Performance Analysis of Variable Step Size MSAGF-MMA using Positive Real Number Power of Decision-Directed Error Size

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Abstract: In this paper, we analyze the performance of variable step size MSAGF-MMA according to a positive real number power of decision-directed error signal size. This algorithm is designed so as to allow the adaptive blind equalization tab coefficients to be updated according to the maximum fixed step size when the decision error signal size is over a certain base value and the equalization tab coefficients to be updated according to the value of the maximum fixed step size multiplied by the power of γ (positive real number) of the decision-directed error size value when it is below that. As a result of a comparative analysis of the performance according to γ power through computer simulation, it is confirmed that the proposed algorithm has a very excellent performance in terms of residual error size as well as convergence rate compared to MMA and MSAGF-MMA. In addition, if γ value is smaller than 1, convergence rate gets relatively faster while the residual error size gets relatively much smaller if it is bigger than 1.

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1. Introduction

Recently, in implementing a high-speed digital communication system, wired and wireless channel band-limited characteristic and multi-path channel environment have caused a problem of inter-symbol interference. This inter-symbol interference is one of the main factors deteriorating the entire communication system performance. To get rid of this, channel equalization technologies are used. Of the channel equalization technologies, recently, adaptive blind equalization methods have been widely used. In particular, Constant Modulus Algorithm [1] is a representative adaptive blind equalization method with an advantage that it has fewer arithmetic operations so that it is easy to implement the system. CMA is an algorithm most widely used in 2-dimensional modulation systems such as QAM or CAP, which has reliable convergence. And yet it has a problem that it has a fairly large residual error after convergence and a disadvantage that it needs a phase compensator for phase correction in the equalizer output terminal. MCMA [2] and MMA [3] produced later are algorithms that can recover inter-symbol interference and irregular phase rotation simultaneously, which improved these disadvantages of CMA. And yet, MCMA or MMA has a disadvantage that even after equalization, it still has a large residual error size in steady-state [4]. This is because it uses a special modulus like CMA. As a method of improving this problem, later various equalization methods such as dual-mode [5] or variable step size adaptive blind equalization method [6] are proposed.

The performance of the adaptive blind equalization method is determined through the convergence rate and the size of a residual error in steady-state after the convergence. We propose an equalization algorithm applying a variable step size to MSAGF-MMA satisfies the two criteria. This algorithm allows the maximum step size used in a tab update formula to affect the tab update in proportion to the error signal size when the eye are open to some degree after equalization, that is, when the error from the signaling point is smaller than 1 as a result of the equalization with the original signaling point. This is devised so as to perform tab update with the maximum step size to find the optimal coefficients at an early phase of equalization and allow the step size to get smaller variably according to the error size to find a more precision, optimal equalization coefficients, so that it has a fast convergence rate and a very small residual error size after the steady-state. We attempts to inquire into the equalization performance of the proposed algorithm according to γ (positive real number value) power of the error signal size.

The paper is composed as follows: Chapter 2 describes a baseband communication system applying a general adaptive blind equalizer; Chapter 3 describes MMA and MSAGF-MMA; Chapter 4 describes variable step size MSAGF-MMA; Chapter 5 carries out a comparative analysis of the performance of the existing algorithm and the proposed algorithm according to γ power through the results of computer simulation; and lastly, Chapter 6 draws conclusions.

2. Adaptive Blind Equalization System

Figure 1 shows a baseband communication system applying an adaptive blind equalizer. Reception signal $x(n)$ is described by

$$x(n) = \sum_{l=0}^{N-1} h(i)a(n-l) + \nu(n) \qquad (1)$$

Where $h(n)$ is a channel impulse response with a length N; transmitting data symbol $a(n)$ is a QAM signal with a constant modulus; and it is defined as a complex signal sequence with a mean $E[|a(n)|] = 0$ that meets i.i.d. (independent & identical distribution). Additive noise $\nu(n)$ is a Gaussian noise with the mean $E[|\nu(n)|] = 0$ and the distribution $E[|\nu(n)|] = \sigma_n^2$, assumed to be an AWGN statistically independent from the transmitting data symbol. As an equalizer, a linear complex FIR filter with stability is used. In Figure 1., equalizer output can be characterized as follows: $y(n) = y_R(n) + jy_I(n) = x^T(n)W(n)$. $W(n) = [w_0(n), w_1(n), ..., w_{N-1}(n)]^T$ is the equalizer's tab coefficient vector; $x(n) = [x_0(n), x_1(n), ..., x_{N-1}(n)]^T$ is the equalizer's input data vector; and N is the equalizer's tab coefficient length. Superscript ^{T} is transposition of the vector while $g(\cdot)$ is a non-linear decision logic.

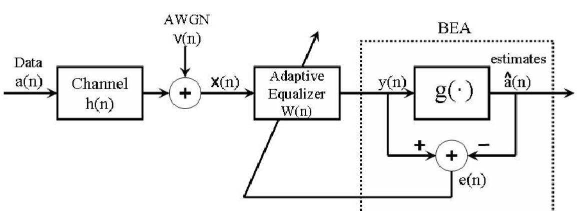

Figure 1. A baseband communication system applying an adaptive blind equalizer

3. MMA and MSAGF-MMA

3-1. MMA

MMA is an algorithm combining RCA [7] and CMA [1] so as to have an advantage that a phase compensator is not necessary after equalization and at the same time to have a relatively fast convergence rate, proposed by J. Yang *et al* [3].

MMA's cost function is $J_{MMA} = E[(|y_R(n)|^2 - \eta^L_{R,MMA})^2 + (|y_I(n)|^2 - \eta^L_{I,MMA})^2]$. $y_R(n)$ and $y_I(n)$ are respectively the real and imaginary parts of Equalizer output $y(n)$, calculated by $\eta^2_{R,MMA} = E[(|a_{R,n}(n)|^4 / |a_{R,n}(n)|^2]$ and $\eta^2_{I,MMA} = E[(|a_{I,n}(n)|^4 / |a_{I,n}(n)|^2]$. In contrast, using a symbol level, it can be calculated as follows: $\eta^2_{R,MMA} = \eta^2_{I,MMA} = (12m^2 - 7)/5$.

The real and imaginary parts in MMA error signal $e_{MMA}(n)$ are given by $e_{R,MMA}(n) = (y_R^2(n) - \eta^2_{R,MMA})\, y_R(n)$ and $e_{I,MMA}(n) = (y_I^2(n) - \eta^2_{I,MMA})\, y_I(n)$ respectively. MMA's tab update equation is adapted by

$$\begin{aligned} W_R(n+1) &= W_R(n) - \mu e_{R,MMA}(n)x^*(n) \\ W_I(n+1) &= W_I(n) - \mu e_{I,MMA}(n)x^*(n) \end{aligned} \qquad (2)$$

3-2. Modified Stop-and-Go Flagged-MMA (MSAGF-MMA)

MSAGF-MMA [8] is an algorithm that improves the equalization performance by investigating if the sign of the error signal estimated by MMA and that of the error signal estimated by a decision-directed algorithm are consistent and using a reliable estimated error signal for a tab update. The modified Stop-and-Go flags $f_{R,MSAG}$ and $f_{I,MSAG}$ used in this algorithm are defined as follows:

$$\begin{aligned} f_{R,MSAG} &= \begin{Bmatrix} 1 & if & \mathrm{sgn}(e_{R,MMA}) = \mathrm{sgn}(e_{R,DD}) \\ 0 & if & \mathrm{sgn}(e_{R,MMA}) \neq \mathrm{sgn}(e_{R,DD}) \end{Bmatrix} \\ f_{I,MSAG} &= \begin{Bmatrix} 1 & if & \mathrm{sgn}(e_{I,MMA}) = \mathrm{sgn}(e_{I,DD}) \\ 0 & if & \mathrm{sgn}(e_{I,MMA}) \neq \mathrm{sgn}(e_{I,DD}) \end{Bmatrix} \end{aligned} \qquad (3)$$

Where $e_{R,DD}$ and $e_{I,DD}$ are the real and imaginary parts of the error signal for the decision-directed algorithm [9].

The real and imaginary parts of the error signal in MSAGF-MMA can be calculated respectively by $e_{R,MSAGF-MMA}(n) = f_{R,MSAG}\, e_{R,MMA}(n)$ and $e_{I,MSAGF-MMA}(n) = f_{I,MSAG}\, e_{I,MMA}(n)$. Accordingly, the tab update equation is written as

$$\begin{aligned} W_R(n+1) &= W_R(n) - \mu e_{R,MSAGF-MMA}(n)x^*(n) \\ W_I(n+1) &= W_I(n) - \mu e_{I,MSAGF-MMA}(n)x^*(n) \end{aligned} \qquad (4)$$

3. Variable Step Size MSAGF-MMA

If the eyes of the estimated signaling points start to be open after equalization is done to some degree, the error size between the original signaling point and the estimated signaling point as a result of equalization will become smaller than 1 and as convergence is made, the error size will change in a direction that it has a smaller value until it reaches a steady state finally. Thus, if the error size value becomes smaller than a base value, the residual error in steady-state will be minimized by allowing the step size of the equalization tab update equation to be changed according to the error size.

We propose an MSAGF-MMA with variable step size $\mu_R(n)$ and $\mu_I(n)$ in proportion to γ (positive real number value) power of the size of each of the real and imaginary parts $|e_{R,DD}|$ and $|e_{I,DD}|$ of the decision error signal if the decision error size value becomes smaller than a base value in the tab update equation. If the decision error size value becomes greater than a base value, the tab update equation has the maximum fixed step size value given by μ_{upper}.

The proposed algorithm's tab update equation is described by

$$W_R(n+1)=\begin{cases} W_R(n)-\mu_{upper}e_{R,MSAGF-MMA}(n)x^*(n) & if \quad |e_{DD}(n)|\geq 1 \\ W_R(n)-\mu_R(n)\cdot e_{R,MSAGF-MMA}(n)x^*(n), & if \quad |e_{DD}(n)|<1 \end{cases}$$

$$W_I(n+1)=\begin{cases} W_I(n)-\mu_{upper}e_{I,MSAGF-MMA}(n)x^*(n) & ,if \quad |e_{DD}(n)|\geq 1 \\ W_I(n)-\mu_I(n)\cdot e_{I,MSAGF-MMA}(n)x^*(n), & if \quad |e_{DD}(n)|<1 \end{cases} \tag{5}$$

Variable step sizes $\mu_R(n)$ and $\mu_I(n)$ are given by

$$\mu_R(n)=\mu_{upper}|e_{R,DD}(n)|^{\gamma}$$
$$\mu_I(n)=\mu_{upper}|e_{I,DD}(n)|^{\gamma} \tag{6}$$

Where μ_{upper} is the maximum fixed step size value while γ is a positive real number value.

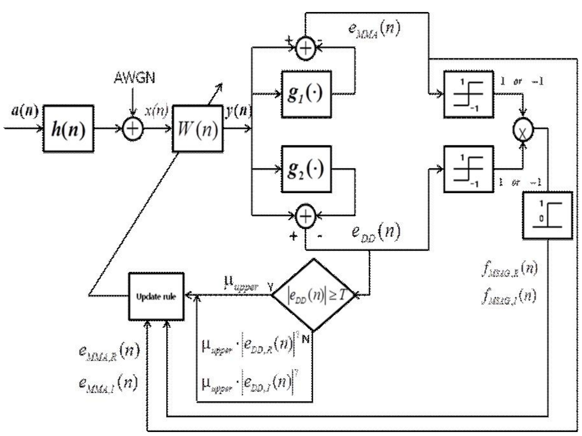

Figure 2. A block diagram for the proposed algorithm

Figure 2 is the entire block diagram of the proposed algorithm. In the figure, $a(n)$ is a transmitting signal; $h(n)$, an impulse response to a channel; and $W(n)$, adaptive blind equalization filter coefficient, respectively. In addition, $x(n)$ represents a reception signal while $y(n)$ is an equalizer output, which represents $e_{MMA}(n)=e_{R,MMA}(n)+je_{I,MMA}(n)$. $g_1(\cdot)$ and $g_2(\cdot)$ represent the non-linear estimator of MMA and the decision-directed algorithm, respectively. Through $g_1(\cdot)$, MMA's error signal $e_{MMA}(n)=e_{R,MMA}(n)+je_{I,MMA}(n)$ is obtained while through $g_2(\cdot)$, the decision-directed algorithm's error signal $e_{DD}(n)=e_{R,DD}(n)+je_{I,DD}(n)$.

3. The Computer Simulation

The proposed VSS MSAGF-MMA performs a computer simulation so as to analyze the impact of γ power of each of sizes $|e_{R,DD}|$ and $|e_{I,DD}|$ of real and imaginary parts in the decision error signal on the equalization performance. The simulation is conducted with a 256-QAM signal for multi-path propagation channel [9]. The simulation uses signal-to-noise ratio 40dB and a complex FIR filter with a tab length of 15 as an equalizer. The central tab of all the equalizer is initialized at $1+j0$ while all other tabs except for the central tab, at $0+j0$ [10]. As performance evaluation indices for a performance analysis, residual ISI (inter-symbol interference) [11] and ensemble averaged-MSE(Mean Square Error) [12] are used. The residual ISI is defined as follows:

$$ISI=\frac{\sum_n |s(n)|^2-|s(n)|^2_{\max}}{|s(n)|^2_{\max}} \tag{7}$$

Where $s(n)=h(n)\otimes w(n)$ and $\otimes$ is a convolution arithmetic operation.

The residual ISI becomes an impulse signal in a time domain in complete equalization, and yet complete equalization is not possible, which represents residual components other than the impulse components.

The estimated ensemble-averaged MSE [11] is defined as

$$estimated \quad MSE(n)=\frac{1}{M}\sum_{k=1}^{M}E\left[\left|y(n)-\hat{a}(n)\right|^2\right] \tag{8}$$

Where $y(n)$ is the equalizer output. $\hat{a}(n)$ is the estimated value of the transmission symbol $a(n)$.

Figure 3 shows a comparison of an ensemble-averaged ISI for the 256-QAM signal in dB, which is obtained through 100 times of Monte Carlo. Here, the experiment is made in a range $0<\gamma\leq 2$ [13].

Through experiments, the maximum fixed step size value μ_{upper} is taken as follows: In MMA, $\mu_{upper}=8.0\times10^{-9}$; in MSAGF-MMA, $\mu_{upper}=2.0\times10^{-8}$; and in the proposed algorithm, a) if $\gamma=1/4$ power, $\mu_{upper}=4.0\times10^{-8}$; b) if $\gamma=1/3$ power, $\mu_{upper}=4.0\times10^{-8}$; c) if $\gamma=1/2$ power, $\mu_{upper}=4.0\times10^{-8}$; d) if $\gamma=2/3$ power, $\mu_{upper}=4.0\times10^{-8}$; e) if $\gamma=1$ power, $\mu_{upper}=4.5\times10^{-8}$; f) if $\gamma=11/10$ power, $\mu_{upper}=5.0\times10^{-8}$; g) if $\gamma=5/4$ power, $\mu_{upper}=5.0\times10^{-8}$; h) if $\gamma=3/2$ power, $\mu_{upper}=5.0\times10^{-8}$; i) if $\gamma=5/3$ power, $\mu_{upper}=4.8\times10^{-8}$; j) if $\gamma=2$ power, $\mu_{upper}=5.0\times10^{-8}$. $\eta^2_{R,MMA}$ and $\eta^2_{I,MMA}$ are set at 152.2 according to Formula (6), and In-phase and Quadrature signals are set so as to have $\pm1,\pm3,\pm5,\pm7,...\pm(2m-1)$, respectively.

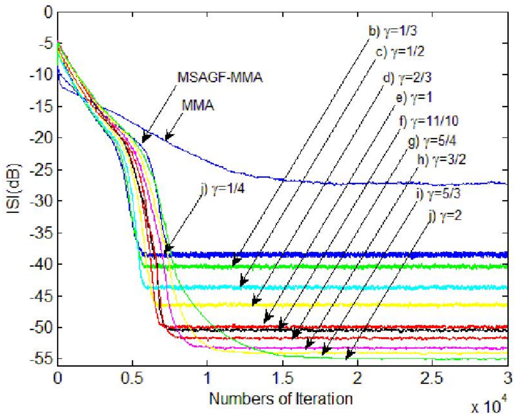

Figure 3. Comparison of an ensemble-averaged ISI for the 256-QAM Signal

Figure 3 shows a comparison of the residual ensemble-averaged ISI for each algorithm. In Figure 3, MMA shows a residual ISI value of –27.2dB on average in steady-state while MSAGF-MMA shows that of about –38.4dB on average. In comparison, the proposed VSS MSAGF-MMA shows a residual ISI value between –38.6dB and –54.9dB on average in steady-state according to γ value as shown in Table 1. As shown in the table, the greater γ value, the smaller the residual ISI value becomes. This is a result from the application of a step size value to the tab update equation smaller than the value when the decision-directed error size is multiplied by γ power greater than 1.

Table 1. Residual ISI values according to γ power in steady-state [dB]

γ power	$\gamma=\frac{1}{4}$	$\gamma=\frac{1}{3}$	$\gamma=\frac{1}{2}$	$\gamma=\frac{2}{3}$	$\gamma=1$
ISI	-38.6	-40.4	-43.7	-46.4	-49.9
γ power	$\gamma=\frac{11}{10}$	$\gamma=\frac{5}{4}$	$\gamma=\frac{3}{2}$	$\gamma=\frac{5}{3}$	$\gamma=2$
ISI	-50.5	-51.7	-53.3	-54.1	-54.9

Figure 3 shows convergence rate characteristic as well. In Fig. 3, MMA maintains the steady-state at iterations about 17020 and on while MSAGF-MMA, at that about 7735. In comparison to these, the proposed VSS MSAGF-MMA maintains the steady-state, according to γ value at iterations from about 5765 to 19000 as shown in Table 2.

Table 2. Iterations according to γ power in steady-state

γ power	$\gamma=\frac{1}{4}$	$\gamma=\frac{1}{3}$	$\gamma=\frac{1}{2}$	$\gamma=\frac{2}{3}$	$\gamma=1$
Iterations	5765	5807	5848	6684	6861
γ power	$\gamma=\frac{11}{10}$	$\gamma=\frac{5}{4}$	$\gamma=\frac{3}{2}$	$\gamma=\frac{5}{3}$	$\gamma=2$
Iterations	7355	7847	9138	12120	19000

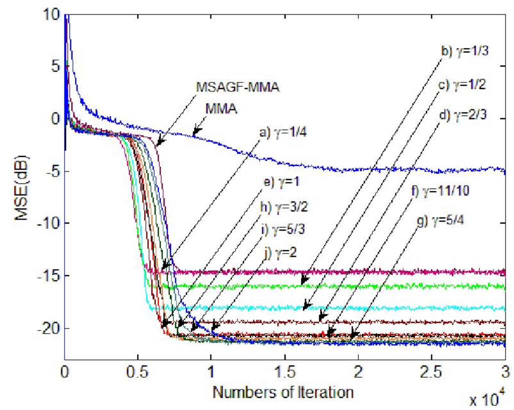

Figure 4. Comparison of an ensemble-averaged MSE.

Figure 4 shows a comparison of the ensemble-averaged MSE value for each algorithm. MMA has an MSE –4.89dB; MSAGF-MMA, -14.62dB; and the proposed VSS MSAGF-MMA, about –14.68dB to –21.50dB on average in steady-state according to γ value as shown in Table 3.

Table 3. MSE according to γ power in steady-state [dB]

γ power	$\gamma=\frac{1}{4}$	$\gamma=\frac{1}{3}$	$\gamma=\frac{1}{2}$	$\gamma=\frac{2}{3}$	$\gamma=1$
MSE	-14.68	-16.02	-18.11	-19.45	-20.63
γ power	$\gamma=\frac{11}{10}$	$\gamma=\frac{5}{4}$	$\gamma=\frac{3}{2}$	$\gamma=\frac{5}{3}$	$\gamma=2$
MSE	-20.75	-21.03	-21.29	-21.41	-21.50

In Figures 3 and 4, if γ value is smaller than 1, the convergence rate gets faster, but the residual error size value gets greater, and if it is greater than 1, the convergence rate gets somewhat slower, but the residual error size becomes much smaller. To sum up with the residual ISI, convergence rate and MSE, if $\gamma=1$, the proposed algorithm has all excellent performance in terms of the convergence rate and the residual error size with a residual ISI value of –49.9dB, iterations of 6861 and an MSE of –20.63dB.

6. Conclusion

In this paper, we carries out a comparative analysis of the equalization performance of a variable step size MSAGF-MMA with an improved equalization performance so as to have a higher convergence rate and a very small residual error value in steady-state according to γ power of the error signal size by which the variable step size is decided. For this, the ensemble-averaged ISI and MSE are calculated for a 256-QAM signal to use as indices of performance comparison. Through computer simulations, the algorithm proposed in a 256-QAM system is the maximum 2.9523 times faster in a range from $\gamma = 1/4$ to $\gamma = 5/3$ than MMA and up to 1.3417 times faster in a range from $\gamma = 1/4$ to $\gamma = 11/10$ than MSAGF-MMA.

On the other hand, it has the residual ISI in steady-state about 11.4-27.7dB lower than MMA and about 0.2-16.5dB lower than MSAGF-MMA. In a comparison of the MSE, the proposed algorithm has a 9.79-16.61dB lower value than MMA and about 0.06-6.98dB lower than MSAGF-MMA. To sum up with the convergence rate, the ISI and the MSE that shows the residual error size, it is shown that the proposed algorithm has very good performance in terms of its convergence rate and residual error size. In the proposed algorithm, it is judged that applying a multiplier smaller than $\gamma = 1$ for a case that needs a faster convergence rate and one greater than $\gamma = 1$ for the one that needs a smaller residual error size to the tab update equation for a variable step size will obtain the desired results of equalization.

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