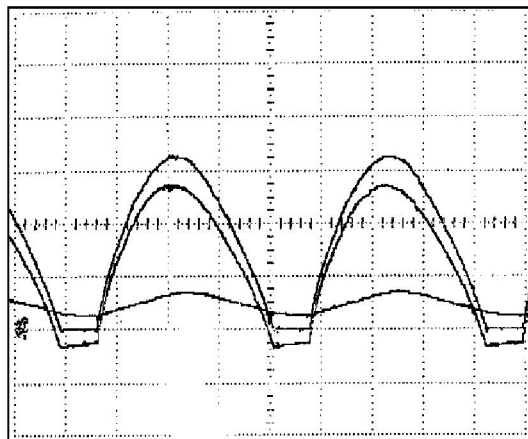


Figure 26. Oscillograms of the converter current, i_{cl} , motor current, i_{c1} , and passive load current, i_{c2} , at $\alpha = \alpha_c = 55.64^\circ$, $N=800$ rpm (time scale = 2 m.sec./div, current scale = 20 amp/div).



Discontinuous converter current mode

Figure 27. Oscillograms of the converter current, i_{dl} , motor current i_{d1} , and passive load current, i_{d2} , at $\alpha = 78.55^\circ$, $N=800$ rpm (time scale = 2 m.sec./div, current scale = 20 amp/div).

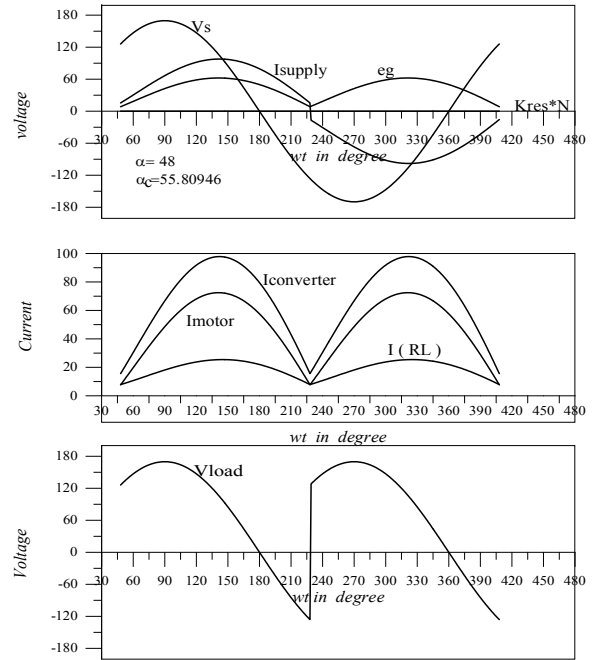


Figure 28. Current and voltage waveforms in the continuous converter current mode of operation for $N=800$ rpm and $\alpha = 48^\circ$.

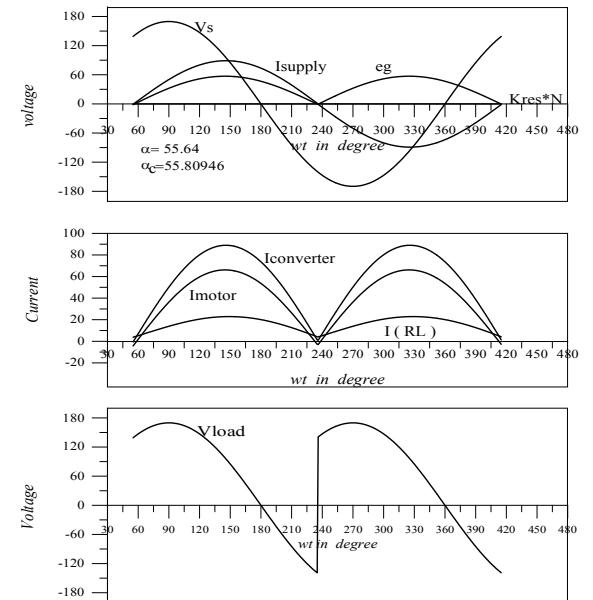


Figure 29. Current and voltage waveforms for $N=800$ rpm and $\alpha = \alpha_c = 55.64^\circ$.

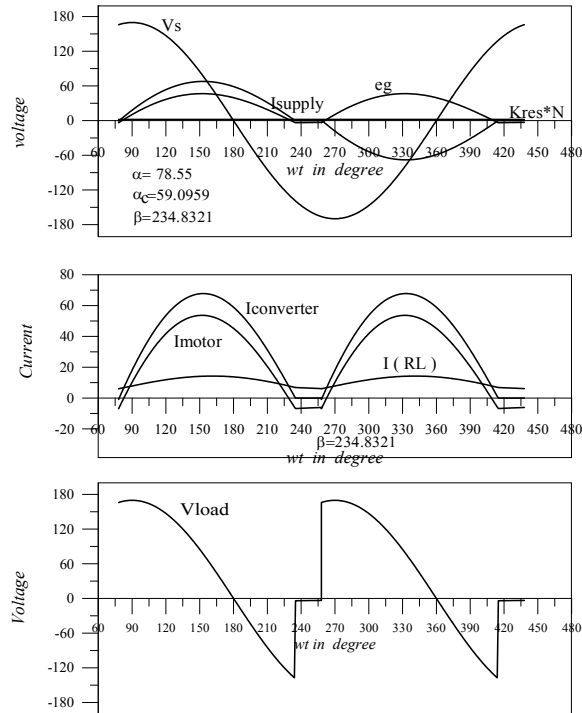


Figure 30 Current and voltage waveforms in the discontinuous converter current mode of operation for $N=800$ rpm and $\alpha = 78.55^\circ$.

4. Conclusions

In this paper, an ac-dc full-controlled converter supplying a series dc motor parallel with an inductive R-L load has been investigated. The steady-state analysis of the system has been derived analytically for each of the two modes of operation (continuous and discontinuous converter current). In each mode, the developed expressions are: converter, motor, passive load currents, motor current ripple factor, torque-speed characteristic, input power factor, and supply current distortion factor. Also the critical firing angle has been derived and investigated. The effect of variation of the passive load phase angle and impedance has been studied. The theoretical and experimental results are compared to check model effectiveness. In view of the analysis and results presented in this paper, the following conclusions have been inferred:

4.1 Critical firing angle α_c :

The critical firing angle α_c increases as the motor speed N is decreased and as the passive load phase angle ϕ_2 is increased. At speeds lower than N_i (where both loads have the same phase angle), α_c increases as the passive load impedance $|Z_2|$ is increased. However, the situation is reverse for speeds

higher than N_i . When $\phi_1 = \phi_2 = \phi$, then $\alpha_c = \phi$, ignoring the effect of K_{res} .

4.2 Supply Current Df:

The best distortion factor Df is obtained at α which is the closer to the critical value α_c .

4.3 Constant Passive Load Impedance:

(a) Constant HP operation:

During this situation, α decreases and the input p.f. increases as the motor speed N is increased.

(b) Constant Speed N operation:

The increase in the value of horsepower level HP leads to a lower α , higher p.f, and a lower Df and motor current R_f . Decreasing the firing angle α increases the input p.f. and decreases the motor current R_f .

(c) Constant Firing Angle α operation:

The increase in the motor speed N leads to decrease of the horsepower level H_p , small variation of p.f, and higher R_f .

4.4 ϕ_2 Variation:

Keeping α , N , and $|Z_2|$ Constants and for the continuous converter current mode of operation, the variation of ϕ_2 has no effect on the T-N characteristics and R_f . In the discontinuous mode of operation, high torque is obtained at low ϕ_2 and high R_f is obtained at high ϕ_2 . The p.f. increases for both mode as ϕ_2 is decreased.

4.5 $|Z_2|$ Variation:

Keeping α , N , and ϕ_2 Constants it is evident that, the variation of $|Z_2|$ has no effect on the T-N characteristics and R_f in the continuous converter current mode of operation. For the discontinuous mode of operation and at speeds lower than N_i , the torque increases, R_f decreases with the decrease of $|Z_2|$, and p.f. increases for both mode as $|Z_2|$ is decreased. The opposite is valid for speeds higher than N_i .

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Appendix

Continuous mode.

The average value of the motor current is obtained from: $I_{clav} = (1/\pi) \int_{\alpha}^{\pi+\alpha} i_{c1} d\omega t$ and is given by:

$$I_{clav} = 2\sqrt{2}(V/\pi Z_1) \cos(\alpha - \phi_1) + (K_{c1} Q_1 / \pi) (1 - e^{(-\pi/Q_1)}) - (K_{res} N / R_1) [1 - (Q_1 / \pi) (1 - e^{(-\pi/Q_1)})] \quad (A-1)$$

The r.m.s value of the motor current is obtained from:

$$I_{clrms} = \sqrt{(1/\pi) \int_{\alpha}^{\pi+\alpha} i_{c1}^2 d\omega t} \text{ and is given by:}$$

$$I_{clrms} = [(V^2 / Z_1^2) + (K_{c1} + (K_{res} N / R_1))^2 (2\sqrt{2} Q_1 / (\pi Z_1 (1 + Q_1^2)))^2 - (4\sqrt{2} V K_{res} N / \pi Z_1 R_1) \cos(\alpha - \phi_1) + (Q_1 / 2\pi) (K_{c1} + (K_{res} N / R_1))^2 (1 - e^{(-2\pi/Q_1)}) - (2Q_1 K_{res} N / \pi R_1) (K_{c1} + (K_{res} N / R_1)) (1 - e^{(-\pi/Q_1)}) + (K_{res} N / R_1)^2]^{1/2} \quad (A-2)$$

The average value of the ic2 is:

$$I_{c2av} = 2\sqrt{2}(V/\pi Z_2) \cos(\alpha - \phi_2) + (K_{c2} Q_2 / \pi) (1 - e^{(-\pi/Q_2)}) \quad (A-3)$$

The rms value of the ic2 is:

$$I_{c2rms} = [(V^2 / Z_2^2) + K_{c2}^2 (2\sqrt{2} V Q_2 / (\pi Z_2 (1 + Q_2^2)))^2 - (1/Q_2) \sin(\alpha - \phi_2) + \cos(\alpha - \phi_2) (1 + e^{(-\pi/Q_2)}) + (Q_2 / 2\pi) K_{c2} (1 - e^{(-2\pi/Q_2)})]^{1/2} \quad (A-4)$$

ac1 and bc1 are the amplitudes of the cosine and sine fundamental components of ics respectively and

are given by: $a_{c1} = (1/\pi) \int_{\alpha}^{\pi+\alpha} i_{cs} \cos \omega t d\omega t$ and is obtained from:

$$a_{c1} = -(\sqrt{2} V / Z_1) \sin \phi_1 + (2Q_1 / (1 + Q_1)) \pi (K_{c1} + (K_{res} N / R_1)) (1/Q_1) \cos \alpha - \sin \alpha (1 + e^{(-\pi/Q_1)}) + (4K_{res} N / \pi R_1) \sin \alpha - (\sqrt{2} V / Z_2) \sin \phi_2 + (2Q_2 / (1 + Q_2)) \pi (K_{c2}) (1/Q_2) \cos \alpha - \sin \alpha (1 + e^{(-\pi/Q_2)}) \quad (A-5)$$

And $b_{c1} = (1/\pi) \int_{\alpha}^{\pi+\alpha} i_{cs} \sin \omega t d\omega t$, and is obtained from:

$$b_{c1} = (\sqrt{2V/Z_1})\cos\phi_1 + (2Q_1/(1+Q_1)\pi)(K_{c1} + (K_{res}N/R_1)) \tag{A-6}$$

$$((1/Q_1)\sin\alpha + \cos\alpha)(1 + e^{(-\pi/Q_1)}) - (4K_{res}N/\pi R_1)\cos\alpha + (\sqrt{2V/Z_2})\cos\phi_2 + (2Q_2/(1+Q_2)\pi)(K_{c2})(1/Q_2)\sin\alpha + \cos\alpha \tag{A-6}$$

$$(1 + e^{(-\pi/Q_2)})$$

Discontinuous mode.

The average value of the motor current is given by: $I_{d1av} = (1/\pi)\int_{\alpha}^{\pi+\alpha} i_{d1} d\omega$. Therefore:

$$I_{d1av} = (\sqrt{2V/Z_1}\pi)[\cos(\alpha - \phi_1) - \cos(\beta - \phi_1)] + (Q_1/\pi) \tag{A-7}$$

$$(K_{d1} + (K_{res}N/R_1))(1 - e^{(-(\beta-\alpha)/Q_1)}) + (K_{res}N/R_1) \tag{A-7}$$

$$((\alpha - \beta)/\pi) + ((-K_{res}N)/(R_1 + R_2))((\pi + \alpha - \beta)/\pi) + (Q/\pi)((-K_{res}N)/(R_1 + R_2)) - I_{d1}(e^{(-(\pi+\alpha-\beta)/Q)} - 1)$$

The r.m.s value of the motor current is found to be:

$$I_{d1rms} = \sqrt{(1/\pi)\int_{\alpha}^{\pi+\alpha} i_{d1}^2 d\omega}$$

$$I_{d1rms} = \{ (2V/\pi Z_1)[(\beta - \alpha)/2 + [\sin 2(\alpha - \phi_1) - \sin 2(\beta - \phi_1)]/4] + \tag{A-8}$$

$$(2\sqrt{2V/\pi Z_1})(K_{res}N/R_1)[\cos(\beta - \phi_1) - \cos(\alpha - \phi_1)] + (2\sqrt{2V/\pi Z_1}) \tag{A-8}$$

$$(Q_1/(1+Q_1))(K_{d1} + (K_{res}N/R_1))e^{(-(\beta-\alpha)/Q_1)} - (1/Q_1) \tag{A-8}$$

$$\sin(\beta - \phi_1) - \cos(\beta - \phi_1) + ((1/Q_1)\sin(\alpha - \phi_1) + \cos(\alpha - \phi_1))] - \tag{A-8}$$

$$(Q_1/2\pi)(K_{d1} + (K_{res}N/R_1))(e^{(-2(\beta-\alpha)/Q_1)} - 1) + (Q_1/\pi) \tag{A-8}$$

$$(2K_{d1}(K_{res}N/R_1) + 2(K_{res}N/R_1))(e^{(-(\beta-\alpha)/Q_1)} - 1) + (K_{res}N/R_1) \tag{A-8}$$

$$(\beta - \alpha)/\pi + ((-K_{res}N)/(R_1 + R_2))((\pi + \alpha - \beta)/\pi) + (2Q/\pi) \tag{A-8}$$

$$[((-K_{res}N)/(R_1 + R_2)) - I_{d1}((-K_{res}N)/(R_1 + R_2))]e^{(-(\pi+\alpha-\beta)/Q)} - 1) \tag{A-8}$$

$$- (Q/2\pi)((-K_{res}N)/(R_1 + R_2)) - I_{d1}(e^{(-2(\pi+\alpha-\beta)/Q)} - 1) \tag{A-8}$$

The average value of the passive load current is:

$$I_{d2av} = (\sqrt{2V/Z_2}\pi)[\cos(\alpha - \phi_2) - \cos(\beta - \phi_2)] + (Q_2/\pi) \tag{A-9}$$

$$K_{d2}(1 - e^{(-(\beta-\alpha)/Q_2)}) + ((K_{res}N)/(R_1 + R_2))((\pi + \alpha - \beta)/\pi) \tag{A-9}$$

$$+ (Q/\pi)((K_{res}N)/(R_1 + R_2)) - I_{d2}(e^{(-(\pi+\alpha-\beta)/Q)} - 1)$$

The r.m.s value of the passive load current is:

$$I_{d2rms} = \{ (2V/\pi Z_2)[(\beta - \alpha)/2 + [\sin 2(\alpha - \phi_2) - \sin 2(\beta - \phi_2)]/4] + \tag{A-10}$$

$$(2\sqrt{2V/\pi Z_2})(Q_2/(1+Q_2))(K_{d2})e^{(-(\beta-\alpha)/Q_2)} - (1/Q_2)\sin(\beta - \phi_2) - \tag{A-10}$$

$$\cos(\beta - \phi_2) + ((1/Q_2)\sin(\alpha - \phi_2) + \cos(\alpha - \phi_2))] - (Q_2/2\pi)(K_{d2}) \tag{A-10}$$

$$(e^{(-2(\beta-\alpha)/Q_2)} - 1) + ((K_{res}N)/(R_1 + R_2))((\pi + \alpha - \beta)/\pi) + \tag{A-10}$$

$$(2Q/\pi)((K_{res}N)/(R_1 + R_2)) - I_{d2}((K_{res}N)/(R_1 + R_2)) \tag{A-10}$$

$$(e^{(-(\pi+\alpha-\beta)/Q)} - 1) - (Q/2\pi)((K_{res}N)/(R_1 + R_2)) - I_{d2} \tag{A-10}$$

$$(e^{(-2(\pi+\alpha-\beta)/Q)} - 1) \tag{A-10}$$

The r.m.s value of the supply current is obtained from:

$$I_{dsrms} = \sqrt{(1/\pi)\int_{\alpha}^{\beta} i_{ds}^2 d\omega}$$

$$I_{dsrms} = \sqrt{(1/\pi)\int_{\alpha}^{\beta} i_{d1}^2 d\omega + (1/\pi)\int_{\alpha}^{\beta} i_{d2}^2 d\omega + (1/\pi)\int_{\alpha}^{\beta} 2i_{d1}i_{d2} d\omega}$$

Then r.m.s value of the supply current is found to be:

$$I_{dsrms} = \{ (2V/\pi Z_1)[(\beta - \alpha)/2 + [\sin 2(\alpha - \phi_1) - \sin 2(\beta - \phi_1)]/4] + \tag{A-11}$$

$$(2\sqrt{2V/\pi Z_1})(K_{res}N/R_1)[\cos(\beta - \phi_1) - \cos(\alpha - \phi_1)] + (2\sqrt{2V/\pi Z_1}) \tag{A-11}$$

$$(Q_1/(1+Q_1))(K_{d1} + (K_{res}N/R_1))e^{(-(\beta-\alpha)/Q_1)} - (1/Q_1) \tag{A-11}$$

$$\sin(\beta - \phi_1) - \cos(\beta - \phi_1) + ((1/Q_1)\sin(\alpha - \phi_1) + \cos(\alpha - \phi_1))] - \tag{A-11}$$

$$(Q_1/2\pi)(K_{d1} + (K_{res}N/R_1))(e^{(-2(\beta-\alpha)/Q_1)} - 1) + (Q_1/\pi) \tag{A-11}$$

$$(2K_{d1}(K_{res}N/R_1) + 2(K_{res}N/R_1))(e^{(-(\beta-\alpha)/Q_1)} - 1) + \tag{A-11}$$

$$(K_{res}N/R_1)(\beta - \alpha)/\pi + (2V/\pi Z_2)[(\beta - \alpha)/2 + [\sin 2(\alpha - \phi_2) \tag{A-11}$$

$$- \sin 2(\beta - \phi_2)]/4] + (2\sqrt{2V/\pi Z_2})(Q_2/(1+Q_2))(K_{d2})e^{(-(\beta-\alpha)/Q_2)} \tag{A-11}$$

$$- (1/Q_2)\sin(\beta - \phi_2) - \cos(\beta - \phi_2) + ((1/Q_2)\sin(\alpha - \phi_2) + \cos(\alpha - \phi_2))] \tag{A-11}$$

$$- (Q_2/2\pi)(K_{d2})(e^{(-2(\beta-\alpha)/Q_2)} - 1) + (2V/\pi Z_2)\cos(\phi_2 - \phi_1) \tag{A-11}$$

$$(\beta - \alpha) - (V/\pi Z_1 Z_2)[\sin 2(\beta - \phi_2 - \phi_1) - \sin 2(\alpha - \phi_2 - \phi_1)] + \tag{A-11}$$

$$(2\sqrt{2V/\pi Z_2})(K_{res}N/R_1)[\cos(\beta - \phi_2) - \cos(\alpha - \phi_2)] + (2\sqrt{2V/\pi Z_2}) \tag{A-11}$$

$$(Q_1/(1+Q_1))(K_{d1} + (K_{res}N/R_1))e^{(-(\beta-\alpha)/Q_1)} - (1/Q_1)\sin(\beta - \phi_2) \tag{A-11}$$

$$- \cos(\beta - \phi_2) + ((1/Q_1)\sin(\alpha - \phi_2) + \cos(\alpha - \phi_2))] + (2\sqrt{2V/\pi Z_1}) \tag{A-11}$$

$$(Q_2/(1+Q_2))(K_{d2})e^{(-(\beta-\alpha)/Q_2)} - (1/Q_2)\sin(\beta - \phi_1) - \cos(\beta - \phi_1)) \tag{A-11}$$

$$+ ((1/Q_2)\sin(\alpha - \phi_1) + \cos(\alpha - \phi_1)) + (2Q_1 Q_2/\pi(Q_1 + Q_2))(K_{d1} + \tag{A-11}$$

$$(K_{res}N/R_1))(K_{d2})(1 - e^{(-2(\beta-\alpha)/Q_2)}) + (2Q_2/\pi) \tag{A-11}$$

$$(K_{res}N/R_1)(K_{d2})(e^{(-(\beta-\alpha)/Q_2)} - 1) \tag{A-11}$$

And, a_{d1} and b_{d1} are the amplitudes of the cosine and sine fundamental components of i_{ds} respectively and are given by:

$$a_{d1} = (\sqrt{2V} / 2\pi Z_1) [\cos(2\alpha - \phi_1) - \cos(2\beta - \phi_1) - 2(\beta - \alpha) \sin \phi_1] + \\ 2(Q_1 / (\pi(1 + Q_1))) (K_{d1} + (K_{res} N / R_1)) [e^{(-(\beta - \alpha) / Q_1)} (-1 / Q_1) \\ \cos \beta + \sin \beta) + ((1 / Q_1) \cos \alpha - \sin \alpha)] - (2K_{res} N / \pi R_1) \\ (\sin \beta - \sin \alpha) + (\sqrt{2V} / 2\pi Z_2) [\cos(2\alpha - \phi_2) - \cos(2\beta - \phi_2) - \\ 2(\beta - \alpha) \sin \phi_2] + 2(Q_2 / (\pi(1 + Q_2))) (K_{d2}) [e^{(-(\beta - \alpha) / Q_2)} \\ (-1 / Q_2) \cos \beta + \sin \beta) + ((1 / Q_2) \cos \alpha - \sin \alpha)], \text{ and} \\ \text{(A-13)}$$

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$$b_{d1} = (\sqrt{2V} / 2\pi Z_1) [\sin(2\alpha - \phi_1) - \sin(2\beta - \phi_1) + 2(\beta - \alpha) \cos \phi_1] + \\ 2(Q_1 / (\pi(1 + Q_1))) (K_{d1} + (K_{res} N / R_1)) [e^{(-(\beta - \alpha) / Q_1)} (-1 / Q_1) \\ \sin \beta - \cos \beta) + ((1 / Q_1) \sin \alpha + \cos \alpha)] + (2K_{res} N / \pi R_1) \\ (\cos \beta - \cos \alpha) + (\sqrt{2V} / 2\pi Z_2) [\sin(2\alpha - \phi_2) - \sin(2\beta - \phi_2) + \\ 2(\beta - \alpha) \cos \phi_2] + 2(Q_2 / (\pi(1 + Q_2))) (K_{d2}) [e^{(-(\beta - \alpha) / Q_2)} \\ (-1 / Q_2) \sin \beta - \cos \beta) + ((1 / Q_2) \sin \alpha + \cos \alpha)] \\ \text{(A-14).}$$