Shock and detonation wave in terms of view of the theory of interaction gasdynamic discontinuities

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Abstract. Historical background is given to the development of the theory of interaction of stationary and non-stationary gas-dynamic discontinuities by scientific school of V.N. Uskov. References are given to the stage works that represent the results of research of interaction of stationary discontinuities in univariate shock wave traveling, oblique running waves, triple shock-wave configurations and, finally, time-dependent (non-stationary) triple configurations. Geometrical meaning of the equations of gas dynamics is shown. The concept of shock and detonation waves as the feature of representation of gas-dynamic characteristics space design is given. The theory given provides methodological principles of design of combustion chambers of jet engines with detonation combustion.

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Introduction

Recently, various ways are discussed of organizing detonation combustion in advanced air jet and rocket engines. To understand clearly the nature of these projects, it is important to represent what is stationary and non-stationary gas-dynamic discontinuity (GDD), shock wave and how they differ from detonation wave, which is also gas-dynamic discontinuity. For definiteness let's consider detonation wave as shock wave, which arose as a result of the chemical reaction of oxidation. And shock wave is gas-dynamic discontinuity, which is formed by interaction of supersonic flow with solid wall, sharp edge or as a result of intersection (interference) of other GDDs.

Shock and detonation waves. General theory and particular

Geometrical Meaning of the Supersonic Flows Gas Dynamics Equations

At present (in the last 19 years), the theory and the mathematical apparatus is formed sufficient for designing optimal shock-wave structures (SWS) in detonation combustion chamber, working both in stationary and pulsed mode. Science team under the direction of V.N.Uskov consistently developed the theory of extreme SWS. First, the theory of interaction of stationary gas-dynamic discontinuities was generalized by V.N.Uskov and S.L.Staryh for the case of second-order discontinuities [1-3]. They have studied a function of flow unevenness behind the discontinuity from the curvature of discontinuity and flow unevenness in front of it. Then, the theory has been complemented in the works of V.N.Uskov, A.V.Omelchenko and M.V.Chernyshev by the theory of interaction of univariate shock wave traveling and interaction of oblique non-stationary waves [4-6]. In parallel, a post graduate student of V.N.Uskov, Tao Gang has developed the theory of optimal triple configurations of shock waves at first in a uniform flow and then in non-uniform flow[7]. Finally, in the works V.N.Uskov, M.V.Chernyshev and P.S.Mostovyh [8-9], it was generalized to the case of triple configurations of shock waves in non-stationary and non-uniform gas flow. The author of this work has developed a theory of low-frequency oscillations of shock waves for flow in a channel with sudden expansion [10-12].

Geometrical Meaning of the Supersonic Flows Gas Dynamics Equations

A here is its own geometry of parameter space at the bottom of every branch of physics. Minkowsky geometry describes space of the Special Theory of Relativity, Riemann geometry describes space of the General Theory of Relativity, symplectic geometry describes space of classical mechanics, etc. In order to be able to take advantage of the achievements of mathematicians in classification and structural stability of shock-wave structures (SWS), one needs to determine gas-dynamic concepts in terms of the symplectic geometry. According to modern concepts, gas-dynamic variables form multi-dimensional hyperspace, and Euler equations that describe the flow of ideal gas assign hypersurface in it, the curvature of which is determined by gas-dynamic unevenness Ni (non-isobaric property, curvature of flow line and vorticity).

$$N_1 = \frac{\partial \ln P}{\partial s}, N_2 = \frac{\partial \vartheta}{\partial s}, N_3 = \varsigma = \frac{\partial \ln P_0}{\partial s}.$$

In axisymmetric case, Euler equations written by unevenness, in natural coordinate system associated with flow lines are as follows:

$$\frac{M^2-1}{\gamma M^2}N_1+\frac{\partial\vartheta}{\partial n}+\frac{\sin\vartheta}{y}=0$$

$$\gamma M^2\frac{\partial\ln V}{\partial s}=-N_1$$

$$M^2N_2=-\frac{\partial\ln P}{\partial n}$$

In these equations n is the length of normal line to flow lines, s is arc length along flow line, P is the pressure, ϑ is the angle of slope of velocity vector, P_0 is the total pressure, ζ is vorticity. The first expression is continuity equation. The second and the third are the projections of equation of motion to the axes of natural coordinate system with streamlines. As is known, supersonic flows may contain areas where the parameters are changing rapidly, abruptly. Within the model of ideal gas, in such cases they speak of the existence of gas-dynamic discontinuities (GDD).

Generalization of the surface concept is the multiformity. The multiformity is an arbitrary set of points represented by a combination of a finite number of regions of Euclidean space, each of which having local coordinates assigned.

Shock and detonation wave as the singularity the projection of the manifold dynamic parameters

For simplicity, let's consider one-dimensional ideal gas equation of motion (Euler equation).

$$\frac{du}{dt}+u\frac{\partial u}{\partial x}=0$$

This equation describes the velocity field of particles freely moving in a straight line. The law of free movement of a particle is $x=\phi(t)=x0+vt$, where v is velocity of the particle. Function ϕ satisfies Newton equation. By definition $\frac{d\phi}{dt}=u(t,\phi)$. Differentiating the last equation by t, we obtain the following equation:

$$\frac{\partial\varphi}{\partial t^2}=\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=0$$

Thus, the description of motion using Euler equations for the field and with the help of Newton equation for the particles are equivalent. It is known that the quasi-linear partial differential equations are solved by building characteristics. Each variety has its own corresponding characteristic field. Characteristics are phase curves of the characteristic field. Equation of characteristics of Euler equation is equivalent to Newton equation. Thus, the problem of wave propagation can be solved by building characteristics, along which material particles are moving. Fig. 1,2 shows how Euler equation is solved by using characteristics.

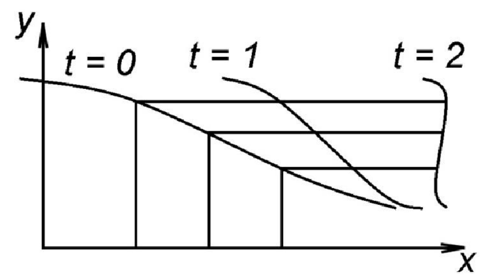

Figure 1. Solution of Euler equation using characteristics

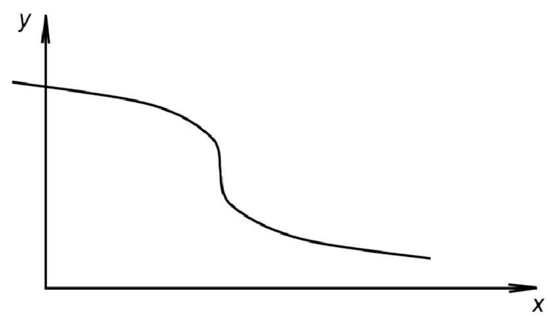

Figure 2. Integral surface y-x ceases to be a function y(x)

Initial function $y=u_0(x)_{t=0}$ is given on y-x plane. Equations of characteristics t'=1, y'=0, x'=y. At points of time t=1, t=2, etc., the solution is built by moving of the values at the initial point of time along the characteristics. Integral surface is non-uniquely projected onto x-t plane (Fig. 2). Mapping of y(x) ceases to be a graph of a function, i.e., there are x values of that have multiple corresponding y values. The curve of critical values of projection (tangent to the surface is vertical) has a reversal point (Fig. 3).

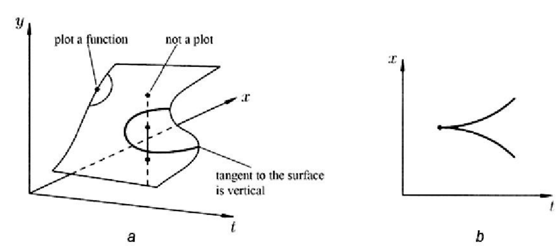

Figure 3. GDD – gas-dynamic discontinuity – the singularity the projection of the manifold dynamic parameters

Violation of uniqueness of the solution can be interpreted as passing of free flow of particles through each other. On the other hand, at higher density of particles, their interaction cannot be neglected. In this case, Euler equation is replaced by Burgers equation which takes into account the interaction of gas particles in a shock wave:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \varepsilon \frac{\partial^2 y}{\partial x^2}$$

For small ε it approximates Euler equation in the areas of smooth change of parameters. To the right and left of the shock wave, the flow is described by Euler equations, in the shock wave (gas-dynamic discontinuity) - by the equation like heat equation.

Thus, shock wave or gas-dynamic discontinuity (GDD) is a feature of representation of gas-dynamic parameters variety design (Fig. 3), their interaction forms shock wave structures (SWS).

In considered one-dimensional case, shock wave can be formed by the piston moving in the tube. If the piston velocity is above a certain critical value, compression waves propagating from its surface overtake each other and uniqueness of solutions is broken. There is a discontinuity of values. Before the wave, pressure and other parameters remain the same as in undisturbed medium, and immediately after the discontinuity the pressure increases abruptly. Shock wave can also occur as a result of a chemical reaction of combustion, in which case it is called a detonation wave. Suppose that in a preliminarily prepared fuel mixture in some way (by a spark, heat transfer or compression) the conditions were created to initiate oxidation reaction (combustion). Fuel mixture can be detonated under certain conditions.

At detonation (rapid combustion), ignition of reacting mixture occurs as a result of compression in a leading shock wave (SW) which precedes the zone of chemical reactions. Shock wave propagates at supersonic speed. The simplest model of detonation combustion is Zeldovich-Neumann model (Fig. 4).

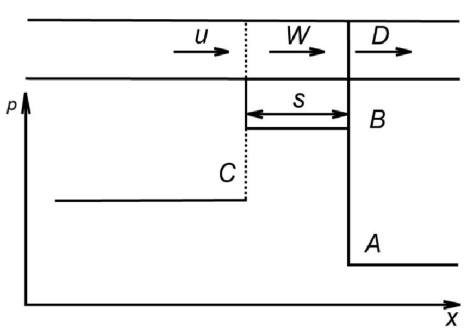

Figure 4. Detonation model of Zeldovich-Neumann

D is detonation wave, w is induction zone, u – the products of combustion, A-B is gas-dynamic discontinuity, s is the zone of free radical formation, C is expansion wave in which chemical reactions of oxidation occur.

Formation of one-dimensional shock or detonation wave is due to the superposition of compression waves (characteristics). In two dimensions, this effect corresponds to the birth of hovering shock wave (HSW) at the point of intersection of the characteristics of one of the family. The advantages of the geometric approach to the study of gas-dynamic discontinuity is illustrated in Figure 5, which shows the results of calculation of HSWs origin coordinates in a supersonic ideal gas jet. The flow in the region between the HSW and the border of the jet, called the compressed layer, in the phase space dynamics variables forms a smooth compact non-stretchable manifold.

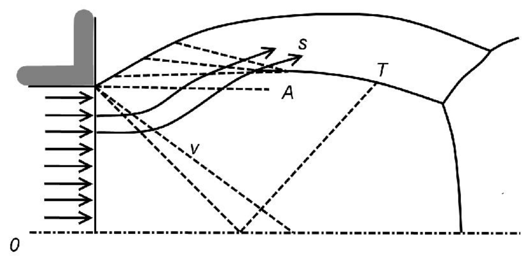

Figure 5. The flow in the vicinity of the nozzle lip

Under the Lagrangian mapping is usually understood projection surface dynamic parameters on the plane. The mapping may have peculiarities and critical points. The set of critical points is called the Lagrangian caustic display. In the space of dynamics variables corresponds to the caustic hanging shock. Caustic at the reversal point (Fig. 6) with a suitable change of variables reduces to the semi-cubical parabola [13]. Characteristics reflected from the boundary jet and intersect at the origin of the HSW (point A in Fig. 5) may be represented by second order polynomials [13].

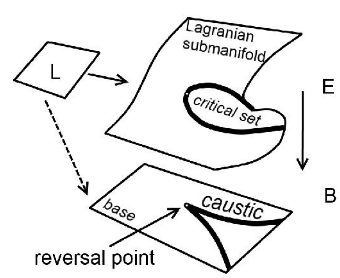

Figure 6. Hanging shock as caustic

L – Lagrangian mapping, E – manifold dynamics variables, B – plane

From the above it follows that the intensity of the HSW at its point of origin is one, and the curvature is equal to the curvature of the discontinuous characteristics of the second family. Passing through the singular point A streamline divides the current flow in the compressed layer into two different areas. Above the streamline S – isentropic flow, lower – the vortex flow. At point A, originates tangential discontinuity of the second order.

Conclusion

The concept of supersonic flows in terms of symplectic geometry gives the researcher an important tool allowing analyzing both compression shocks and shock and detonation waves. This allows building complex configurations of shock and detonation waves according to certain selected criterion of optimality.

In the space of gas-dynamic parameters any supersonic flow forms a multi-dimensional manifold. Projection of the manifold of the physical plane may have singularities such as caustics. In the space of physical variables and coordinate caustics correspond to gas-dynamic discontinuities. Caustics can have critical points. In the one case, it corresponds to the time-dependent formation of a shock wave that can be created by the movement of plunger or increase in pressure in the area of chemical reactions. In the space of higher dimension of the critical point of the caustic corresponds to the point of origin of hanging shocks. The interaction of gasdynamic discontinuities forms a shock wave structures, which are geometrical sense exhaustive classification. The geometric representation of the shock-wave structures provides a convenient mathematical tool that allows you to explore the dynamics of the shock waves.

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