Routing in the conditions of uncertainty with the use of fuzzy temporal graphs

Leonid Samoilovich Bershtein[1], Stanislav Leonidovich Belyakov[1], Alexander Vitalievich Bozhenyuk[1], Igor Naymovich Rozenberg[2]

[1]Southern Federal University, Nekrasovsky Street, 44, Taganrog, 347922, Russia
[2]Public corporation "Research and development institute of railway engineers", Nizhegorodskaya, 27/1, Moscow, 109029, Russia

Abstract. In this paper the routing problem in a transport network with fuzzy parameters changing in time is considered. In this connection, the concept of fuzzy temporal graph is introduced. Which one is a generalization of a fuzzy graph on the one hand, and a temporal graph on the other hand. The incidence of graph vertices is changed in the discrete time in fuzzy temporal graph. Fuzzy temporal graph is offered to use as a model in geographical information system. The problem is analyzed and the method of allocation fuzzy temporal external stable sets is offered. The features of modern geographical information systems connected with operative mapping are considered. The model of working area of precedent of routing is entered. Procedures of a logic conclusion for estimation of affinity and degree of analogousness of precedents are described.
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Introduction

There is quite an extensive class of problems related to the management of material flows in management of technical and socio-economic systems. The purpose of flow control is to move the product from one point another with minimal costs, subject to certain limitations assigned. An important feature of the implementation of the management process is incomplete, unclear information and multifactor description of the transport medium. This creates difficulties in constructing the best routing strategy, that is, the difficulty in solving the problem of choosing the transport path of the product in the transport network.

This paper analyzes the routing model, using the concept of fuzzy temporal graphs and spatial analysis in intelligent geographic information systems. The combination of formalisms graph models with the procedures of extraction of reliable information about the real state of the transport network can significantly improve the quality of the solutions of the routing.

Basic representation of fuzzy temporal graphs

The graphs theory draws the big attention of experts of various areas of knowledge. Traditionally graphs theory is used for representation of relations between elements of difficult structures of the various natures [1]. Thus the given relations between elements are constants and do not vary in time. Such graphs have been named by "static" [2]. In a case when relations between elements of some structure change in time, there is actual use by the graph model, in which communication between vertices of the count change in time, that is, temporal graph [3]. In a case when in temporal graph, relationships between vertices are fuzzy, we come to concept fuzzy temporal graphs [4]. However using fuzzy temporal graphs as models of various systems has difficulties. These it is connected by that the majority of isomorphic transformations of graphs change their external representation, not changing their signature. In this connection, the questions connected with consideration invariants of fuzzy temporal graphs are topical. In the given work the concept of external stable fuzzy set of fuzzy temporal graph is introduced. It is invariant concerning isomorphic transformations considered fuzzy temporal graph. External stable fuzzy set allows making structural analysis of fuzzy temporal graph.

The fuzzy temporal graph [5] is called the three $\widetilde{G}=(X,\{\widetilde{\Gamma}_t\},T)$, where X – set of graph vertices, $|X|=n$; $T=\{1,2,\ldots,N\}$ – set of the natural numbers defining (discrete) time; $\{\widetilde{\Gamma}_t\}$ – family of fuzzy equivalence, or fuzzy mappings of set of vertices X in itself during time moments $t\in T$. That is:

$$(\forall x\in X)(\forall t\in T)\,[\widetilde{\Gamma}_t(x)=\{<\mu_t(y)/y>\}],$$
$$y\in X,\ \mu_t\in[0,1].$$

Graphically fuzzy temporal graph we can present in the form of the directed graph (Fig. 1.) on which edges the fuzzy set on set of time *T* is specified.

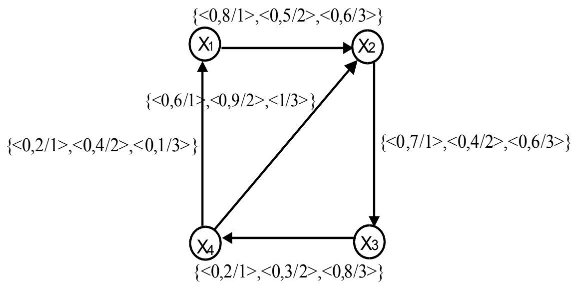

Fig.1 Example fuzzy temporal graph

Thus, fuzzy temporal graph is reduced to family *T* fuzzy sugraphs on the same set of vertices *X*.

Let's consider fuzzy sugraph $\widetilde{G}_t = (X, \widetilde{U}_t)$ of temporal fuzzy graph $\widetilde{G}$ =(*X*,{ $\widetilde{\Gamma}_t$ },*T*), where X – set of vertices, $\widetilde{U}_t = \{\mu_t(x_i, x_j) \mid (x_i, x_j) \in X^2\}$ - fuzzy set of edges at the moment time *t* with membership function $\mu_t : X^2 \to [0,1]$. Consider fuzzy sugraph $\widetilde{G}' = (X', \widetilde{U}')$, with $X' \subseteq X, \widetilde{U}' \subseteq \widetilde{U}_t$. Let X' - arbitrary subset of vertices set *X*. For each vertex $y \in X \setminus X'$ define the value

$$\gamma(y) = \max_{x \in X'} \{\mu_t(y, x)\}. \qquad (1)$$

The set X' we call fuzzy external stable set with the external stable degree $\beta(X') = \min_{y \in X \setminus X'} \gamma(y)$. Considering expression (1), we receive:

$$\beta(X') = \min_{y \in X \setminus X'} \max_{x \in X'} \{\mu_t(y, x)\}.$$

Let's enter now concept of the minimum fuzzy external domination set of fuzzy temporal graph, which is expansion of set for fuzzy graphs. [6-8].

Subset $X' \subseteq X$ is called minimum fuzzy external domination set of sugraph $\widetilde{G}_t$ with the degree $\beta(X')$, if the condition $\beta(X'') < \beta(X')$ is carried out for any subset $X'' \subset X$.

Let $\tau_k = \{X_{K_1}, X_{K_2}, \ldots, X_{K_l}\}$ - family of all minimum fuzzy external domination steady *k* vertices sets with degrees of external stability $\beta^0_{X^1_k}, \beta^0_{X^2_k}, \ldots, \beta^0_{X^l_k}$ accordingly. Let's designate through $\beta^{\min}_k = \max\{\beta^0_{X^1_k}, \beta^0_{X^2_k}, \ldots, \beta^0_{X^l_k}\}$. If the family $\tau_k = \varnothing$, then define $\beta^{\min}_{X_k} = \beta^{\min}_{X_{k+1}}$. Value $\beta^{\min}_k$ means, that in the graph $\widetilde{G}$ there is a sugraph with *k* vertices with degree of external stability $\beta^{\min}_k$ and there is no other sugraph with *k* vertices, whose degree of internal stability would be more sizes $\beta^{\min}_k$.

Set

$$\widetilde{B}_t = \{<\beta^{\min}_{X_1}/1>, <\beta^{\min x}_{X_2}/2>, \ldots, <\beta^{\min}_{X_n}/n>\}$$

is fuzzy set of external stable of fuzzy temporal sugraph $\widetilde{G}$ =(*X*,{ $\widetilde{\Gamma}_t$ },*T*) at the moment *t*. Set $\widetilde{T} = \underset{t=1,T}{\&} \widetilde{B}_t$ we name fuzzy set of external stable of fuzzy temporal graph $\widetilde{G}$ =(*X*,{$\widetilde{\Gamma}_t$},*T*).

Consider the method of finding all minimal fuzzy externally stable sets with the highest degree of external stability. Let *P* is a fuzzy set externally stable fuzzy graph $\widetilde{G} = (X, \widetilde{U})$ with the degree $\beta(P)$ of external stability. Then for an arbitrary vertex $x_i \in X$, one of the following conditions must be true.

a) $x_i \in P$;

b) if $x_i \notin P$, then there is a vertex x_j such that it belongs to the set *P*, and membership function $\underset{t \in T}{\&} \mu_t(x_i, x_j) \geq \beta(P)$.

In other words, the following statement is true:

$$(\forall x_i \in X)[x_i \in P \vee (\exists x_j)(x_j \in P \& (\underset{t \in T}{\&} \mu_t(x_i, x_j)) \geq \beta(P))]. \qquad (2)$$

To each vertex $x_i \in X$ we assign Boolean variable p_i that takes the value 1, if $x_i \in P$ and 0 otherwise. We assign a fuzzy variable $\xi_{ij} = \underset{t \in T}{\&} \mu_t(x_i, x_j)$. for the proposition

$\underset{t \in T}{\&} \mu_t(x_i, x_j) \geq \beta(P)$. Passing from the quantifier form of proposition (2) to the form in terms of logical operations, we obtain a true logical proposition:

$$\Phi_P = \underset{i}{\&}(p_i \vee \underset{j}{\vee}(p_j \& \xi_{ij})).$$

Supposing $\xi_{ii} = 1$ and considering that the equality $p_i \vee \underset{j}{\vee} p_i \& \xi_{ij} = \underset{j}{\vee} p_j \xi_{ij}$ is true for any x_i, we finally obtain:

$$\Phi_P = \underset{i}{\&} \underset{j}{\vee}(\xi_{ij} p_j) . \qquad (3)$$

We open the parentheses in the expression (3) and reduce the similar terms the following rules:

$a \vee a\&b=a$; $a\&b \vee a\& \bar{b}=a$; $\xi'\&a \vee \xi''\&a\&b=\xi'\&a$, if $\xi' \geq \xi''$. (4)

Here, $a,b \in \{0,1\}$ and $\xi', \xi'' \in [0,1]$.

Then the expression (3) may be rewrite as:

$$\Phi_T = \underset{i=\overline{1,l}}{\vee}(p_{1_i} \& p_{2_i} \& \ldots \& p_{k_i} \& a_i) . \qquad (5)$$

If in expression (5) further simplification on the basis of rules (4) is impossible, then for each member of the disjunctive set of all vertices corresponding to the variables that are present in it, gives the minimal externally stable set with the calculated degree of external stability a_i .

Find minimal externally stable sets for the graph shown in Fig. 1. Expression (3) for this graph takes the form:

$$\Phi_P = (p_1 \vee 0{,}5p_2)\&(p_2 \vee 0{,}4p_3)\&(p_3 \vee 0{,}2p_4)\&(p_4 \vee 0{,}1p_1 \vee 0{,}6p_2).$$

Multiplying the first bracket of the second, third and the fourth and absorption using the rules (4), we obtain:

$$\Phi_P = (p_1p_2 \vee 0{,}4p_1p_3 \vee 0{,}5p_2)\&(p_3p_4 \vee 0{,}6p_2p_3 \vee 0{,}1p_1p_3 \vee 0{,}2p_4).$$

Multiplying brackets obtained, we finally obtain:

$$\Phi_P = p_1p_2p_3p_1 \vee 0{,}6p_1p_2p_3 \vee 0{,}2p_1p_2p_4 \vee 0{,}4p_1p_3p_4 \vee 0{,}1p_1p_3 \vee 0{,}5p_2p_3 \vee 0{,}2p_2p_4 .$$

The last equality implies that the fuzzy set external sustainability temporal graph $\widetilde{G}$ is:

$$\widetilde{B} = \{<0/1>, <0{,}5/2>, <0{,}6/3>, <1/4>\}$$

.

This set of means in particular that in this graph at any time there is a subset of the two vertices, in which the remaining two vertices are displayed with a degree of not less than 0.5, there is a subset of the three vertices, in which the fourth vertex is displayed with the degree of 0.6.

Mapping fuzzy situations in GIS

The application of this formalism to the practice allows assessing the properties of transport networks and solving the problem of routing, transferring experience from one network to the other. However, data on the initial timing diagrams of edge weights change remains an important issue. Consider the approach to its solution using GIS.

Cartographic production standards require updating and adjustment of cards with a period of 5-7 years or more. This completely eliminates the transient phenomena fixation. In addition, when mapping areas not fundamentally seeks to display "fast" processes and phenomena. Every geographical map is a generalization of the observations made over a long period of time. Live Update GIS map data is quite difficult to provide a number of objective reasons. In particular, the individual steps of creating maps are not formalized, because they assume the use of knowledge, skills and experience of expert cartographers. This increases the time map updates. This implies that traditional mapping does not give the desired effect principle.

True picture of the dynamic real world can be created only with the involvement of operational information from disparate systems that capture real events and associated telecommunications. Here the Internet can play a special role. For example, data on the anomalous nature of the situation can be obtained as a message from the news flow (RSS), published from space or aerial photography of the Web-camera video streams of electronic media, from personal blogs from specialized communities in social networks, as well as from mapping services. Very often this information is not metric and no explicit gridding. However, its value in the case of responsible decision-making is very high.

We can assume that improving image recognition systems, text and speech in the near future will lead to a new function of geographic information systems - mapping is not fully certain situations. Corresponding subsystem will be a landmark in the figurative form of objects and phenomena, the description of which is derived from the information sources on the Internet. Obviously, that requires a special approach to procedures of construction and evaluates the quality of these cards. Its basis should be the use of the principles of intellectualization of information systems.

Possible implementation of the two situations in the map is shown in Fig.2 and Fig.3.

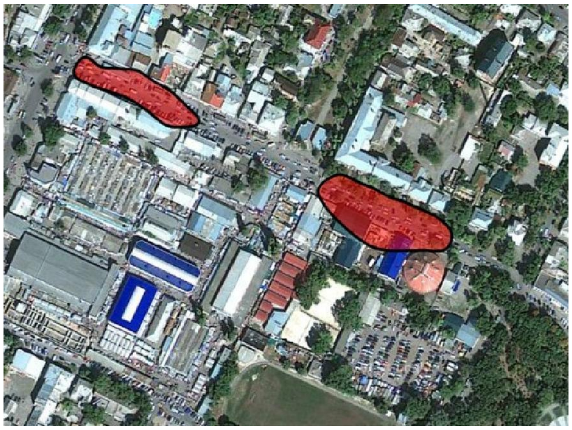

Fig.2. Satellite image

On the part of the map, the corresponding satellite images are plotted two areal objects displayed traffic jams. We can see that the mapping situations on basic geographic basis agreed with the surrounding objects. This is to be expected due to inaccuracies template description of the situation and simplified algorithm to display it on the map. However, there is obvious usefulness of such information about the real world. Featured mapping objects exist on the map for a limited time, which also evaluated the program recognition.

Fig.3. Not certain situation on the map

Experience-based routing in GIS

Organized in a similar way GIS arises a new way to use the experience of routing. Assume that there is an experience of passage along the route $P_t = \{(p_0, t_0), (p_1, t_1), ..., (p_n, t_n)\}$, here (p_1, t_1) is a couple "section of track - time interval passage way". Using the methodology of case analysis [9, 10], the knowledge gained can be extended to the same trajectory in any other time period.

Such variant is valid only in one case, when the behavior of the transport network does not depend on time. To improve the reliability of the route estimate we can bring additional information about the behavior of traffic flows. For example, assume that there is a change in behavior of the cyclical daily, weekly or monthly basis, seasonally. If $P_{\Delta t}$ is a route designed based on the precedent *P*, then the possibility of its practical using is determined by restriction $M(P, P_{\Delta t}) > \varepsilon$.

Here the value $M(x, y)$ is a metric of precedents proximity, the value ε is the distance between the levels of membership precedents, above which is an area of their equivalence. The value $M(P, P_{\Delta t})$ is estimated by expert knowledge.

The disadvantage of this model is the lack of evaluation routes analysis of asynchronous events [11]. These include events with unpredictable moments of onset and completion. For example, the repair of the road or even emerged near the highway emergency may cause temporary changes in behavior portion of the transport network.

For this purpose, it seems appropriate to use the concept of workspace precedent, which is defined as

$$A_P = P \cup E.$$

Here set *P* is a set of objects describing the route on the map, set *E* is a set of objects of the route environment that affect its behavior.

The set *E* is constructed as a result of expert analysis of precedent in the cartographic form.

Thus, the analysis of the temporal behavior of any portion of the transport network is associated with areas on the map. Objects, phenomena and processes should be displayed by cartographic objects with spatially referenced.

Logical reasons, which are formed on the basis of membership degrees of fuzzy temporal graph arcs, are based on the following mechanisms. Fuzzy equivalence of precedents is defined as:

$$M(E_m, E_n) > \varepsilon \Rightarrow P_m \equiv P_n.$$

Here $M(x, y)$ is a metric proximity to areas surrounding precedents. As in the previous case, the distance is estimated using expert knowledge.

The analogy between the precedents ($P_m \cong P_n$) can be set as:

$$M(\tilde{E}_m, \tilde{E}_n) > \varepsilon \Rightarrow P_m \cong P_n.$$

Here

$$(\widetilde{E}_m \subset E_m)\,\&\,(\widetilde{E}_n \subset E_n)\,\&\,(\widetilde{E}_m \cap \widetilde{E}_n \neq \varnothing)$$

In other words, the analogy is defined structural way. In this case, there is a speech of the coincidence of selected subsets of object instances and their types.

Conclusion

Temporal graph model provides a new route to solve the problem the shortage of information. This is due to the possibility to evaluate the integral indices of transport networks and experience to carry routing. This approach plays a special role in the case-analysis in a GIS environment. The definition externally stable sets improves the quality of adopted GIS solutions for natural incompleteness, uncertainty and ambiguity of cartographic information.

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Corresponding Author:
Dr.Bershtein Leonid Samoilovich
Southern Federal University
Nekrasovsky Street, 44, Taganrog, 347922, Russia

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