The flood process mathematical modelling an their prediction methods based on static data

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Abstract. The flood water level estimation and floods forecasting method is presented based on the statistical information about flood situations in different regions. The mathematical model of flood development is constructed based on the theory of incorrect problems regularization technique for the problem of function's reconstruction using the limited information about its behaviour in the defined area. To define the unknown parameters based on the statistical data the technique of linear regression and the least square method are used. The method of flood investigation experimental tools using optimization is offered based on the associative analysis technique. The designed model practical using method is constructed based on the flood process experimental studying and statistical data analysis. The presented method allows to estimate and to forecast the flood process development, to determine the maximal floods intensity and duration.

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Introduction

In the analysis of real phenomena flood situation the relationship between time (t) and the level of floodwater (l) can be schematically depict (figure 1):

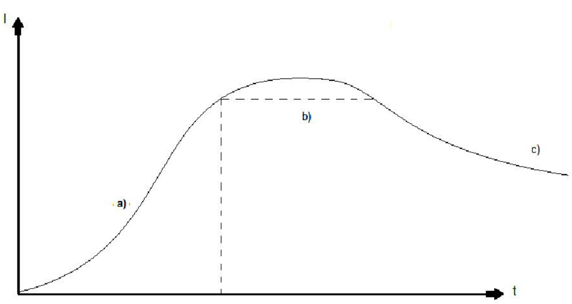

Figure 1. Scheme of the relationship between time and the level of flood water

Any floods can characterized by the development period a) the critical or maximal intensity period b) and the flood recession period c). The scheme shown in figure 1 does not deal with cases where a short span of time has a number of flood peaks, in which case the phenomenon shown in figure 1 can be modelled by imposing a number of other flood peak. It is necessary to reproduce the analytical framework functions shown in figure 1. From a mathematical point of view, this function should be characterized by the following conditions:

$$\begin{cases} f(0)=0; \\ \exists\ \mathrm{x}_0 : f^{'}(0)=0;\ \ f^{"}(\mathrm{x}_0)<0; \\ \lim\limits_{\mathrm{x}\to\infty} f(\mathrm{x})=0. \end{cases} \quad (1)$$

Obviously, these conditions corresponds to many functions, and therefore, the task of constructing a model of flooding from a mathematical point of view is incorrect [7] for its regulation need to develop specific algorithms using additional information about the type of function that satisfies conditions (1).

The simplest among the known function is a function of:

$$f(t)=t\cdot \mathrm{e}^{-t}, \quad (2)$$

By direct verification establishes that the function (2) are satisfied all three conditions (1):

1. $f(0)=0\cdot e^{-0}=0$;
2. $\exists f(\mathrm{x}_0): f^{'}(\mathrm{x}_0)=0$;

$(t\cdot \mathrm{e}^{-t})=\mathrm{e}^{-t}-\mathrm{e}^{-t}+t\cdot \mathrm{e}^{-t}=0\rightarrow t=1$;

$f^{"}(t)=-\mathrm{e}^{-t}-\mathrm{e}^{-t}+t\cdot \mathrm{e}^{-t}=(-2+t)\cdot \mathrm{e}^{-t}$;

$f^{"}(1)=-\mathrm{e}^{-1}<0$;

3. $\lim\limits_{t\to\infty}\frac{t}{e^t}=\lim\limits_{t\to\infty}\frac{1}{e^t}=0$.

However, the function (2) and its using it to describe the flooding associated with some problems

– it is difficult to change and choose its form under certain experimental results and analysis of statistical data on floods. To solve this problem you can suggest function:

$$y = f(t) = t \cdot e^{-at}, \quad a > 0, \qquad (3)$$

which, obviously, also satisfies the specified conditions figure 2:

1. $f(0) = 0 \cdot e^{-a \cdot 0} = 0$;
2. $\exists f(x_0) : f'(x_0) = 0$;

$$(t \cdot e^{-at})' = e^{-at} - a \cdot t \cdot e^{-at} = (1 - a \cdot t) \cdot e^{-at} = 0;$$

$$t = \frac{1}{a};$$

$$f''(t) = -a \cdot e^{-at} + (1 - a \cdot t) \cdot (-a) \cdot e^{-at} = e^{-at} \cdot (-2 \cdot a + a^2 \cdot t) = 0;$$

$$f''(\frac{1}{a}) = -e^{-1} \cdot (-2 \cdot a + a) = -a e^{-1} < 0;$$

3. $\lim\limits_{t \to \infty} \frac{t}{e^{at}} = \lim\limits_{t \to \infty} \frac{1}{a \cdot e^{at}} = 0$.

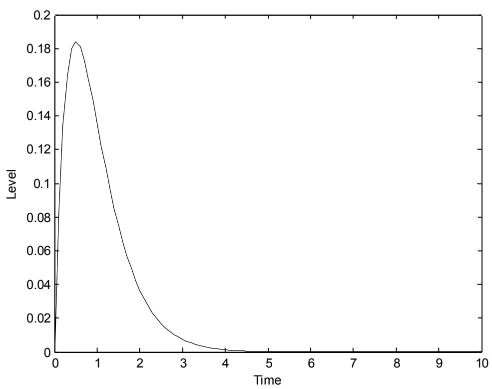

Figure 2. The graph of the function

$$y = f(t) = t \cdot e^{-at}, \quad a > 0$$

The addiction (3) has an option $a > 0$ that allows you to get a wide spectrum of the urves but all this curves are topologically (in their spatial arrangement) similar, that does not allow to enter many parameters that affect the level of elevation of flood waters. Therefore, it is proposed the following two-parameter model for the function of type (1) (figure 3):

$$y = t^n \cdot e^{-at}, a > 0, n > 0, \qquad (4)$$

for which all the conditions (1) are fulfilled:

1. $(t^n \cdot e^{-at})' = n \cdot t^{n-1} \cdot e^{-at} - a \cdot t^n \cdot e^{-at} =$;

$$= t^{n-1} \cdot (n \cdot e^{-at} + a \cdot t \cdot e^{-at}) = t^{n-1} \cdot e^{-at} \cdot (n - a \cdot t) = 0 \to t = \frac{n}{a}$$

$$(t^n \cdot e^{-at})'' = -a \cdot e^{-at} \cdot t^{n-1} \cdot (n - a \cdot t) + e^{-at} \cdot t^{n-2} \cdot (n-1) \cdot (n - a \cdot t) + e^{-at} \cdot t^{n-1} \cdot (-a) =$$
$$= e^{-at} \cdot t^{n-2} \cdot [(-a) \cdot t \cdot (n - a \cdot t) + (n-1) \cdot (n - a \cdot t) - a \cdot t] =$$
$$= e^{-at} \cdot t^{n-2} \cdot [-a \cdot t \cdot n + a^2 \cdot t^2] + (n^2 - n - n \cdot a \cdot t + a \cdot t) - a \cdot t] =$$
$$= e^{-at} \cdot t^{n-2} \cdot [-a \cdot t \cdot n + a^2 \cdot t^2 + n^2 - n - n \cdot a \cdot t] =$$
$$= e^{-at} \cdot t^{n-2} \cdot [a^2 \cdot t^2 - 2 \cdot a \cdot t \cdot n + n^2 - n] = e^{-at} \cdot t^{n-2} \cdot [(a \cdot t - n)^2 - n];$$

$$f''(\frac{n}{a}) = -e^{-n} \cdot (\frac{n}{a})^{n-2} \cdot [(n-n)^2 - n] = -n \cdot e^{-n} (\frac{n}{a})^{n-2} < 0;$$

2. $f(0) = 0$;

$\lim\limits_{t \to \infty} \frac{t^n}{e^{at}} = 0$ using the L'Hopital rule [8].

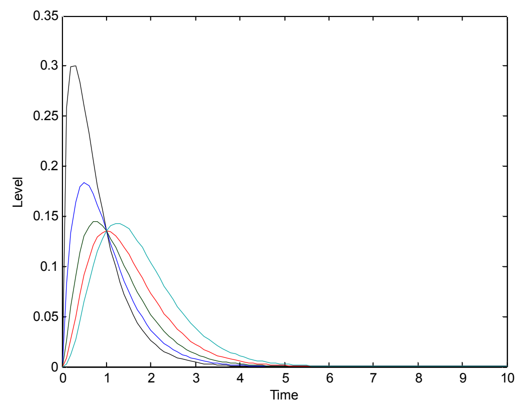

Figure 3. The functions graphics family

$$y = t^n \cdot e^{-at}, a > 0, n > 0$$

Using, for example, the relationship (4), it is possible to recover the differential equation of the process described by each of the functions (2)-(4). It is necessary to use the following property of linear systems: if the function $y_1(x)$ satisfies the differential equation of the first order type $f(y, y') = 0$, then the following condition holds [9]:

$$\begin{vmatrix} y(x) & y'(x) \\ y_1(x) & y_1'(x) \end{vmatrix} = 0. \qquad (5)$$

Then, when substituted in the determinant (5) functions $y_1(x)$ according to the properties of determinants, equation (5) is satisfied identically. Through these values, it is possible to obtain the differential equation upshot of which is relevant functions: the function (2), equation (5) takes the form:

$$\begin{vmatrix} y' & y \\ e^{-t} - t\cdot e^{-t} & t\cdot e^{-t} \end{vmatrix} = 0$$

or after the construction of these terms:

$$y'\cdot t - (1-t)\cdot y = 0\ , \qquad (6)$$

for the functions like (3):

$$\begin{vmatrix} y & y' \\ t\cdot e^{-at} & e^{-at} - t\cdot a\cdot e^{-at} \end{vmatrix} = 0,$$

or:

$$y'\cdot t - (1-t)\cdot y = 0\ , \qquad (7)$$

and for the functions like (4):

$$\begin{vmatrix} y & y' \\ t^n\cdot e^{-at} & n\cdot t^{n-1}e^{-at} - t^n\cdot a\cdot e^{-at} \end{vmatrix} = 0,$$

where derived:

$$y'\cdot t^n\cdot e^{-at} - t^n\cdot e^{-at}\cdot(\frac{n}{t} - a)\cdot y = 0\ ;$$

$$y' = \left(\frac{n}{t} - a\right)\cdot y. \qquad (8)$$

For each of the equations (6)-(8), and in particular for the equation (8), we can put the initial conditions as:

$$y(t_0) = y_0, \qquad (9)$$

setting conditions as (9) takes into account the initial level of the flood:

$$y = y_0\cdot e^{-at_0}\cdot e^{-at}\cdot t_0\cdot t^n = t_0\cdot\left(\frac{t}{t_0}\right)^n\cdot e^{-a(t-t_0)}.$$

Gaining importance as establishing physical sense of coefficients a and n that may be a function of the any unknown value:

$$\begin{cases} n = n\left(\mathrm{x}_1\ \mathrm{x}_2\ \ldots\ \mathrm{x}_\mathrm{k}, t\right); \\ a = a\left(\mathrm{x}_1\ \mathrm{x}_2\ \ldots\ \mathrm{x}_\mathrm{k}, t\right), \end{cases} \qquad (10)$$

where values x_k – permeability of soil, humidity, moisture intakes, relief features, etc., and the variable t – time. The question is – how determined the numerical values (10) for different types of floods and elected as variables in (10), which are the most significant influence on the specified dependency. It should also give an interpretation components of equation (8). The function $y(t)$ – level of flood waters at some point in time that is proportional to itself with alternating form factor:

$$k = \frac{n}{t} - a, \qquad (11)$$

where n and a – empirically determined coefficients. If $k > 0$ – the flood intensity increases, if $k < 0$ – the one's decreases.

Suppose that for some flood is known statistics about its course (y_i, t_i). The feature of the model type (8) is that according to (y_i, t_i) $i = 1,\ldots,N$, where N – number of observations that during heavy floods can be very significant because the control of the flood water is regularly, to estimate the parameters n and a it is possible to use the linear regression tools. For this purpose, the function (4) can be written as (using the logarithm operation):

$$\ln y = n\ln t - at,$$
$$\frac{\ln y}{t} = n\frac{\ln t}{t} - a, \qquad (12)$$

introducing the notation $\tilde{y} = \frac{\ln y}{t};\ \tilde{t} = \frac{\ln t}{t};\ \tilde{k} = n;\ \tilde{b} = -a$, it is possible to receive:

$$\tilde{y} = \tilde{k}\tilde{t} + \tilde{b}, \qquad (13)$$

is, the linear regression equation. Using the known formula (7) for the coefficients of the linear regression it is possible to obtain:

$$\tilde{k} = \frac{N\sum\tilde{y}_i\tilde{t}_i - \sum\tilde{y}_i\sum\tilde{t}_i}{N\sum\tilde{t}_i^2 - \left(\sum\tilde{t}_i\right)^2};$$
$$\tilde{b} = \frac{1}{N}\left(\sum y_i - \tilde{k}\sum\tilde{t}_i\right), \qquad (14)$$

where, using the connection formulas is obtained:

$$n = \tilde{k};\ \tilde{b} = -a, \qquad (15)$$

so, the parameters n and a can be determined uniquely, which suggests a built of regularization algorithm for the incorrect problems in terms of function (1) reconstruction. To determine the factors that affect on the process the technique [9] associative analysis is used, which allows to identify the variables and parameters that influence the process of raising the flood waters. Let the results of experimental research and statistical analysis is the quantitative characterization of some parameters x_i, range of variation which can be divided into two segments, which correspond roughly equally probable quantity value x_i^c. It is possible to construct the table 1.

Table 1. Associative analysis

	$x_i<x_i^c$	$x_i>x_i^c$
$f<f_0$	A	B
$f>f_0$	C	D

where f – the flood waters level, f_0 – some average value that divides the range of variation of flood waters into intervals, in which the value f shared about equally by the number realizations. A, B, C, D – the number of results compare which correspond to the such value f and x_i^c.

It is necessary to calculate:

$$A+B=n_1;$$

$$C+D=n_2;$$

$$A+C=n_3;$$

$$B+D=n_4.$$

Obviously, the total number of experiments is either n_1+n_2 or n_3+n_4. The contingency ratio is calculated as:

$$\psi=\frac{AD-BC}{\sqrt{(A+B)(C+D)(A+C)(B+D)}}.(16)$$

If $\psi>0,3$ [psi>0,3], the relationship between variables is confirmed, it should be studied more in detail, if $\psi<0,3$ [psi<0,3], the relationship between these variables can be considered as insignificant.

Forecasting technique

It is propose the following method of the flood waters level estimation and prediction of their development. After the study of the floods that occurred in the investigated region the relations using the method (12)-(15) was obtained:

$$f_i=t^{n_i}\cdot e^{-a_i t},\ i=1,...,k,\quad (17)$$

where k – the number of investigated floods. The parameters $x_1,x_2,...,x_m$, which make the influence on the flood formation and development are defined. Such parameters are – the water-physical soil properties (permeability, saturation by the water), humidity, wind speed and direction, etc. It is believed that on the basis of experimental studies, these values are known for each of the flood – $x^i{}_1,x^i{}_2,...,x^i{}_m$. Following the procedure [10] of the associative analysis (16) for each of the variables x_s, $s=1,...,M$ the contingency ratio (16) is calculated and the level of communication between the corresponding values x_i and f is set. Thus the number of variables x_i, $i=1,...,M$, that affect on f, is decreased, and further it will be consider only x_j, $j=1,...,M_0$, $M_0\le M$, affecting on the process. These results allow us to optimize the process of experimental research – the number of parameters x_j, $j=1,...,M_0$, which is necessary to develop methods of experimental measurement and monitoring, is reduced. In the process of the defined region flooding possibility studying the value of $x^p{}_j$, $j=1,...,M_0$ are defined, then the formula (17) and the early defined for all the investigated floods with level f_i parameters x_j, $j=1,...,M_0$, which affect on the f_i, are used. The next value is determined:

$$\arg\min_i\left\{\sum(x_j^i\text{-}x_j^p)^2\right\}=j^*,\quad (18)$$

to obtain a more accurate prediction the value found j_s^*,s=1,....N_0, $N_0(N_0=2)$, which is the nearest to j^* is founded. It is used the formulae (17) corresponding to the defined j_s^*. Using the appropriate depending on f_i (17) found for j_s^* (in the most simple variant j_s^* is the only and $j_s^*=j^*$, which was defined using (18)), taking to consideration the corresponding graphs it is possible to estimate the potential level of the flood and one's intensity.

Conclusion

The method of the flood water level estimation and the flood situations forecasting is designed and discussed. Directions for further research may be associated with the development of experimental tools to determine the parameters x_i, that impact on the flooding process formation.

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