Structural Stability of Flat Electroencephalography

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Abstract: Flat Electroencephalography is a way of viewing electroencephalography signals on the first component of Fuzzy Topograhic Topological Mapping (FTTM), a model which was designed to solve neuromagnetic inverse problem. This novel method is well known for its ability to preserves the orientation and magnitude of EEG sensors and signals. However, this preservation renders Flat EEG to contain unwanted signals captured during recording from the surroundings. Consequently, its accuracy in depicting actual electrical activity inside the brain is affected. Present of artifacts would pose a serious problem if it is large. Thusly, this study will investigate the persistence of Flat EEG to surrounding "noises" from dynamic viewpoint by means of structural stability. Basically, it will be showed that Flat EEG in the presence of "noises" may still reflects the actual electrical activity inside the brain, if the contaminated Flat EEG falls within a class of dynamical systems.
[Tahir A. and Tan LK. **Structural Stability of Flat Electroencephalography.** *Life Sci J* 2014;11(8):165-170]. (ISSN:1097-8135). http://www.lifesciencesite.com. 21

Keywords: Flat Electroencephalography, FTTM, Persistence, Artifacts, Dynamic, Structural Stability, Diffeomorphisms

1. Introduction

Epilepsy is a general term referring to a type of brain disorder that is characterized by seizures. Formally, it is defined by the Commission on Epidemiology and Prognosis and International League Against Epilepsy 1993 as "the occurrence of at least two unprovoked seizure" (Panayiotopoulos, 2010), that is, it usually occur more than once and is not prompted. This brain disorder is not contagious i.e., cannot be transmit or spread from one person to another. However, it can happen to anyone in the world at any age and regardless of gender. Statistically, the number of people with epilepsy in the world is at least 50 millions in total which is roughly 1.5% of the world populations (Jahnecke *et al.*, 2007).

During epilepsy, a miniature brainstorm take place within human brain by a group of brain cells called neurons. The electrical potential produce by these neurons are recordable non-invasively via electroencephalogram. In other words, recorded electrical potentials on EEG are the reflection of neuronal activity inside the brain (Bergey and Franaszczuk, 2001). This harmless and painless electrophysiological process are referred as electroencephalography and recorded signals are often portrayed on an electroencephalograph (EEG signal) for further analysis.

EEG is used extensively to diagnose epilepsies, classify the type of seizure occurring and locate the source of electrical activity (Sanei and Chambers, 2007) (Figure 1). According to (Popp and Deshaies, 2007; Yudofsky and Hales, 2008; Gilhus *et al.*, 2011), this is one of the most important laboratory tests in identifying epilepsies. Perhaps the best reason for its wide acceptance is that EEG allows neurologists to analyze and locate damaged brain tissue and also to make planning prior to surgery to avoid or lessen the risk of injury on important parts of the brain. Recently, obtaining the graphic electrical activity inside the brain has in general become a necessary part of surgical (Miller and Cole, 2011).

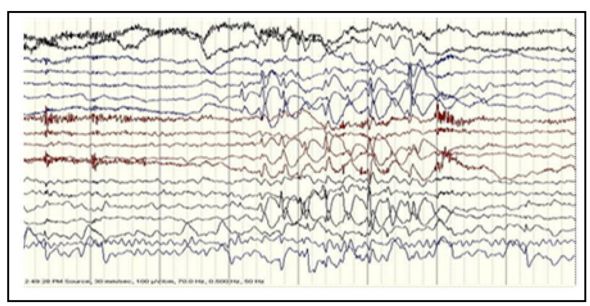
Figure 1. EEG signal

2. Literature Review

Fuzzy Topographic Topological Mapping (FTTM) is a fuzzy and topological based model for solving neuromagnetic inverse problem (Figure 2). Consisting of four components i.e., Magnetic Contour Plane (MC), Base Magnetic Plane (BM), Fuzzy Magnetic Field (FM) and Topographic Magnetic Field (TM), each of these components are homeomorphic to one another (Liau, 2001). For a recorded data, the model is capable of portraying current sources topographically in three dimensions space. The advantage of this method is that, it does not need priori information and it is not time consuming (Tahir *et al.*, 2005).

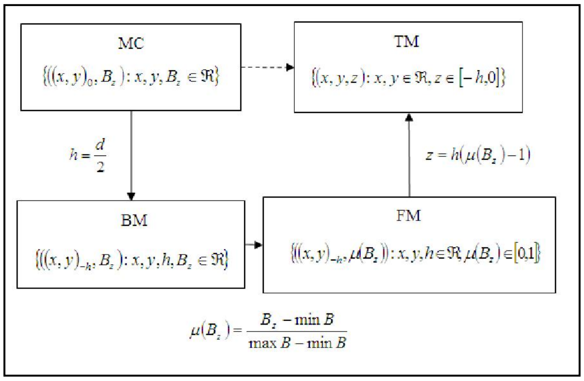

Figure 2. FTTM

Furthermore, Flat electroencephalography (Flat EEG) (Figure 3) is a way of viewing EEG signals on the first component of Fuzzy Topographic Topological Mapping (FTTM). Thus, theoretically, by FTTM model, EEG signals can be portrayed in 3-dimension space. Built in (Fauziah, 2008) this method consists of a flattening procedure (a stereographic projection) which serves as the transformation from EEG to MC (Figure 4). The main scientific value of this method lies in its ability to preserve the orientation and magnitude of EEG signals to MC, allowing it to be compressed and analyzed.

Epileptic seizure and Flat EEG was respectively modelled as dynamical system in (Fauziah, 2008) and (Tahir and Tan, 2010). Generally, a dynamical system is a system whose temporal evolution from some initial state is dictated by a set of rules (Eduard *et al.*, 1999). Another way to understand this is, it consists of a set of variables that describe its state and a law that describe the evolution of the state variables with time, i.e., how the state of the system in the next moment of time depends on the input and its state in the previous moment of time (Eugene, 2007).

In the theory of dynamical system, stability is one of the basic issues. There are two types of stability, local stability and global stability. Lyapunov is an example of local stability while structural stability is an example of global stability. Structural stable dynamical systems were introduced by Andronov and Pontryagin (Morris, 1990). By saying a dynamical system to be structurally stable, it means that small perturbation on the system does not change the global geometric structure.

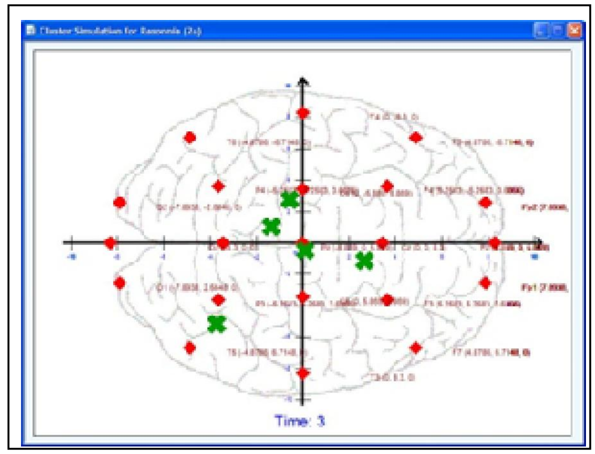

Figure 3. A random Flat EEG

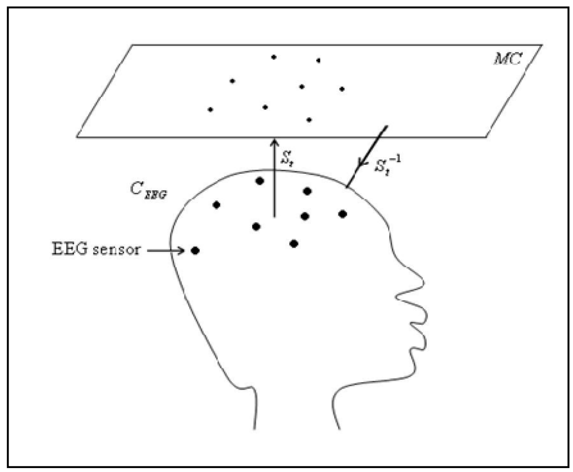

Figure 4. Stereographic projection

EEG signal which provides a way to view electrical activity inside human brain are often contaminated with unwanted signals (or "noises"). Since the transformation from EEG to MC is conformal i.e., preserves magnitude and orientation it renders. Flat EEG may contains unwanted signals too (see Figure 4). These disturbances are known formally as artifacts or noises. They are unavoidable, because they are everywhere. For examples, during the process of recording, appliances cables and amplifier tools emits tiny electric and magnetic fields. Besides, the blinking of eyes or movement of body also interfere the recording process. These artifacts are captured unintentionally during recording process, which reduce the accuracy of Flat EEG in representing the actual electrical activity within the brain (Figure 5).

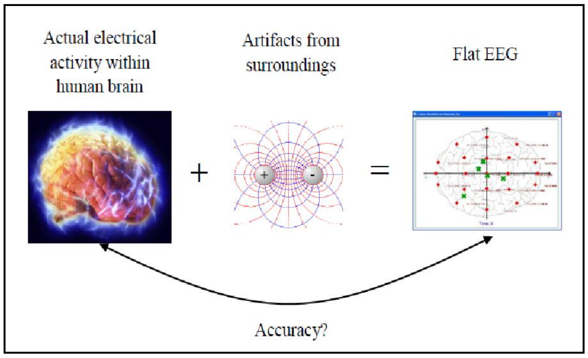

Figure 5. Existence of artifacts within Flat EEG.

However, in some sense and depending on the degree of contamination, Flat EEG may still be a reliable platform in representing electrical activity inside the brain. Knowing that dynamics are embedded within Flat EEG, it would be appealing to know the type or class of perturbations ("noises" from surroundings) that does not change the dynamic structure of Flat EEG. That is, it would be interesting to discover to what extent these artifacts would still retains the global dynamic structure of the actual electrical activity inside the brain. Consequently, the persistence of Flat EEG to surrounding artifacts (or more appropriately, perturbations) from dynamic viewpoint will be investigated by means of structural stability.

3. Material and Methods

Checking whether a system is structurally stable is equivalent to asking whether the system persist to small perturbation. Essentially, small perturbation of a structurally stable system retains its global dynamic structure. In terms of dynamic equivalence, those perturbed systems topologically conjugate to the structurally stable system. Thusly, indicates that the notion of topological conjugacy (see Figure 6) is necessary in addressing issue which pertaining to structural stability.

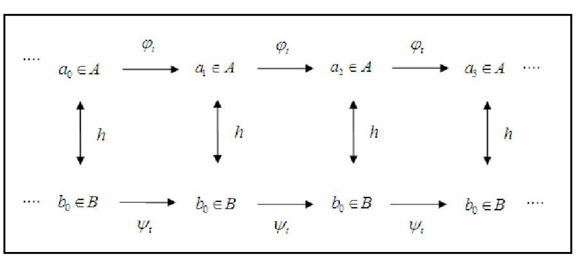

Figure 6. Pictorial representation of topological conjugacy.

At present, no flow describing Flat EEG can be written explicitly. As such, to speak of topological conjugacy, we are obliged to rely upon on a theorem from (Tahir and Tan, 2011) which allows conjugacy to be spoken even if the function cannot be written explicitly. However, the space of flows for consideration must be reduced to suits the requirements of the theorem. Consequently, space of flows considered will only be those with no periodic trajectory. Basically, the flow of Flat EEG modelled in (Tahir and Tan, 2010) can be generally define as $\varphi_t(x)$ where, $\varphi : \Re \times X \to X$ such that the following two properties are fulfilled:

i. $\varphi_0(x) = x \quad \forall x \in X = \Re^n$, and

ii. for all t and $s \in \Re$

$$\varphi_t \circ \varphi_s = \varphi_{t+s}$$

Here, for any $x_k \in X = \Re^n$, $\varphi_t(x)$ is generally defined as $\varphi_i(x_k) = x_i$ i.e., the state of the system which initiate from x_k at time i is x_i.

According to (Stephen, 2000), a flow is C^r-diffeomorphism. Therefore, Flat EEG's flow can be viewed as an element belonging to the set $C^r(\Re^n, \Re^n)$, or more specifically, in the set $Diff^r(\Re^n)$. Here, $Diff^r(\Re^n)$ and $C^r(\Re^n, \Re^n)$ respectively denotes the set of all C^r functions mapping from $\Re^n$ to $\Re^n$ and set of all C^r-diffeomorphism mapping from $\Re^n$ to $\Re^n$. For the class of dynamical systems discussed in (Tahir and Tan, 2011), we denote it as $Diff_\nabla^r(\Re^n)$. Thus, flows that are considered in this article will be the set $Diff_\nabla^r(\Re^n)$, such that

$Diff_\nabla^r(\Re^n) \subset Diff^r(\Re^n) \subset C^r(\Re^n, \Re^n)$ (see Figure 7 and Figure 8).

Formally, the definition of structural stability is as follow

Definition 1 (Artur, 1979): Let M be a C^∞-manifold without boundary and $Diff^r(M)$ be the space of C^r-diffeomorphisms of M with the C^r-topology, $r \geq 1$. A diffeomorphism $f \in Diff^r(M)$ is C^r-structurally stable if there exists a neighborhood U of f in $Diff^r(M)$, such that if $g \in U$ there exist a homeomorphism h of M satisfying $fh = hg$.

Clearly, the state space of the dynamical system of Flat EEG is a C^∞-manifold (by the atlas $\{(\Re^n, \mathrm{I}_d)\}$ i.e., atlas with single chart such that I_d be the identity mapping). This atlas is certainly valid as $\Re^n$ itself is an smooth manifold (Knapp, 2005). It is also boundaryless. Boundary points of a set are points obtained subtracting the interior of the set from its closure (Davis, 2005). Since both the interior and closure of the set $\Re^n$ with disjoint union topology is $\Re^n$ itself, thus the subtraction will results in an empty set, implying that $\Re^n$ has no boundary points, that is, boundaryless.

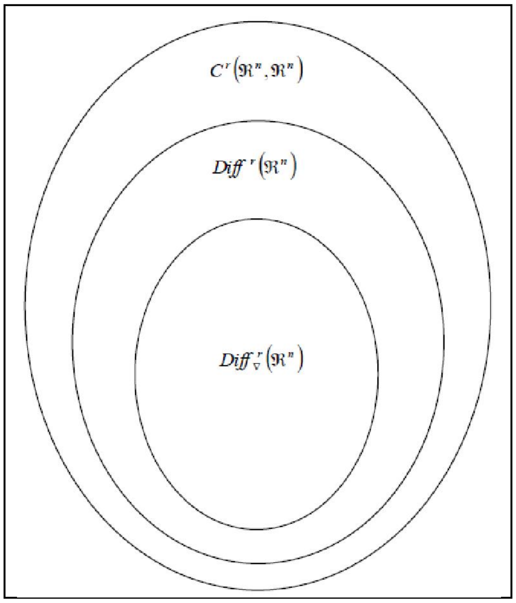

Figure 7. Space of flows for consideration.

Notion of topology will be used to give prescription on deciding the "closeness" between any two flows.

A C^r topology is a topology generated by C^r measure which formally is defined as

Definition 2 (Stephen, 2000): Two elements of $C^r(\Re^n, \Re^n)$ are said to be C^r ε-close $(k \le r)$, or just C^k close if they, along with their first k derivatives, are within ε as measure in some norm.

Basically, this measure defines two flows to be C^k ε-close if the pointwise distance of their value and corresponding derivatives value for all points up to the first k order (where $k \le r$) is ε-close when measured using some norm (Figure 9). With this topology, $Diff^r_\nabla(\Re^n)$ is a topological space. From this point onwards, $Diff^r_\nabla(\Re^n)$ with the C^r topology will be denoted as $(Diff^r_\nabla(\Re^n), \tau_{C^r})$.

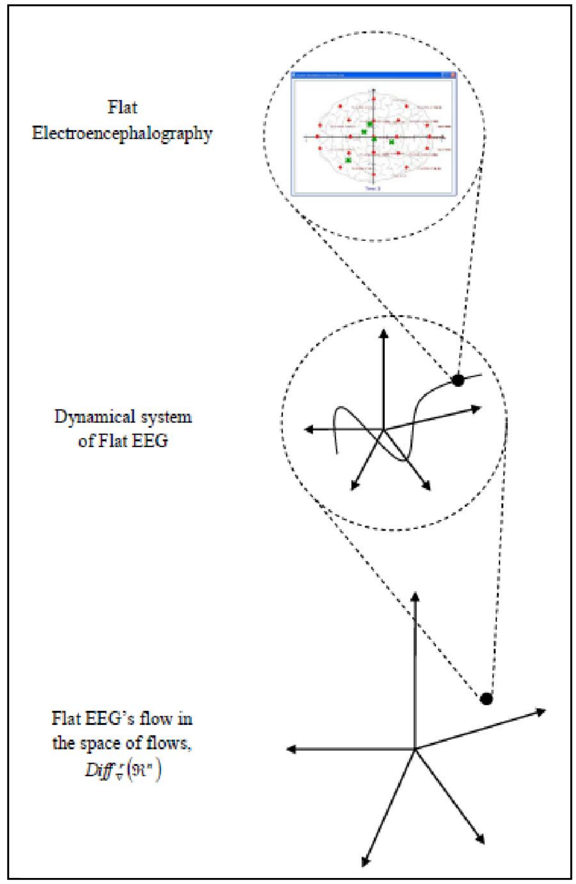

Figure 8. Fat EEG's flow in the space of flows.

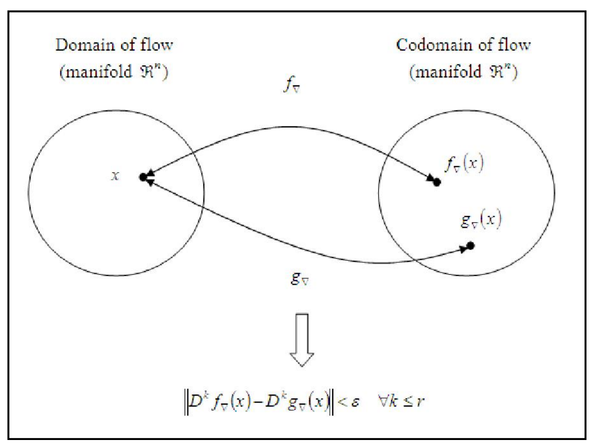

Figure 9. C^r measure between two flows.

Now, suppose that Flat EEG's flow is denoted as f, then it remains to find a neighborhood containing f, such that all flows contained in the neighborhood topologically conjugate to f. Basically, any neighborhood satisfy the requirement since the space of flows considered in this article forms an equivalence class under topological conjugacy relation (Tahir and Tan, 2011). In other words, for any neighbourhood and two flows in this space, they can be conjugated topologically (Figure 10). Consequently, it leads to the following theorem

Theorem 1: Flat EEG that is modeled as dynamical system is structurally stable in the space $\left(Diff_{\nabla}^{r}\left(\Re^{n}\right), \tau_{C^{r}}\right)$.

Proof: Since $Diff_{\nabla}^{r}\left(\Re^{n}\right)$ forms an equivalence class under topological conjugacy relation [16].

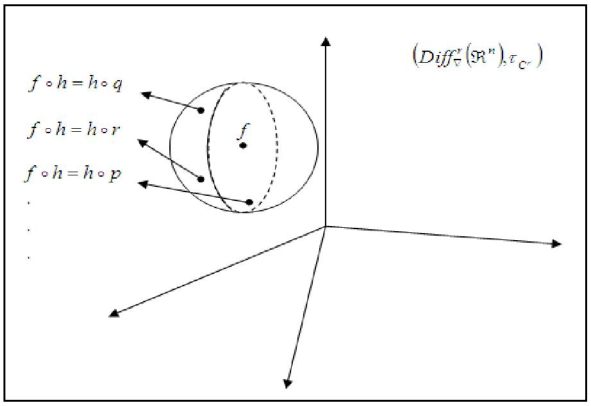

Figure 10. A structurally stable diffeomorphism f in $\left(Diff_{\nabla}^{r}\left(\Re^{n}\right), \tau_{C^{r}}\right)$.

4. Results

Using the notion of stability, this study showed that Flat EEG is structurally stable within a class of dynamical system endowed with C^{r} topology. In particular, the class of dynamical systems which has no periodic orbit, equilibrium points and has equal state space dimension with the dynamical system of epileptic seizure.

5. Discussions

With the fact that Flat EEG is structurally stable, this implies that small perturbations on the system do not change the global geometric and dynamic structure of the electrical activity inside the brain. More specifically, the referred class of perturbed systems is those that form an equivalence class with the dynamic system of Flat EEG and epileptic seizure. In the language of topological conjugacy, the dynamics of small perturbed systems are still equivalent topologically.

6. Conclusion

Although Flat EEG is frequently contaminated, from topological viewpoint, it can still be a reliable platform to represents the dynamics of electrical activity within human brain. That is, in the presence of artifacts, Flat EEG could offer significant descrptions of electrical activities in the brain during seizure. Hence, localizing epileptic foci via Flat EEG is reliable.

Acknowledgements:

The authors would like to thank their family members for their continuous support. Authors also appreciate the financial support receive from grant Vot. RDU 141307.

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4/26/2014