MIMO Correlation-Based Analytical Channel Modeling

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Abstract: Multiple-input multiple-output (MIMO) scheme is an important tool to achieve a high data rate communication and higher signal to noise ratio (SNR). In order to facilitate the communication system design, understanding and modeling the channel is crucial. Different approaches are available for modeling the MIMO channel based on the required utilization that range between physical and analytical approaches. In the case of analytical modeling, correlative-based modeling approach is common. Several papers illustrating this technique are available. In the case of ultra-wideband (UWB)-MIMO channel correlative modeling, few papers are available. This paper presents a brief elaboration on the correlative-based modeling, reviewing the most common available models and explains the practical way of implementing such modeling method. In addition, the case of modeling MIMO-UWB channel is presented where proposed methods are explained.

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1. Introduction

The radio communication channel is expressed as a linear operator that interacts with the transmitted signal and produce the received signal. The relation between the transmitter (Tx), receiver (Rx), and the channel is represented as [1, 2]

$$y = Hx \tag{1}$$

where y is the received signal, x is the transmitted signal, and H is the channel operator.

In the case of a multiple-input multiple-output (MIMO) channel, the channel operator is comprised of channel elements that describe the channel behavior between each pair of antennas. Consequently, the MIMO channel operator is represented as a channel matrix that contains all channel elements. For example, for 2×2 MIMO communication, the channel matrix is written as

$$\boldsymbol{H} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \tag{2}$$

where H represents a MIMO channel matrix, $h_{11}, h_{12}, h_{21}, h_{22}$ represents channel elements regarding a signal transmission between Rx1 and Tx1, Rx1 and Tx2, Rx2 and Tx1, and Rx2 and Tx2, respectively.

In MIMO communication, the chosen way to exploit spatial diversity, the spatial multiplexing possibility and the beamforming gain feasibility to achieve are all depend on the spatial structure of the MIMO channel (matrix or tensor) elements [3]. Therefore, the statistical description of the MIMO channel should express the spatial structure.

Full characterization of the performance of multiple-input multiple-output (MIMO) systems is facilitated by accurate channel models that capture the spatial behavior of true MIMO channels [4]. These channel models should give an approximated performance of the real channel that can be extracted through channel measurements.

To model the MIMO channel, analytical channel modeling can be used based on the correlation between the channel elements. The synthesis equation of the channel can be found later, and it should approximate the real channel behavior. In this paper, we are going to explain the method of modeling the MIMO channel based on the analytical approach and specifically based on the correlation-based approach.

The contribution of this paper is illustrated as follows. The paper reviews the correlation-based analytical channel modeling approach in modeling the MIMO channel. A review of previous models is included. Important issues with this approach such as the number of parameters and the methods of validation are shown. Furthermore, the proposed method of using this approach in modeling MIMO-UWB channel is illustrated.

The paper is organized as follows. Section 2 explains the analytical and physical MIMO modeling. Section 3, contains an illustration of the notation of symbols used in the paper. A review of correlative-based MIMO models is available in Section 4. The calculations of model complexity in terms of the number of required parameters are shown in Section

5, while metrics of model validation are available in Section 6. Section 7 illustrates MIMO-UWB modeling. Finally, the paper is concluded in Section 8.

2. Analytical vs. Physical MIMO Channel Modeling

Physical modeling characterizes the MIMO propagation channel based on the electromagnetic propagation of the signal from the transmitter (Tx) array to the receiver (Rx) array. The propagation parameters can be the angle of arrival (AoA), angle of departure (AoD), and the delay. In addition, antenna polarization can be added [5]. Usually, physical models are independent of antenna configuration, such as, mutual coupling and antenna pattern.

Analytical Modeling characterizes the channel response in an analytical or mathematical way in contrast to the wave propagation in the physical modeling. All the individual channel impulse response elements are summed into a channel matrix that quantifies the response of the MIMO system [5]. Analytical modeling ignores the physical proprieties of the scatterers in the environment and focuses on the correlation of the spatial channel coefficients [3]. In this type of channel modeling, the channel synthesis equation is found based on the full correlation matrix. Later approaches focused on using one-sided correlation matrices that leads to decrease the number of parameters in the models and hence decrease complexity.

It is good to understand the channel from two perspectives: the physical and the analytical views. Each type of modeling, based on the mentioned views, has its own advantage and utilization in terms of its application in the communication system. Understanding the physical side of the channel is important for the development of new communication systems, while the analytical side can be useful for the design of signal processing algorithms[3].

Increasing the correlation leads to a reduction in the MIMO channel capacity. The high correlation between MIMO channel elements reduces the channel rank. In this case, a MIMO channel with full correlated elements can be approximated to be just a one SISO channel in the case of perfect correlated channel [6]. Therefore, large capacity can be obtained through the decorrelation of MIMO channel elements [7]. Consequently, creating parallel sub-channels is achieved based on that. Therefore, the importance of the correlation between the antenna channel elements shows the importance of the correlation-based analytical MIMO modeling.

3. Notations

In this paper, we will use $vec(\cdot)$ to vectorize the matrix or tensor by stacking their column one after the other. The subscripts $\bullet^T$, $\bullet^H$, and $\bullet^*$ represent the matrix transpose, the conjugate transpose (Hermitian) of a matrix, and the complex conjugate respectively. $\odot$ is the matrix element wise product , while $\otimes$ is the Kronecker product. Scalars are denoted by lowercase letters, vectors by bold lowercase letters, matrices by bold uppercase letters, and third-order tensors by calligraphic letters.

4. Review of MIMO correlation-based channel models

In the MIMO system which employs multiple antennas at the transmitter and/or receiver side, the correlation between the transmitting and receiving antennas is an important aspect of MIMO channels [8]. This section reviews the MIMO channel models that have been modeled based on the correlation-based channel modeling.

In the case of modeling the MIMO channel based on the channel spatial correlation, full correlation matrix need to be extracted based on channel measurements. Full correlation matrix represents the correlation between all measured channel elements. Then the synthesis channel can be found by [9]

$$\boldsymbol{h}_{synthesis} = \boldsymbol{R}_h^{1/2}\boldsymbol{g}_h \tag{3}$$

where $\boldsymbol{h}_{synthesis}$ is the synthesis channel vector (modeled channel), $\boldsymbol{R}_h$ is the full correlation matrix of the channel vector, $\boldsymbol{g}_h$ represents an independent and identically distributed (IID) complex Gaussian random variable.

For a narrowband MIMO channel (3) is modified to

$$\boldsymbol{H}_{synthesis} = unvec(\boldsymbol{R}_h^{1/2}\boldsymbol{g}_h) \tag{4}$$

where $\boldsymbol{H}_{synthesis}$ is the narrowband synthesis MIMO channel matrix

The simplest analytical MIMO channel model is the IID model. In this model, it is assumed that all the channel elements in the MIMO channel matrix are uncorrelated (i.e., they are IID) [5]. Based on that, higher and unpractical values of capacity are resulted due to neglecting the correlation values.

The Kronecker model [10], models the MIMO correlation channel matrix as the Kronecker product of the spatial correlation matrices at the transmitter and the receiver. In this model, the joint spatial structure is neglected. So, no coupling between transmit and receive sides is considered.

One sided correlation matrices at Tx and Rx are generated based on measured channel, and the Kronecker model between these matrices is used. The Correlation matrix resulting from the Kronecker product is written as

$$\boldsymbol{R}_{Kron} = \boldsymbol{R}_{Tx} \otimes \boldsymbol{R}_{Rx} \qquad (5)$$

where $\boldsymbol{R}_{Kron}$ is the Kronecker correlation matrix, $\boldsymbol{R}_{Tx}$ is the transmit one-sided correlation matrix, and $\boldsymbol{R}_{Rx}$ is the receive one-sided correlation matrix

The Kronecker model adopts the independence between the scatterers around the transmitter and the one around the receiver. The model synthesis equation is

$$\boldsymbol{H}_{Kron} = \boldsymbol{R}_{Rx}^{1/2}\boldsymbol{G}(\boldsymbol{R}_{Tx}^{1/2})^{1/2} \qquad (6)$$

where $\boldsymbol{H}_{Kron}$ is the synthesis channel resulted from the Kronecker model.

Weichselberger model [11] has been developed for the MIMO narrowband communication channels. In this model, the average coupling between the eigenmodes of two link ends is used to model the correlation properties in the MIMO channel. The parameters of this model are the eigenbases (eigenmodes) of the transmission and reception correlation matrices and the coupling between the direction of arrival (DoA) and the direction of departure (DoD). This model assumes the channel stationarity, and it is developed for narrowband MIMO channels. The Weichselberger model synthesis equation is defined as:

$$\boldsymbol{H}_{Weichselberger} = \boldsymbol{U}_{Rx}(\tilde{\boldsymbol{\Omega}} \odot \boldsymbol{G})\boldsymbol{U}_{Tx} \qquad (7)$$

where $\boldsymbol{U}_{Rx}$ and $\boldsymbol{U}_{Tx}$ are the eigenmodes of transmit and receive correlation matrices. $\tilde{\boldsymbol{\Omega}}$ is the element wise square root of power coupling matrix $\boldsymbol{\Omega}$.

In [12], a wideband MIMO channel tensor is defined. It is a three dimensions tensor. The third dimension that has been added to the channel matrix is the delay. Here, each channel element has different delayed versions. As the bandwidth of the signal increases the number of delay taps can be increased also. The third order wideband channel tensor is expressed as:

$$\mathcal{H} \in \mathbb{C}^{M_{Rx} \times M_{Tx} \times D} \qquad (8)$$

where M_{Rx} is the number of receiving antennae, M_{Tx} is the number of transmit antennae, and D is the number of delay taps. This model assumes the stationarity of the MIMO channel. The channel synthesis equation is expressed as

$$\mathcal{H}_{struct} = \mathcal{W} \times_1 \boldsymbol{U}_{Rx} \times_2 \boldsymbol{U}_{Tx} \times_3 \boldsymbol{U}_{Del} \qquad (9)$$

where U_{Rx}, U_{Tx} and U_D are the eigenmodes for the transmit, receive, and delay eigenmodes, respectively.

A correlation tensor model developed for time-variant frequency selective MIMO channels is proposed in [13]. It focuses on the space and frequency dimensions. Temporal block-wise stationarity has been assumed. In this model, the channel is represented as a fourth order tensor.

$$\mathcal{H} \in M_R \times M_T \times N_f \times N_t \qquad (10)$$

where N_f represents the frequency samples and N_t represents the time samples.

The authors proposed that for each channel response there are different frequency and time samples. Frequency samples reflect the frequency selectivity of the wideband channel, and the time sampling represents the time varying channel where each channel response changes with time.

5. Evaluation of model complexity

The Number of parameters needed in a channel model is an important aspect. The channel model should not be so complex with a huge number of parameters. On the other hand, enhancing the model accuracy can be done through increasing the number of required parameters. Therefore, a tradeoff should be taken in regard between the number of the parameters and the model accuracy in order to approximate the real channel behavior without turning the model into a complex state. Table 1 shows the number of parameters in some of the mentioned models compared to the case of using the full correlation matrix.

Table 1: Required parameters in some specific models for 2×2 MIMO system

Modeling method	Number of parameters
Full Correlation approach (Narrowband)	16
Kronecker	8
Weichselberger	8
Structured	20

As it can be seen in Table I, in the case of the Kronecker model, the number of parameters needed in the model is $M_{Rx}^2 + M_{Tx}^2$, while it is $(M_{Rx}M_{Tx})^2$ when the full correlation matrix in (4) is used. In order to understand how these numbers are obtained, Let a system of 2×2 MIMO is assumed. In the case of using the Kronecker model, (6) will be used. The number of parameters in each one sided correlation matrix is 4. Therefore, two of one sided correlation matrices will lead into 8 parameters in the Kronecker model. This number represents the equation $M_{Rx}^2 + M_{Tx}^2$ for 2×2 MIMO. The result shows that when the channel is calculated based on the Kronecker model, half of the required parameters are needed in comparison to the full correlation matrix approach. This number of parameters increases sharply as the number of antennas increases, leading to more complex system. Consequently, one sided correlation matrix approach shows good choice to overcome growing complexity with the number of antennas, and that's why it has been used in all succeeded models.

In the case of using 4x4 antennas, the complexity of Weichselberger and structured will be increased sharper than the Kronecker. For example, for 4x4 the number of parameters needed in the Weichselberger model is 40, while it is 32 in the case of the Kronecker model. However, it is noted that, the accuracy of the model can be more important than complexity, such as in the case of the Weichselberger model that provides more spatial structure accuracy compared to the Kronecker model.

6. Metrics of Model validation

Any developed channel model needs to be validated through comparing its results with the real channel measurements. Model parameters are extracted from measured channels. Then the channel synthesis equation is generated. Finally, metrics obtained based on the channel measurements are compared with the one resulted from the modeled channel [14].

Several metrics are used to evaluate the channel model. As illustrated in [14], it is more advantageous to have a single metric that can show the whole goodness of a particular MIMO channel model for validation purposes. However, one parameter cannot reflect the whole of the channel properties. Therefore, there are some metrics that can be used to validate MIMO channel models. Some of these metrics are: capacity or mutual information, correlation matrices and diversity measure.

7. MIMO-UWB modeling using the correlative-based approach

In addition to increasing channel capacity; in terms of ultra-wideband (UWB) communication; using MIMO can also lead to an increase in the transmission range [15, 16]. This gives many benefits for UWB communication and assists in competing with other wireless technologies that offer less data rate but longer transmission ranges. Therefore, channel modeling for MIMO-UWB is crucial for this purpose.

In order to model the MIMO-UWB channel, the frequency selectivity of this channel should be taken in regard. In this case, each channel can be represented with many delay taps as in the case of wideband channels such as the structured model. However, the number of delay taps in UWB channel is much more.

In [17], a MIMO-UWB channel model has been developed. The channel has been represented as a three order tensor as

$$H^{(UWB)} \in \mathbb{C}^{M_{Tx}\times M_{Rx}\times F} \qquad (11)$$

where F is the number of frequency components in a channel vector.

It can be seen that the UWB channel has been represented in terms of its frequency dependency. The synthesis equation in this model is written as

$$\boldsymbol{H}[f] = \boldsymbol{R}_{Rx}^{1/2}\,\boldsymbol{G}\,\boldsymbol{R}_{Tx}^{1/2} \qquad (12)$$

where $\boldsymbol{H}[f]$ is the channel matrix at a specific frequency component, and $\boldsymbol{G}$ is a matrix with IID complex Gaussian entries.

The model follows the Kronecker approach of having independence between transmit and receive correlation matrices, i.e., no coupling between transmit and receive sides.

It is noted that the modeling approaches in MIMO-UWB case focused on the indoor environment. In the case of outdoor environment, the UWB channel is sparse. Therefore, there is a need for modeling MIMO-UWB channel with taking the sparsity issue in consideration. This assists in the development of more practical models for outdoor environment.

Based on the previous analysis, in order to model a MIMO-UWB channel for outdoor or any sparse case. Several steps are proposed:

1. The UWB channel snapshot can be seen as a row of a number of impulse responses based on delay component *H[d]* where $d \in \{1,\ldots,D\}$ and D is the maximum number of delay taps (delayed component of the channel)
2. Based on the measured campaign conducted in [18], channel sparsity is observed. For example, in a particular channel snapshot *H [d]* where *D = 1632*, the number of non-zero elements is about 30 only.
3. The coupling between Tx and Rx sides need to be addressed as it has been used in Weichselberger and the structured model. This will provide an accurate spatial structure of a particular environment.

8. Conclusion

In this paper, correlation-based analytical modeling approach used in modeling MIMO channel has been illustrated. Several common models have been reviewed. In addition, the complexity and methods of validation has been elaborated. Steps to be used in modeling the MIMO-UWB channel has been proposed where the channel is represented as a third order tensor. The sparsity of the channel and coupling between Tx and Rx sides are also included.

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References

1. P. Bello, "Characterization of randomly time-variant linear channels," Communications Systems, IEEE Transactions on, vol. 11, pp. 360-393, 1963.
2. F. Hlawatsch and G. Matz, Wireless Communications Over Rapidly Time-Varying Channels: Academic Press, 2011.
3. W. Weichselberger, "Spatial structure of multiple antenna radio channels," Institut für Nachrichtentechnik und Hochfrequenztechnik, 2003.
4. J. Wallace and B. Maharaj, "Accurate MIMO channel modeling: Correlation tensor vs. directional approaches," 2007, pp. 3750-3754.
5. P. Almers, E. Bonek, A. Burr, N. Czink, M. Debbah, V. Degli-Esposti, et al., "Survey of channel and radio propagation models for wireless MIMO systems," EURASIP Journal on Wireless Communications and Networking, vol. 2007, pp. 56-56, 2007.
6. W. Q. Malik, "Spatial correlation in ultrawideband channels," Wireless Communications, IEEE Transactions on, vol. 7, pp. 604-610, 2008.
7. K. I. Pedersen, J. B. Andersen, J. P. Kermoal, and P. Mogensen, "A stochastic multiple-input-multiple-output radio channel model for evaluation of space-time coding algorithms," in Vehicular Technology Conference, 2000. IEEE VTS-Fall VTC 2000. 52nd, 2000, pp. 893-897.
8. Y. S. Cho, J. Kim, W. Y. Yang, and C. G. Kang, MIMO-OFDM wireless communications with MATLAB: Wiley, 2010.
9. N. Costa and S. Haykin, Multiple-input, multiple-output channel models: theory and practice vol. 65: Wiley-Interscience, 2010.
10. J. P. Kermoal, L. Schumacher, K. I. Pedersen, P. E. Mogensen, and F. Frederiksen, "A stochastic MIMO radio channel model with experimental validation," Selected Areas in Communications, IEEE Journal on, vol. 20, pp. 1211-1226, 2002.
11. W. Weichselberger, M. Herdin, H. Ozcelik, and E. Bonek, "A stochastic MIMO channel model with joint correlation of both link ends," Wireless Communications, IEEE Transactions on, vol. 5, pp. 90-100, 2006.
12. N. Costa, S. Member, S. Haykin, and L. Fellow, "A Novel Wideband MIMO Channel Model and Experimental Validation," IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, vol. 56, pp. 550-562, 2008.
13. M. Weis, G. Del Galdo, and M. Haardt, "A correlation tensor-based model for time variant frequency selective MIMO channels," in Proceedings of International ITG/IEEE Workshop on Smart Antennas (WSA'07), 2007.
14. H. Ozcelik, N. Czink, and E. Bonek, "What makes a good MIMO channel model?," 2005, pp. 156-160 Vol. 1.
15. Z. Qiyue, A. Tarighat, and A. H. Sayed, "Performance analysis of multiband OFDM UWB communications with application to range improvement," Vehicular Technology, IEEE Transactions on, vol. 56, pp. 3864-3878, 2007.
16. Y. K. Zahedi, U. A. K. C. Okonkwo, R. Ngah, T. A. Rahman, and N. A. Mamat, "Time-Scale Domain Characterization of Ultra-wideband 2x2 MIMO Channel and Range Extension for Infostation Environment " 2012.
17. J. Adeane, W. Malik, I. Wassell, and D. Edwards, "Simple correlated channel model for ultrawideband multiple-input multiple-output systems," Microwaves, Antennas & Propagation, IET, vol. 1, pp. 1177-1181, 2007.
18. Y. Zahedi, R. Ngah, U. A. K. Chude-okonkwo, S. Nunoo, and M. Mokayef, "Modeling the RMS Delay Spread in Time-Varying UWB Communication Channels," in International Conference on Intelligent and Advanced Systems, Kuala Lumpur, Malaysia, 2014,in press.

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