# Modeling and analysis of complex systems on the basis of fuzzy graph models

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**Abstract.** The using of fuzzy directed graphs for modeling and the analysis of functioning of difficult systems is offered in this paper. The basic problems inherent in modeling of complex systems are considered here. The graphs model is described from the point of view of the tool of modeling and the analysis of complex systems, for example social and economic systems. The questions of the analysis of a complex system like finding of fuzzy directed graph vitality degree in the case, when the vitality degree is understood as a degree of its strong connectedness are considered here. The example of finding vitality degree of fuzzy direct graph is considered as well.
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**Keywords:** complex system, fuzzy model, fuzzy graph, fuzzy way, accessible degree, fuzzy transitive closure, fuzzy reciprocal transitive closure, vitality degree

## Introduction

One of the most important characteristics of complex systems is their sensitivity and reliability to a various sort to revolting influences of environment. Today this characteristic is not very studied (investigated). Therefore, it is most relevant for the study in order to understand how to build a model of the system to be able to predict its development with a view to optimal control. Similar questions often rise in modern researches [1, 2, 3]. Studying of development of complex systems is one of the important modern problems more investigated at a stage of supervision over such systems. The urgency of research is caused, including such factors, as accumulation of new knowledge and data about modeling of complex systems for last 10-15 years.

## Theoretical analyses

In the course of studying of influence of external factors it is possible to allocate some accents for development of system. First, this is studying of possibility of identification of sensitivity of system to such influences, both at each level of hierarchy, and in a separate subsystem. Secondly, this is investigating the nature and character of such influences, their versions. And, in the third, this is researching of robustness of system, including ability to resist to such influences.

Property robustness of complex system can be considered as a certain average between such concepts as "reliability" and "safety". We will understand as robustness property of the complex system consisting in its ability to resist to influence of unforeseen external factors and to keep in advance set trajectory of development, ability of system to adapt for new situations, not stopping to "carry out" in full or in part the criterion function.

This property is certainly a complex integral one, which is assessed both quantitatively and qualitatively.

Complex system is a complex of various subsystems in a hierarchical view, which combines a sufficiently large number of interconnected and interacting variables of different nature.

Any complex system, in particular social and economic (SES) is the evolving object, developing under the influence of many changing factors, both internal, and external. By the nature structure SES is dynamical, and reflects evolution of system in time and space.

Classification of existing and most widespread approaches and the tools used at modeling and forecasting of functioning SES is brought in [4]. The idea of the hierarchical or so-called stratified description of system is put in a basis of modeling SES [5] and applied in [6].

Modeling on everyone stratum is offered to be conducted with use of the device of the directed graphs, which allows formalizing the expert knowledge. One of most important points of process of modeling is revealing of connections between objects, their orientation and influence.

Graphs models are most applicable to the systems, which problems ill-structured, and their basic advantage is possibility to be combined with other methods at different stages of research.

Graphs mathematical models, are intended for formalization of the description of complex object, a problem or functioning of system and revealing of relationships of cause and effect between their elements as a result of influence on these elements or changes of character of connections. These models are well suited to the following systems:

1) Which are characterized by multi aspect processes occurring in them and their coherence;

2) Which are characterized by absence of the sufficient quantitative information on dynamics of processes;

3) Which are characterized by variability of processes in time, etc.

The list of these characteristics also defines complex system.

For investigating emergent properties it is possible to use methodology of pulse modeling [7].

So, the model can be set using fuzzy directed graph. Fuzzy directed graph [8] is a pair of sets $\widetilde{G}=(X,\ \widetilde{U})$ in which $X=\{x_i\}$, $i\in I=\{1,2, ..., n\}$ is a crisp set of vertices (or concepts), and $\widetilde{U}=\{<\mu_U<x_i,x_k>/<x_i,x_k>>\}$ is a fuzzy set of edges (or arcs ), where $<x_i,x_k>\in X^2$, and $\mu_U<x_i,x_k>$ is a degree of membership oriented edge $<x_i,x_k>$ to fuzzy set of directed edges $\widetilde{U}$.

The path of fuzzy graph $\widetilde{l}(x_i,x_j)$ is called a direct sequence of fuzzy edges from vertex $x_i$ to vertex $x_j$, in which the final vertex of any edge is the first vertex of the following edge.

The conjunctive strength of the path of a fuzzy graph is defined by the formula:

$$\mu_{\widetilde{l}}(x_i,x_j)=\underset{<x_k,x_t>\in\widetilde{l}(x_i,x_j)}{\&}\mu_U(x_k,x_t).$$

Fuzzy direct graph conveniently given in the form $\widetilde{G}=(X,\ \widetilde{\Gamma})$, where $X=\{x_i\}$, $i\in I=\{1,2, ..., n\}$, and $\widetilde{\Gamma}$ - fuzzy valued mapping of the vertex set $X$ into itself, ie, $\widetilde{\Gamma}: X\rightarrow X$, which is given in the form of fuzzy images of the elements $x\in X$, ie, $\widetilde{A}(x_i)=\{<\mu_\Gamma(x_j)/x_j>\}$, $x_j\in\Gamma(x_i)$, here $\Gamma(x_i)$ is a crisp set of images of the vertices $x_i\in X$.

The vertices $X$ of graph $\widetilde{G}=(X,\ \widetilde{\Gamma})$ may be any objects of social, economic, political, ecological systems, or their combination, for example, a total national product and a financing of education, the purposes and necessary means of their achievement. The revealed connections between tops and its importance will be fuzzy arches.

Traditionally, there are two types of causality: the positive and negative in the graph model. Positive connection ("+" sign above the arc) means an increase in the value-factor causes and increases the value of the factor-investigation. Negative connection ("-" sign above the arc) means a reduction in value of the factor-causes and reduces the value of the factor-investigation. Thus, the graph model can be set using the adjacency matrix $W=\{w_{ij}\}$, $w_{ij}$ $\{-1,0,1\}$. However, the following is a more informative. In this paper we propose to transfer the accepted designation of force of connection on arches, from set: {ill connection − W, average connection − M, strong connection − S} in a value from an interval [0, 1]. For example, 0 - connection is absent, [0.1, 0.3] - connection weak, [0.4, 0.6] - connection average, [0.7, 1] - connection strong. In our opinion it much more expediently also answers the requirement fuzzy graphs.

Besides, the three for concepts {-1, 0, 1} which would designate value and importance of this or that top is offered. I.e., if the increase (or reduction) in one concept conducts to increase (or, to reduction) in other we appoint value = 1. If there is no relation between concepts, it is assigned the value equal to 0. If the increase (or reduction) in one концепте conducts to reduction (or to increase) another, it is assigned the value equal to 1.

Fuzzy graphs are widely used as models SES. The majority of isomorphic transformations of fuzzy graphs change their external representation. In this connection, definition invariants of fuzzy graphs are used. In particular, in papers [9, 10] the concepts of fuzzy bases and antibases are considered for the fuzzy graphs, and the method of their calculation is offered for the analysis of complex model as a whole, and in particular, for the decision of the research problem of sensitivity.

In paper [11] application of the device of fuzzy logic is described and possibility of use of the device of fuzzy sets is considered. The way of a finding of transitions between reference situations in the multilevel complex systems, based on comparison of fuzzy intervals is offered, the algorithm of comparison of fuzzy sets on an individual interval is in detail stated.

In paper [12] the approach using fuzzy sets for modeling of force of operating influence at different types of connections between the previous and subsequent purposes of functioning at various levels of hierarchy SES is offered. The choice of the administrative decision in complex system by means of a method of the analysis of hierarchies is always accompanied by uncertainty. The situation when the weight over an arch of the fuzzy graph is designated by a fuzzy interval is considered. For search of decisions in a fuzzy hierarchical control system where separate tops are presented by fuzzy intervals with borders on different scales, the approach which is based on definition of degree of fuzzy equality of fuzzy numbers is inapplicable. Therefore it is offered to apply the approach based on comparison of fuzzy intervals.

The estimation and the analysis concern the same problems received graphs models from the point of view of their vitality. In the crisp graphs vitality is understood as its sensitivity to damages from the point of view of removal of some edges or vertices [13]. In case of fuzzy graphs, depending on tasks in view, under vitality different concepts, including degree of strong connectivity of the fuzzy graph [14, 15] can be understood.

**Technique**

Let $\widetilde{L}(x_i,x_j)$ be a family of the fuzzy graph paths from vertex $x_i$ to vertex $x_j$. Then the value $\tau(x_i,x_j) = \max_{\widetilde{l} \in \widetilde{L}} \{\mu_l(x_i,x_j)\}$ defines the reachability degree of vertex $x_j$ for vertex $x_i$.

We will consider the degree of fuzzy graph vitality as a degree of strong connection, so it will be defined by the formula:

$$V(\widetilde{G}) = \mathop{\&}_{x_i \in X} \mathop{\&}_{x_j \in X} \tau(\mathrm{x_i}, \mathrm{x_j}).$$

It means there is a path between each pair of graph vertices with the conjunctive strength not less than value $V$.

Let a fuzzy graph $\widetilde{G}=(X, \widetilde{U})$ is given. Let's define fuzzy multiple-valued reflections $\widetilde{\Gamma}^1$, $\widetilde{\Gamma}^2$, $\widetilde{\Gamma}^3$,..., $\widetilde{\Gamma}^k$ [10] as:

$\widetilde{\Gamma}^1(x_i) = \{< \mu_{\Gamma^1(x_i)}(x_j)/(x_j) >\}$ , here $(\forall x_j \in X)[\mu_{\Gamma^1(x_i)}(x_j) = \mu_U < x_i, x_j >]$,

$\widetilde{\Gamma}^2(x_i) = \widetilde{\Gamma}\{\widetilde{\Gamma}(x_i)\}$,

$\widetilde{\Gamma}^3(x_i) = \widetilde{\Gamma}\{\widetilde{\Gamma}^2(x_i)\}$, ...,

$\widetilde{\Gamma}^k(x_i) = \widetilde{\Gamma}\{\widetilde{\Gamma}^{k-1}(x_i)\} = \{< \mu_{\Gamma^k(x_i)}(x_j)/x_j >\}$,

here $(\forall x_j \in X)[\mu_{\Gamma^k(x_i)}(x_j) = \bigvee_{\forall x_l \in X} \mu_{\Gamma^{k-1}(x_i)}(x_l) \,\&\, \mu_U < x_l, x_j >]$

It is obvious, that $\widetilde{\Gamma}^k(x_i)$ is a fuzzy subset of vertices, which it is accessible to reach from $x_i$, using fuzzy ways of length $k$.

Fuzzy transitive closure $\widetilde{\widetilde{\Gamma}}(x_i)$ is fuzzy multiple-valued reflection:

$$\widetilde{\widetilde{\Gamma}}(x_i) = \widetilde{\Gamma}^0(x_i) \cup \widetilde{\Gamma}(x_i) \cup \widetilde{\Gamma}^2(x_i) \cup ... = \bigcup_{j=0}^{\infty} \widetilde{\Gamma}^j(x_i)$$

.

Here, by definition:

$\widetilde{\Gamma}^0(x_i) = \{ <1/x_i> \}$.

In other words, $\widetilde{\widetilde{\Gamma}}(x_i)$ is fuzzy subset of vertices, which it is accessible to reach from $x_i$ by some fuzzy way with the greatest possible conjunctive durability. As we consider final graphs, it is possible to put, that:

$$\widetilde{\widetilde{\Gamma}}(x_i) = \bigcup_{j=0}^{n-1} \widetilde{\Gamma}^j(x_i).$$

Similarly, we define the fuzzy reciprocal transitive closure as [10]:

$$\widetilde{\widetilde{\Gamma}}^-(x_i) = \widetilde{\Gamma}^0(x_i) \cup \widetilde{\Gamma}^{-1}(x_i) \cup \widetilde{\Gamma}^{-2}(x_i) \cup ... = \bigcup_{j=0}^{\infty} \widetilde{\Gamma}^{-j}(x_i) = \bigcup_{j=0}^{(n-1)} \widetilde{\Gamma}^{-j}(x_i)$$

In other words, $\widetilde{\widetilde{\Gamma}}^-(x_i)$ is fuzzy subset of vertices, from which it is accessible to reach vertex $x_i$ by some fuzzy way with the greatest possible conjunctive durability.

To find the vitality degree of a fuzzy graph we use the method of Malgrange to find the maximal strongly connected components of fuzzy graphs [16].

To do this, take an arbitrary vertex $x_i \in X$ and find for it fuzzy transitive closure and fuzzy reciprocal transitive closure, which are fuzzy sets on the vertex set $X$. Then we find their intersection:

$$\widetilde{C}(x_i) = \widetilde{\widetilde{\Gamma}}(x_i) \cap \widetilde{\widetilde{\Gamma}}^-(x_i) = \{<\alpha_1/x_1>, <\alpha_2/x_2>, ..., <\alpha_i/x_i>, ..., <\alpha_n/x_n>\}$$

,

where $\alpha_j \in [0,1]$.

If the carrier of fuzzy set $\widetilde{C}(x_i)$ coincides with the set of vertices $X$, then the fuzzy graph $\widetilde{G}=(X,\widetilde{\Gamma})$ is strong connection, and the degree of its vitality will be defined as $V(\widetilde{G}) = \min\{\alpha_1, \alpha_2, ..., \alpha_n\}$. Otherwise the value $V = 0$.

**Example**. Let the fuzzy graph is given in Fig.1:

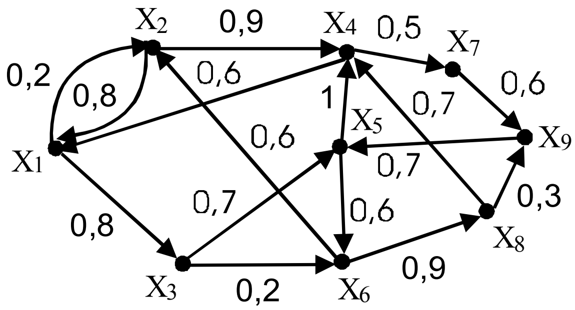


**Fig.1. Example of fuzzy graph**

For the considered graph fuzzy transitive closure has the form:

$\tilde{\tilde{\Gamma}}(x_1)$ = {<1/$x_1$>, <0,6/$x_2$>, <0,8/$x_3$>, <0,7/$x_4$>, <0,7/$x_5$>, <0,6/$x_6$>, <0,5/$x_7$>, <0,6/$x_8$>, <0,5/$x_9$>},

and fuzzy reciprocal transitive closure has the form:

$\tilde{\Gamma}^{-}(x_1)$ = {<1/$x_1$>, <0,8/$x_2$>, <0,6/$x_3$>, <0,6/$x_4$>, <0,6/$x_5$>, <0,6/$x_6$>, <0,6/$x_7$>, <0,6/$x_8$>, <0,5/$x_9$>}.

Further we find:

$$\tilde{C}(x_1) = \tilde{\Gamma}(x_1) \cap \tilde{\Gamma}^{-}(x_1) =$$

$\tilde{C}(x_1) = \tilde{\Gamma}(x_1) \cap \tilde{\Gamma}^{-}(x_1)$ = {<1/$x_1$>, <0,6/$x_2$>, <0,6/$x_3$>, <0,6/$x_4$>, <0,6/$x_5$>, <0,6/$x_6$>, <0,5/$x_7$>, <0,6/$x_8$>, <0,5/$x_9$>}.

Whence, the size of degree of vitality of the considered fuzzy graph equals 0.5.

**Conclusion**

The considered approach can be used for construction of formal procedure of the analysis of complex system as findings of vitality of the fuzzy directed graph.

In summary it would be desirable to note the important advantage of the given approach. The fuzzy sets theory, fuzzy logic and fuzzy graph theory represent theoretic and analytical base more all approaching for modeling of complex systems in the conditions of incompleteness of the information, so-called conditions of uncertainty. By means of fuzzy graph it is possible to construct the model uniting subsystems of various indicators: economic, ecological, social, educational, etc. And the part of these indicators can have statistical base, and the part can be estimated and is qualitative. The model in the form of the fuzzy graph allows to generate the quantitative and qualitative forecast of change of system, and also to choose the best variants of influence on investigated system. Secondly, by means of such model it is possible to see forecast of change of system, both quantitative, and qualitative. And in the third, the decision graphs problems, such as the finding of fuzzy ways, a component of strong connectivity, degree of vitality allows to solve problems of the analysis of ways and cycles and research of sensitivity of cognitive model.

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