A note on s-topological groups

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Abstract: In this paper, properties of s-topological groups are investigated. s-regularity and s-compactness have been studied for s-topological groups. Relation between topologized groups has been established. Counter examples are given to show that reverse implications are not true in general.

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1. Introduction

To study the properties of topologized groups by weakening the continuity conditions in the sense of N. Levine is the basic interest of the article. Semi continuity [9] and irresolute mapping [6] were the consequence of the study of semi open sets in topological spaces. In [2] Bohn defined and investigated the notion of s-topological groups and in [15] Siddique et al. defined the notion of S-topological groups. In [14] Siab et al. defined and studied the notions of irresolute topological groups and Irr-topological groups by using irresolute mappings [6]. In this paper we will continue the study of the properties of s-topological groups. Investigations on s-compactness and s-regularity are presented on s-topological groups and irresolute topological groups. Relations between S-topological groups, s-topological groups, irresolute topological groups and Irr-topological groups are investigated. Counter examples are given to show that the reverse implications are not true in general.

2. Definitions and Preliminaries

Throughout this paper X and Y are always topological spaces on which no separation axioms are assumed. For a subset A of a space X the symbols $Int(A)$ and $Cl(A)$ are used to denote the interior of A and the closure of A respectively. If $f : X \to Y$ is a mapping between topological spaces X and Y and B is a subset of Y, then $f^{-1}(B)$ denotes the pre image of B. Our other topological notations and terminology are standard as in [7]. If $(G,*)$ is a group, then e denotes its identity element, and for a given $x \in G$, $\ell_x : G \to G, y \mapsto x * y$, and $r_x : G \to G$, $y \mapsto y * x$, denote the left and the right translations by x, respectively. The operation $*$ we call the multiplication mapping $m: G \times G \to G$, and the inverse operation $x \mapsto x^{-1}$ is denoted by i.

In 1963, N. Levine [9] defined semi open sets in topological spaces. Since then many mathematicians explored different concepts and generalized them by using semi open sets (see [1, 6, 8, 12, 13]). A subset A of a topological space X is said to be semi open if there exists an open set U in X such that $U \subset A \subset Cl(U)$, or equivalently if $A \subset Cl(Int(A))$. $SO(X)$ denotes the collection of all semi open sets in X. The complement of a semi open set is said to be semi closed; the semi closure of $A \subset X$, denoted by $sCl(A)$, is the intersection of all semi closed subsets of X containing A [4,5]. $x \in sCl(A)$ if and only if for any semi open set U containing $x, U \cap A \neq \emptyset$.

Clearly, every open (closed) set is semi open (semi closed). It is known that the union of any collection of semi open sets is again a semi open set, while the intersection of two semi open sets need not be semi open. The intersection of an open set and a semi open set is semi open. If $A \subset X$ and $B \subset Y$ are semi open in spaces X and Y, then $A \times B$ is semi open in the product space $X \times Y$. Basic properties of semi open sets are given in [9], and of semi closed sets and the semi closure in [4, 5].

Recall that a set $U \subset X$ is a semi open neighbourhood of a point $x \in X$ if there exists $A \in SO(X)$ such that $x \in A \subset U$. A set $A \subset X$ is semi open in X if and only if A is a semi open neighbourhood of each of its points. If a semi open neighbourhood U of a point x is a semi open set we say that U is a semi open neighbourhood of x. Let (G, τ) be a topological space and $A \subset G$. Then $x \in G$ is called a semi interior point of A if there exists a semi open set U such that $x \in U \subset A$. The set of all semi interior points of A is called a semi interior of A, denoted by $sInt(A)$. A topological space (G,τ) is semi-T_2 [10] if for every distinct points x,y in G, there are semi open neighbourhoods U containing x and V containing y such that $U \cap V = \varphi$. A topological space (G,τ) is s-regular [11] if for any $x \in G$, and any closed set $F \subset G$ such that $x \notin F$, there is a semi open neighbourhood U of x and a semi open neighbourhood V containing F such that $U \cap V = \varphi$. A space G is s-compact [3], if every semi open cover of G has a finite subcover. Every compact space is s-compact but converse is not always true. For some applications of semi open sets see [12].

A mapping $f: X \to Y$ between topological spaces X and Y is called:

(1) semi continuous (resp. irresolute) if for each open (resp. semi open) set $V \subset Y$ the set $f^{-1}(V)$ is semi open in X. Equivalently, the mapping f is semi continuous (irresolute) if for each $x \in X$ and for each open (semi open) neighbourhood V of $f(x)$, there exists a semi open neighbourhood U of x such that $f(U) \subset V$. Every irresolute mapping is semi continuous;

(2) pre semi open [6] if for every semi open set A of X, $f(A)$ is semi open in Y;

(3) semi homeomorphism [6] if f is bijective, irresolute and pre semi open;

(4) S-homeomorphism [15] if f is bijective, semi continuous and pre semi open. Every semi homeomorphism is an *S-homeomorphism.*

3. Properties of topologized groups.

Definition 3.1. [15] A triple $(G,*,\tau)$ is said to be an S-topological group if $(G,*)$ is a group, (G,τ) is a topological space, and (a) the multiplication mapping $m: G \times G \to G$ defined by $m(x,y) = x * y$, for every $x,y \in G$, is semi continuous, (b) the inverse mapping $i: G \to G$ defined by $i(x) = x^{-1}$, for every $x \in G$, is semi continuous.

Definition 3.2. [2] An s-topological group is a group $(G,*)$ with a topology τ such that for each $x,y \in G$ and each neighbourhood W of $x * y^{-1}$ there are semi open neighbourhoods U of x and V of y such that $U * V^{-1} \subset W$.

Definition 3.3. [14] A triple $(G,*,\tau)$ is said to be an Irr-topological group if $(G,*)$ is a group, (G,τ) is a topological space, and (a) the multiplication mapping $m: G \times G \to G$ defined by $m(x,y) = x * y$, for every $x,y \in G$, is irresolute, (b) the inverse mapping $i: G \to G$ defined by $i(x) = x^{-1}$, for every $x \in G$, is irresolute.

Definition 3.4. [14] A triple $(G,*,\tau)$ is an irresolute-topological group with a group $(G,*)$ and a topology τ such that for each $x,y \in G$ and each semi open neighbourhood W of $x * y^{-1}$, there are semi-open neighbourhoods U of x and V of y such that $U * V^{-1} \subset W$.

Lemma 3.5. [2] If $(G,*,\tau)$ is an s-topological group, then
(1) $A \in SO(G)$ if and only if $A^{-1} \in SO(G)$;
(2) If $A \in SO(G)$ and $B \subset G$, then $A * B$ and $B * A$ are both semi open in G.

Lemma3.6. [14] If $(G,*,\tau)$ is an irresolute topological group, then
(1) $A \in SO(G)$ if and only if $A^{-1} \in SO(G)$;
(2) If $A \in SO(G)$ and $B \subset G$, then $A * B$ and $B * A$ are both semi open in G.

Lemma 3.7. [14] Every irresolute topological group is an Irr-topological group.

Lemma 3.8. [15] Every s-topological group is an S-topological group.

Lemma 3.9. [15] Each left (right) translation of s-topological groups is S-homeomorphism.

Lemma 3.10. Each left (right) translation of irresolute topological groups is semi homeomorphism.

Proof. Let $(G,*,\tau)$ be an irresolute topological group. We prove the statement only for left translation. Of course, left translation is bijective mapping. We prove directly that for any $x \in G$ the left translation $\ell_x : G \to G$ is irresolute. Let y be an arbitrary element in G and let W be a semi open neighbourhood of $\ell_x(y) = x * y = x * (y^{-1})^{-1}$. By definition of irresolute topological group, there are semi open neighbourhoods U and V containing x and y^{-1}, respectively, such that $U * V^{-1} \subset W$. In particular, we have $x * V^{-1} \subset W$. By Lemma 3.6, the set V^{-1} is a semi open neighbourhood of y, so that the last inclusion actually says that ℓ_x is irresolute at y. Since $y \in G$ was an arbitrary element in G, therefore, ℓ_x is irresolute on G. We prove now that ℓ_x is pre semi open mapping. Let A be a semi open set in G. Then by Lemma 3.6, the set $\ell_x(A) = x * A$ is semi open in G, which means that ℓ_x is a pre semi open mapping. Thus left translation is semi homeomorphism. Similarly we can prove that the right translation is semi homeomorphism.

Theorem 3.11. Let $(G,*,\tau)$ be an s-topological group and let β_e be the base at identity element e of G. Then:
(i) for every $U \in \beta_e$, there is an element $V \in SO(G,e)$ such that $V^2 \subset U$.
(ii) for every $U \in \beta_e$, there is an element $V \in SO(G,e)$ such that $V^{-1} \subset U$.
(iii) for every $U \in \beta_e$, there is an element $V \in SO(G,e)$ such that $V * x \subset U$, for each $x \in U$.

Proof (i). Let $U \in \beta_e$. This implies that $e \in U \subset G$ and $U_{e*e^{-1}} = U$. Since $(G,*,\tau)$ is an s-topological group therefore, there exists $V \in SO(G,e)$ and by Lemma 3.5,

$V^{-1} \in SO(G,e)$ such that $V * V \subset U$. Therefore, $V^2 \subset U$.
(ii) Since $(G,*,\tau)$ is an s-topological group. Therefore, for every $U \in \beta_e$, there exists $V \in SO(G,e)$ such that $i(V) = V^{-1} \subset U$ because the identity function $i: G \to G$ is semi continuous by Theorem 7 [2].
(iii) Since $(G,*,\tau)$ is an s-topological group, therefore by Lemma 3.9, $\ell_x : G \to G$ and $r_x : G \to G$ are S-homeomorphisms. Thus for each $U \in \beta_e$ containing x, there exists $V \in SO(G,e)$ such that $r_x(V) = V * x \subset U$.
The following example shows that Theorem 3.11 (i) is not true for S-topological groups.

Example 3.12. $G = \{0,1,2\}$ is a group under addition modulo 3, and $\tau = \{\varphi, G, \{0\}, \{0,1\}\}$ is a topology on G. $SO(G) = \{\varphi, G, \{0\}, \{0,1\}, \{0,2\}\}$. Then $(G, +_3, \tau)$ is an S-topological group. For any $U \in \beta_e$ (base at the identity element e other than $\{e\}$ and G), we can not find any semi open set V containing e, satisfying $V^2 \subset U$.

Lemma 3.13. If $(G,*,\tau)$ is an irresolute topological group, then the inverse mapping $i: G \to G$ defined by $i(x) = x^{-1}$ for all $x \in G$ is a semi homeomorphism.

Theorem 3.14. Let $(G,*,\tau)$ be an irresolute topological group and μ_e be the collection of all semi open neighbourhoods of the identity element e of G. Then:
(i) For every $U \in \mu_e$, there is an element $V \in \mu_e$ such that $V^2 \subset U$.
(ii) For every $U \in \mu_e$, there is an element $V \in \mu_e$ such that $V^{-1} \subset U$.
(iii) For every $U \in \mu_e$, there is an element $V \in \mu_e$ such that $V * x \subset U$, for each $x \in U$.
(iv) For every U$\in \mu_e$, and $x \in G$, there is an element $V \in \mu_e$ such that $x * V * x^{-1} \subset U$.

Proof (i). Let $U \in \mu_e$ This implies that $e \in U \subset G$ and $U_{e*e^{-1}} = U$. Since $(G,*,\tau)$ is an irresolute topological group therefore, there exist $V, V^{-1} \in \mu_e$ such that $V * V \subset U$. Therefore, $V^2 \subset U$.
(ii). Since $(G,*,\tau)$ is an irresolute topological group. Therefore, for every $U \in \mu_e$, there exists $V \in SO(G,e)$ such that $i(V) = V^{-1} \subset U$ because the identity function $i: G \to G$ is irresolute by Lemma 3.13.
(iii). Since $(G,*,\tau)$ is an irresolute topological group and, therefore by Lemma 3.10, ℓ_x:G→G and $r_x : G \to G$ are semi homeomorphisms. Thus for each $U \in \mu_e$ containing x, there exists $V \in SO(G,e)$ such that $r_x(V) = V * x \subset U$.
(iv). It follows from the fact that ℓ_x and $r_{x^{-1}}$ are semi homeomorphisms of G and $\ell_x(e) = x$, so that there exists $V \in \mu_e$ such that $\ell_x(V) = x * V$, and $x * V$ is a semi open neighbourhood of x, and $r_{x^{-1}}(x * V) = x * V * x^{-1} \subset U$.

Theorem 3.15. If $(G,*,\tau)$ is an irresolute topological group, then it is an s-topological group.

Proof. Let $x, y \in G$ and let $W \subset G$ be an open neighbourhood of $x * y^{-1}$. Then $W \subset G$ is a semi open neighbourhood of $x * y^{-1}$. Since $(G,*,\tau)$ is an irresolute-topological group, there are semi open neighbourhoods $U \subset G$ of x and $V \subset G$ of y such that $U * V^{-1} \subset W$. This implies that $(G,*,\tau)$ is an s-topological group.

Theorem 3.16. If $(G,*,\tau)$ is an Irr-topological group, then it is an S-topological group.

Proof. Since irresolute mapping is a semi continuous mapping, therefore an Irr-topological group $(G,*,\tau)$ is an S-topological group.

Remark 3.17:

topological group	⇒	s-topological group
⇓		⇓
Irr-topological group	⇒	S-topological group
⇑		⇑
Irresolute topological group	⇒	s-topological group

The following example shows that S-topological group may not be an s-topological group.

Example 3.18. Let $G = \{-1,1\}$ be a group under multiplication and $\tau = \{\varphi, \{1\}, G\}$ a topology on G. In [15], it is shown that $(G,*,\tau)$ is an S-topological group but not a topological group. Here $(G,*,\tau)$ is not an s-topological group because the only semi open neighbourhood of -1 is G and if we choose $\{1\}$ for $x = y = -1$, as an open neighbourhood of $x * y^{-1}$. Now, the only possibility for the selection of semi open neighbourhoods in G containing -1,-1 is G, G but, then $G * G \nsubseteq \{1\}$. This shows that $(G,*,\tau)$ is not an s-topological group
The following example shows that $(G,*,\tau)$ is an S-topological group but non of the followings: s-topological group, irresolute topological group and topological group.

Example 3.19. Suppose $G = \{0,1,2\}$ is a group under addition modulo 3 and topology on G is $\tau = \{\varphi, G, \{0\}, \{0,1\}\}$. Then $(G, +_3, \tau)$ is an S-topological group but not an s-topological group, therefore not an irresolute topological group. $(G, +_3, \tau)$ is not a topological group.The following example shows that Irr-topological group may neither be irresolute topological group nor topological group.

Example 3.20. $G = \{1, \omega, \omega^2\}$ is a group under multiplication, where ω is a complex cube root of unity and topology on G is $\tau = \{\varphi, \{1\}, G\}$. Then $(G,*,\tau)$ is an Irr-topological group but not an irresolute topological group, and $(G,*,\tau)$ is not a topological group.

Theorem 3.21. Let V be a semi open neighbourhood of e in s-topological group $(G,*,\tau)$. Then $V \subset sCl(V) \subset V^2$.

Proof. Note that $s \in sCl(V)$ if and only if every semi open neighbourhood of s meets V. Since $s * V^{-1}$ is a semi open neighbourhood of s, it must meet V. Thus there is $t \in V$ of the form $s * v^{-1}$, where $v \in V$. But then $s = t * v \in V * V = V^2$ and $sCl(V) \subset V^2$. Thus $V \subset sCl(V) \subset V^2$.

Theorem 3.22. Let $(G,*,\tau)$ be an s-topological group. Then for every subset A of G and every open neighbourhood U of the neutral element e, $sCl(A) \subset A * U$.

Proof. Since $(G,*,\tau)$ is an s-topological group. Then for every open neighbourhood U of e, by Theorem 3.11, there exists $V \in SO(G,e)$ such that $V^{-1} \subset U$. Let $x \in sCl(A)$ and $x * V$ is a semi open neighbourhood of x. Therefore, there is $a \in A \cap x * V$, that is $a \in x * V$. This implies that $a = x * b$ for some $b \in V$. Then $x = a * b^{-1} \in a * V^{-1} \subset A * U$. Hence $sCl(A) \subset A * U$.

Corollary 3.23. Let $(G,*,\tau)$ be an irresolute topological group. Then for every subset A of G and every open neighbourhood U of the neutral element e, $sCl(A) \subset A * U$.

Theorem 3.24. Let $(G,*,\tau)$ be an s-topological group and β_e a base of the space (G,τ) at the neutral element e. Then for every subset A of G, $sCl(A) = \cap \{A * U : U \in \beta_e\}$.

Proof. In view of Theorem 3.22, we only have to verify that if $x \notin sCl(A)$, then there exists $U \in \beta_e$ such that $x \notin A * U$. Since $x \notin A$, then by definition there exists a semi open neighbourhood W of e such that $x * W \cap A = \varphi$. Take U in β_e satisfying the condition $U^{-1} \subset W$. Then $x * U^{-1} \cap A = \varphi$, that is $\{x\} \cap A * U = \varphi$. This implies that $x \notin A * U$.

Theorem 3.25. If $(G,*,\tau)$ is an s-topological group, then (G,τ) is s-regular and semi-T_2 space.

Proof. Suppose that $F \subset G$ is closed and $s \notin F$. Multiply by s^{-1} allows us to assume that $s = e$. Since F is closed, $W = G - F$ is an open neighbourhood of e. By Theorem 3.11, there exists $V \in SO(G,e)$ such that $V^2 \subset W$. Thus by Theorem 3.22, $sCl(V) \subset W$. Then $U = G - sCl(V)$ is a semi open neighbourhood containing F which is disjoint from V. This proves that $(G,*,\tau)$ is s-regular. That is, $e \in V \in SO(G)$ and $e \neq y \in F \subset U \in SO(G)$ such that $V \cap U = \varphi$. Hence G is semi-T_2 space.

Theorem 3.26. Let $(G,*,\tau)$ be an irresolute topological group, F a semi closed subset of G, and K is an s-compact subset of G such that $F \cap K = \varphi$. Then there is semi open neighbourhood V of e such that $F \cap V * K = \varphi$ (and a semi open neighbourhood $V'$of e such that $F \cap K * V' = \varphi$).

Proof. Let $x \in K$, so $x \in G - F$, and $G - F = F^c$ is a semi open neighbourhood of x. Therefore, $F^c * x^{-1}$ is a semi open neighbourhood of e. By Theorem 3.14, there is a semi open neighbourhood W_x of e such that $W_x * W_x \subset F^c * x^{-1}$. Now $K \subset \cup_{x \in K} W_x * x$. Since K is s-compact so $K \subset \cup_{i=1}^{n} W_{x_i} * x_i = \cup_{i=1}^{n} W_i * x_i$. Take $V = \cap_{i=1}^{n} W_i$. For $x \in K$, $x \in W_i * x_i$ for some i. By construction of V, we have $V \subset W_i$ for each i. This implies $V * x \subset W_i * x \subset W_i * W_i * x_i \subset F^c$. In other words, $F \cap V * x = \varphi$. Since this is true for any $x \in K$, we now have $F \cap V * K = \varphi$.

Theorem 3.27. If K is s-compact, then $y * K^{-1}$ is s-compact in an s-topological group $(G,*,\tau)$.

Proof. Let $\{U_\alpha : \alpha \in I\}$ be a cover of $y * K^{-1}$, where $U_\alpha \in SO(G)$. Then $y * K^{-1} \subset \cup_{\alpha \in I} U_\alpha$. This implies that $K^{-1} \subset y^{-1} \cup_{\alpha \in I} U_\alpha = \cup_{\alpha \in I} y^{-1} * U_\alpha$. This implies that $K \subset \cup_{\alpha \in I} y * U_{\alpha^{-1}}$. Since K is s-compact, then there exists a finite set I_0 of I such that $K \subset \cup_{\alpha \in I_0} y * U_{\alpha^{-1}}$. This implies that $y^{-1} * K \subset \cup_{\alpha \in I_0} U_{\alpha^{-1}}$. This implies that $y * K^{-1} \subset \cup_{\alpha \in I_0} U_\alpha$. That is $y * K^{-1}$ has a finite subcover in G. Hence $y * K^{-1}$ is s-compact.

Theorem 3.28. Let $(G,*,\tau)$ be an s-topological group, K an s-compact subset of G, and F a semi closed subset of G. Then $F * K$ and $K * F$ are semi closed subsets of G.

Proof. If $F * K = G$ we are done, so let $y \in G - F * K$. This means $F \cap y * K^{-1} = \varphi$. Since K is s-compact so by Theorem 3.27, $y * K^{-1}$ is s-compact. By Theorem 3.26, there is a semi open neighbourhood V of e such that $F \cap V * y * K^{-1} = \varphi$. That is, $F * K \cap V * y = \varphi$. Since $V * y$ is semi open neighbourhood of y contained in $G - F * K$, we have $F * K$ is semi closed and similar arguments for the proof of $K * F$.

Theorem 3.29. A non-empty subgroup H of an s-topological group G is semi open if and only if its semi interior is non empty.

Proof. Assume that $x \in sInt(H)$. Then by definition there is a semi open set V such that $x \in V \subset H$. For every $y \in H$, we have $y * V \subset y * H = H$. Since V is semi open so is $y * V$, we conclude that $H = \cup \{y * V : y \in H\}$ is a semi open set as the union of semi open sets is semi open. Converse is very easy approach.

Theorem 3.30. If $U \in SO(G)$, then the set $L = \cup_{n=1}^{\infty} U^n$ is a semi open set in an s-topological group $(G,*,\tau)$.

Proof. Since U is semi open in an s-topological group $(G,*,\tau)$. Then by Lemma 3.5, $U * U = U^2 \in SO(G)$. $U^2 * U = U^3 \in SO(G)$ and similarly $U^4, U^5, \ldots$all are semi open sets in G. Thus the set $L = \cup_{n=1}^{\infty} U^n$ being the union of semi open sets is a semi open set.

Lemma 3.31. If $(G,*,\tau)$ is an s-topological group, then the inverse map $i: G \to G$ defined by $i(x) = x^{-1}$ for all $x \in G$ is an S-homeomorphism.

Theorem 3.32. Let A be a subset of an s-topological group $(G,*,\tau)$. Then $(sInt(A))^{-1} = sInt(A^{-1})$.

Proof. By Lemma 3.31, the inverse mapping $i: G \to G$ is an S-homeomorphism, then $sInt(i(A)) = sInt(A^{-1}) = i(sInt(A)) = (sInt(A))^{-1}$.

Theorem 3.33. Let $(G,*,\tau)$ be an irresolute topological group. Then for any symmetric subset A of G, the closure of A is also symmetric in G.

Proof. By Lemma 3.13, the inverse mapping $i: G \to G$ defined by $i(x) = x^{-1}$, for each $x \in G$ is a semi homeomorphism. Hence $i(Cl(A)) = (Cl(A))^{-1} = Cl(A^{-1}) = Cl(A)$ because $A^{-1} = A$. That is, $(Cl(A))^{-1} = Cl(A)$.

Definition 3.34. Suppose U is a semi open neighbourhood of the neutral element e of an s-topological group $(G,*,\tau)$. A subset A of G is called U-semi disjoint if $b \notin a * U$, for any disjoint $a, b \in A$.

Definition 3.35. A collection τ_s of subsets of a topological space (G,τ) is semi discrete, provided each $x \in G$ has a semi open neighbourhood that intersects at most one member of τ_s.

Theorem 3.36. Let U and V be semi open neighbourhoods of the neutral element e in an s-topological group $(G,*,\tau)$ such that $V^4 \subset U$ and $V^{-1} = V$. If a subset A of G is U-semi disjoint, then the family of semi open sets $\{a * V: a \in A\}$ is semi discrete in G.

Proof. It suffices to verify that, for every $x \in G$, a semi open neighbourhood $x * V$ of x intersects at most one element of the family $\{a * V: a \in A\}$. Suppose to the contrary that, for some $x \in G$, there exists distinct elements $a, b \in A$ such that $x * V \cap a * V \neq \varphi$, and $x * V \cap b * V \neq \varphi$. Then $x^{-1} * a \in V^2$ and $b^{-1} * x \in V^2$, where $b^{-1} * a = (b^{-1} * x)(x^{-1} * a) \in V^4 \subset U$. This implies that $a \in b * U$. This contradicts the assumption that A is U-semi disjoint.

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