# Asymptotic behavior of an anti-competitive system of rational difference equations*

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**Abstract:** In this paper, we investigate the global behavior and rate of convergence of following anti-competitive system of rational difference equations: $x_{n+1}=\frac{\alpha y_n}{\beta+\gamma x_n^r}, y_{n+1}=\frac{\alpha_1 x_n}{\beta_1+\gamma_1 y_n^r}, n=0,1,\cdots,$ where the parameters $\alpha,\beta,\gamma,\alpha_1,\beta_1,\gamma_1,r\in(0,\infty)$ and the initial conditions $x_0,y_0\in(0,\infty)$. Some numerical examples are given to verify our theoretical results.

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## 1. Introduction

Difference equations manifest themselves as mathematical models describing real life situations in probability theory, queuing problems, statistical problems, stochastic time series, combinatorial analysis, number theory, geometry, electrical networks, quanta in radiation, genetics in biology, economics, psychology, sociology, etc. It is an indisputable fact that difference equations appeared much earlier than differential equations and were instrumental in paving the way for the development of the latter. It is only recently that difference equations have started receiving the attention they deserve. Perhaps this is largely due to the advent of computers where differential equations are solved by using their approximate difference equation formulations. For basic theory of difference equations we refer [1-4]. In literature there are many papers on qualitative behavior of biological models [11-15]. Recently there has been a lot of interest in studying the global attractivity, boundedness character, periodicity and the solution form of nonlinear difference equations.

Cinar [5] investigated the periodicity of the positive solutions of the system of rational difference equations:

$$x_{n+1}=\frac{1}{y_n}, y_{n+1}=\frac{y_n}{x_{n-1}y_{n-1}}.$$

Stević [6] studied the system of two nonlinear difference equations:

$$x_{n+1}=\frac{u_n}{1+v_n}, y_{n+1}=\frac{w_n}{1+s_n},$$

where $u_n,v_n,w_n,s_n$ are some sequences $x_n$ or $y_n$.

Zhang et al. [7] studied the dynamics of a system of rational third-order difference equation:

$$x_{n+1}=\frac{x_{n-2}}{B+y_n y_{n-1} y_{n-2}}, y_{n+1}=\frac{y_{n-2}}{B+x_n x_{n-1} x_{n-2}}, n=0,1,\cdots.$$

where $A,B,x_{-2},x_{-1},x_0,y_{-2},y_{-1},y_0\in(0,\infty)$.

Elabbasy et al. [8] studied the asymptotic behavior of two dimensional rational system of difference equations:

$$x_{n+1}=\frac{a_1 x_n}{a_2+a_3 y_n^r}, y_{n+1}=\frac{b_1 y_n}{b_2+b_3 x_n^r}, n=0,1,\cdots$$

where the parameters $a_1,a_2,a_3,b_1,b_2,b_3,r\in(0,\infty)$ and the initial conditions $x_0,y_0\in(0,\infty)$.

Our aim in this paper is to investigate the asymptotic behavior of following anti-competitive system of rational difference equations:

$$x_{n+1}=\frac{\alpha y_n}{\beta+\gamma x_n^r}, y_{n+1}=\frac{\alpha_1 x_n}{\beta_1+\gamma_1 y_n^r}, n=0,1,\cdots, \quad (1)$$

where the parameters $\alpha,\beta,\gamma,\alpha_1,\beta_1,\gamma_1,r\in(0,\infty)$ and initial conditions $x_0,y_0\in(0,\infty)$.

## 2. Linearized stability

Let us consider two-dimensional discrete dynamical system of the form:

$$\left.\begin{array}{c} x_{n+1}=f(x_n,y_n) \\ y_{n+1}=g(x_n,y_n), n=0,1,\cdots \end{array}\right\} \quad (2)$$

where $f:I\times J\rightarrow I$ and $g:I\times J\rightarrow J$ are continuously differentiable functions and $I,J$ are some intervals of real numbers. Furthermore, a solution $\{(x_n,y_n)\}_{n=0}^{\infty}$ of system (2) is uniquely determined by initial conditions $(x_0,y_0)\in I\times J$. An equilibrium point of system (2) is a point $(\bar{x},\bar{y})$ that satisfies

$\bar{x}=f(\bar{x},\bar{y})$,
$\bar{y}=g(\bar{x},\bar{y})$.

**Definition 1.** Let $(\bar{x},\bar{y})$ be an equilibrium point of the system (2).

(i) An equilibrium point $(\bar{x},\bar{y})$ is said to be stable if for every $\varepsilon>0$ there exists $\delta>0$ such that for every initial conditions $(x_0,y_0)$, if $\|(x_0,y_0)-(\bar{x},\bar{y})\|<\delta$ implies $\|(x_n,y_n)-(\bar{x},\bar{y})\|<\varepsilon$ for all $n>0$, where $\|\cdot\|$ is usual Euclidian norm in $\mathbb{R}^2$.

(ii) An equilibrium point $(\bar{x}, \bar{y})$ is said to be unstable if it is not stable.
(iii) An equilibrium point $(\bar{x}, \bar{y})$ is said to be asymptotically stable if there exists $\eta > 0$ such that $\|(x_0, y_0) - (\bar{x}, \bar{y})\| < \eta$ and $(x_n, y_n) \to (\bar{x}, \bar{y})$ as $n \to \infty$.
(iv) An equilibrium point $(\bar{x}, \bar{y})$ is called global attractor if $(x_n, y_n) \to (\bar{x}, \bar{y})$ as $n \to \infty$.
(v) An equilibrium point $(\bar{x}, \bar{y})$ is called asymptotic global attractor if it is a global attractor and stable.

**Definition 2.** Let $(\bar{x}, \bar{y})$ be an equilibrium point of a map $F = (f(x,y), g(x,y))$ where $f$ and $g$ are continuously differentiable functions at $(\bar{x}, \bar{y})$. The linearized system of (2) about the equilibrium point $(\bar{x}, \bar{y})$ is given by

$$X_{n+1} = F(X_n) = F_J X_n,$$

Where $X_n = \begin{pmatrix} x_n \\ y_n \end{pmatrix}$ and $F_J$ is Jacobean matrix of the system (2) about the equilibrium point $(\bar{x}, \bar{y})$.

Let $(\bar{x}, \bar{y})$ be an equilibrium point of the system (1), then

$$\bar{x} = \frac{\alpha \bar{y}}{\beta + \gamma\, \bar{x}^r}, \bar{y} = \frac{\alpha_1 \bar{x}}{\beta_1 + \gamma_1\, \bar{y}^r}.$$

Hence, $O = (0,0)$ be a unique equilibrium point of the system (1).

The Jacobian matrix about the fixed point $(\bar{x}, \bar{y})$ is given by

$$F_J(\bar{x}, \bar{y}) = \begin{pmatrix} -\frac{\alpha\gamma r \bar{y} \bar{x}^{r-1}}{(\beta + \gamma\, \bar{x}^r)^2} & \frac{\alpha}{\beta + \gamma\, \bar{x}^r} \\ \frac{\alpha_1}{\beta_1 + \gamma_1\, \bar{y}^r} & -\frac{\alpha_1 \gamma_1 r \bar{x} \bar{y}^{r-1}}{(\beta_1 + \gamma_1\, \bar{y}^r)^2} \end{pmatrix}.$$

**Lemma 1.** [3] For the system $X_{n+1} = F(X_n), n = 0,1,\cdots,$ of difference equations such that $\bar{X}$ be a fixed point of $F$. If all eigenvalues of the Jacobian matrix $F_J$ about $\bar{X}$ lie inside the open unit disk $|\lambda| < 1$, then $\bar{X}$ is locally asymptotically stable. If one of them has a modulus greater than one, then $\bar{X}$ is unstable.

## 3. Main results

**Theorem 1.** Let $\{(x_n, y_n)\}$ be a positive solution of the system (1), then for every $m \geq 0$ the following result hold:

$$(i)\ 0 \leq x_n \leq \begin{cases} \left(\frac{\alpha}{\beta}\right)^{m+1} \left(\frac{\alpha_1}{\beta_1}\right)^{m} y_0, & n = 2m+1, \\ \left(\frac{\alpha\alpha_1}{\beta\beta_1}\right)^{m+1} x_0, & n = 2m+2. \end{cases}$$

$$(ii)\ 0 \leq y_n \leq \begin{cases} \left(\frac{\alpha}{\beta}\right)^{m} \left(\frac{\alpha_1}{\beta_1}\right)^{m+1} x_0, & n = 2m+1, \\ \left(\frac{\alpha\alpha_1}{\beta\beta_1}\right)^{m+1} y_0, & n = 2m+2. \end{cases}$$

Proof. It follows from induction. ■

**Lemma 2**. Let $\alpha < \beta$ and $\alpha_1 < \beta_1$, then every solution $\{(x_n, y_n)\}_{n=0}^{\infty}$ of the system (1) is bounded.

Proof. Assume that

$$\lambda_1 = \max\left\{ \frac{\beta_1}{\alpha_1} y_0, x_0 \right\}$$

and

$$\lambda_2 = \max\left\{ \frac{\beta}{\alpha} x_0, y_0 \right\}.$$

Then, from theorem 1 one can see that $0 \leq x_n < \lambda_1$ and $0 \leq y_n < \lambda_2$ for all $n = 0,1,\cdots$. ■

**Theorem 2.** If $\alpha < \beta$ and $\alpha_1 < \beta_1$, then unique equilibrium point $O = (0,0)$ of the system (1) is locally asymptotically stable.

Proof. The linearized system of (1) about the equilibrium point $O = (0,0)$ is given by

$$X_{n+1} = F_J(0,0) X_n,$$

where

$X_n = \begin{pmatrix} x_n \\ y_n \end{pmatrix}$ and

$$F_J(0,0) = \begin{pmatrix} 0 & \frac{\alpha}{\beta} \\ \frac{\alpha_1}{\beta_1} & 0 \end{pmatrix}.$$

The characteristic polynomial of $F_J(0,0)$ is given by

$$P(\lambda) = \lambda^2 - \frac{\alpha\alpha_1}{\beta\beta_1}. \quad (3)$$

The roots of $P(\lambda)$ are $\lambda_{1,2} = \pm\sqrt{\frac{\alpha\alpha_1}{\beta\beta_1}}$. Hence, by lemma 1 the unique equilibrium point $(0,0)$ is locally asymptotically stable if $\alpha < \beta$ and $\alpha_1 < \beta_1$.

**Lemma 3.** [2] Let $I = [a,b]$ and $J = [c,d]$ be real intervals, and let $f: I \times J \longrightarrow I$ and $g: I \times J \longrightarrow J$ be continuous functions. Consider the system (2) with initial conditions $(x_0, y_0) \in I \times J$. Suppose that following statements are true:

(i) $f(x,y)$ is non-increasing in $x$, and non-decreasing in $y$.

(ii) $g(x,y)$ is non-decreasing in $x$, and non-increasing in $y$.

(iii) If $(m_1, M_1, m_2, M_2) \in I^2 \times J^2$ is a solution of the system

$$m_1 = f(M_1, m_2), M_1 = f(m_1, M_2)$$
$$m_2 = g(m_1, M_2), M_2 = g(M_1, m_2)$$

such that $m_1 = M_1$, and $m_2 = M_2$. Then, there exist one positive equilibrium point $(\bar{x}, \bar{y})$ of the system (2) such that $\lim_{n\to\infty}(x_n, y_n) = (\bar{x}, \bar{y})$.

**Theorem 3.** The unique equilibrium point $O = (0,0)$ of the system (1) is global attractor.

Proof. Let $f(x,y) = \frac{\alpha y}{\beta + \gamma x^r}$ and $g(x,y) = \frac{\alpha_1 x}{\beta_1 + \gamma_1 y^r}$. Then, it is easy to see that $f(x,y)$ is non-increasing in $x$, and non-decreasing in $y$. Moreover, $g(x,y)$ is non-decreasing in $x$, and non-increasing in $y$. Let $(m_1, M_1, m_2, M_2)$ be s solution of the system

$$m_1 = f(M_1, m_2), M_1 = f(m_1, M_2),$$
$$m_2 = g(m_1, M_2), M_2 = g(M_1, m_2).$$

Then, one has $m_1 = \frac{\alpha m_2}{\beta + \gamma {M_1}^r}, M_1 = \frac{\alpha M_2}{\beta + \gamma {m_1}^r}. \quad (4)$

and

$$m_2 = \frac{\alpha_1 m_1}{\beta_1 + \gamma_1 {M_2}^r}, M_2 = \frac{\alpha_1 M_1}{\beta_1 + \gamma_1 {m_2}^r}. \quad (5)$$

From equations (4) and (5), one get

$$\frac{m_1}{M_1} = \frac{m_2}{M_2} \cdot \frac{\beta+\gamma {m_1}^r}{\beta+\gamma {M_1}^r}. \quad (6)$$

$$\frac{m_2}{M_2} = \frac{m_1}{M_1} \cdot \frac{\beta_1+\gamma_1 {m_2}^r}{\beta_1+\gamma_1 {M_2}^r}. \quad (7)$$

Setting

$$\frac{m_1}{M_1} = a_1 \leq 1, \frac{m_2}{M_2} = a_2 \leq 1. \quad (8)$$

In view of (8), equations (6) and (7) then implies that

$$\left.\begin{aligned}(a_1 - a_2)\beta &= (a_2 a_1^{r-1} - 1)a_1 \gamma {M_1}^r \\ (a_2 - a_1)\beta_1 &= (a_1 a_2^{r-1} - 1)a_2 \gamma_1 {M_2}^r\end{aligned}\right\}. \quad (9)$$

Note that right-hand side of (9) are less than or equal to zero, and thus

$a_1 - a_2 \leq 0, a_2 - a_1 \leq 0$.

This implies that $a_1 \leq a_2 \leq a_1$ which hold if and only if $a_1 = a_2$. In view of (9), it follows that $a_1 = a_2 = 1$. Thus $m_1 = M_1, m_2 = M_2$. Hence, from Lemma (3) the unique equilibrium point $O = (0,0)$ is a global attractor.■

**Lemma 4.** Under the conditions $\alpha < \beta$ and $\alpha_1 < \beta_1$, the unique equilibrium point $O = (0,0)$ of the system (1) is globally asymptotically stable.

Proof. The proof follows from theorem 2, and theorem 3.■

**4. Rate Of Convergence**

In this section we will determine the rate of convergence of a solution that converges to unique equilibrium point (0, 0) of the system (1).

The following results give the rate of convergence of solutions of a system of difference equations

$$X_{n+1} = \big(A + B(n)\big)X_n, \quad (10)$$

Where $X_n$ is an $m$ −dimensional vector, $A \in C^{m\times m}$ is a constant matrix, and $B: Z^+ \to C^{m\times m}$ is a matrix function satisfying

$$\|B(n)\| \to 0 \quad (11)$$

as $n \to \infty$, where $\|.\|$ denotes any matrix norm which is associated with the vector norm

$\|B(n)\| = \sqrt{x^2 + y^2}$

**Proposition 1.** (Perron's theorem) [10] Suppose that condition (11) holds. If $X_n$ is a solution of (10), then either $X_n = 0$ for all large $n$ or

$$\rho = \lim_{n\to\infty}(\|X_n\|)^{\frac{1}{n}} \quad (12)$$

exist and is equal to the modulus of one the eigenvalues of matrix $A$.

**Proposition 2.** (Perron's theorem) [10] Suppose that condition (11) holds. If $X_n$ is a solution of (10), then either $X_n = 0$ for all large $n$ or

$$\rho = \lim_{n\to\infty}\frac{\|X_{n+1}\|}{\|X_n\|} \quad (13)$$

exist and is equal to the modulus of one the eigenvalues of matrix $A$.

Let $\{(x_n, y_n)\}$ be any solution of the system (1) such that $\lim_{n\to\infty} x_n = \bar{x}$, $\lim_{n\to\infty} y_n = \bar{y}$. To find the error terms, one has from the system (1)

$$x_{n+1} - \bar{x} = \frac{\alpha y_n}{\beta + \gamma x_n^r} - \frac{\alpha \bar{y}}{\beta + \gamma\, \bar{x}^r}$$
$$= -\frac{\alpha\, \gamma \bar{y}(x_n^r - \bar{x}^r)}{(\beta+\gamma x_n^r)(\beta+\gamma\, \bar{x}^r)(x_n - \bar{x})}(x_n - \bar{x}) + \frac{\alpha}{\beta+\gamma x_n^r}(y_n - \bar{y}),$$

and

$$y_{n+1} - \bar{y} = \frac{\alpha_1 x_n}{\beta_1 + \gamma_1\, y_n^r} - \frac{\alpha_1 \bar{x}}{\beta_1 + \gamma_1\, \bar{y}^r}$$
$$= \frac{\alpha_1}{\beta_1+\gamma_1\, y_n^r}(x_n - \bar{x}) - \frac{\alpha_1\gamma_1 \bar{x}(y_n^r - \bar{y}^r)}{(\beta_1+\gamma_1\, y_n^r)(\beta_1+\gamma_1\, \bar{y}^r)(y_n - \bar{y})}(y_n - \bar{y}).$$

Let $e_n^1 = x_n - \bar{x}$ and $e_n^2 = y_n - \bar{y}$, one has

$e_{n+1}^1 = a_n e_n^1 + b_n\, e_n^2$,

$e_{n+1}^2 = c_n e_n^1 + d_n\, e_n^2$.

Where

$$a_n = -\frac{\alpha\, \gamma \bar{y}(x_n^r - \bar{x}^r)}{(\beta+\gamma x_n^r)(\beta+\gamma\, \bar{x}^r)(x_n - \bar{x})}, b_n = \frac{\alpha}{\beta+\gamma x_n^r},$$

$$c_n = \frac{\alpha_1}{\beta_1+\gamma_1\, y_n^r}, d_n = -\frac{\alpha_1\gamma_1 \bar{x}(y_n^r - \bar{y}^r)}{(\beta_1+\gamma_1\, y_n^r)(\beta_1+\gamma_1\, \bar{y}^r)(y_n - \bar{y})}.$$

Moreover ,

$$\lim_{n\to\infty} a_n = -\frac{\alpha\gamma r \bar{y}\bar{x}^{r-1}}{(\beta+\gamma\, \bar{x}^r)^2}, \lim_{n\to\infty} b_n = \frac{\alpha}{\beta+\gamma\, \bar{x}^r},$$

$$\lim_{n\to\infty} c_n = \frac{\alpha_1}{\beta_1+\gamma_1\, \bar{y}^r}, \lim_{n\to\infty} d_n = -\frac{\alpha_1\gamma_1 r \bar{x}\bar{y}^{r-1}}{(\beta_1+\gamma_1\, \bar{y}^r)^2}.$$

Now the limiting system of error terms can be written as

$$\begin{pmatrix} e_{n+1}^1 \\ e_{n+1}^2 \end{pmatrix} = \begin{pmatrix} -\frac{\alpha\gamma r \bar{y}\bar{x}^{r-1}}{(\beta+\gamma\, \bar{x}^r)^2} & \frac{\alpha}{\beta+\gamma\, \bar{x}^r} \\ \frac{\alpha_1}{\beta_1+\gamma_1\, \bar{y}^r} & -\frac{\alpha_1\gamma_1 r \bar{x}\bar{y}^{r-1}}{(\beta_1+\gamma_1\, \bar{y}^r)^2} \end{pmatrix} \begin{pmatrix} e_n^1 \\ e_n^2 \end{pmatrix},$$

which is similar to linearized system of (1) about the equilibrium point $O = (0, 0)$. Using the proposition (1), one has following result.

**Theorem 4.** Assume that $\{(x_n, y_n)\}$ be a positive solution of the system (1) such that $\lim_{n\to\infty} x_n = \bar{x}$ and $\lim_{n\to\infty} y_n = \bar{y}$ where

$(\bar{x}\,, \bar{y}) = (0,0)$.

Then, the error term $e_n = \begin{pmatrix} e_n^1 \\ e_n^2 \end{pmatrix}$ of every solution of (1) satisfies both of the following asymptotic relations

$$\lim_{n\to\infty}(\|e_n\|)^{\frac{1}{n}} = |\lambda F_J(\bar{x}\,, \bar{y})|, \lim_{n\to\infty}\frac{\|e_{n+1}\|}{\|e_n\|} = |\lambda F_J(\bar{x}\,, \bar{y})|,$$

where $\lambda F_J(\bar{x}\,, \bar{y})$ are the characteristic roots of Jacobian matrix $F_J(\bar{x}\,, \bar{y})$ about $(0,0)$ .

**5. Examples**

In order to verify our theoretical results we consider several interesting numerical examples in this section. These examples represent different types of qualitative behavior of solutions to the system of nonlinear difference equations (1). All plots in this section are drawn with mathematica.

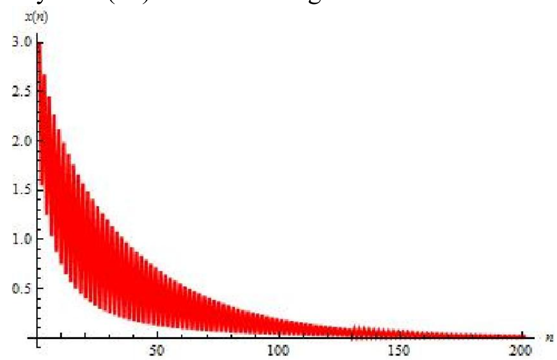
**Example 1.** Consider the system (1) with initial conditions $x_0 = 2.99, y_0 = 1.96$ . Moreover, choosing $\alpha = 115, \beta = 120, \gamma = 2.9, \alpha_1 = 116, \beta_1 =$

$117, \gamma_1 = 0.04, r = 1.994$. Then, the system (1) can be written as

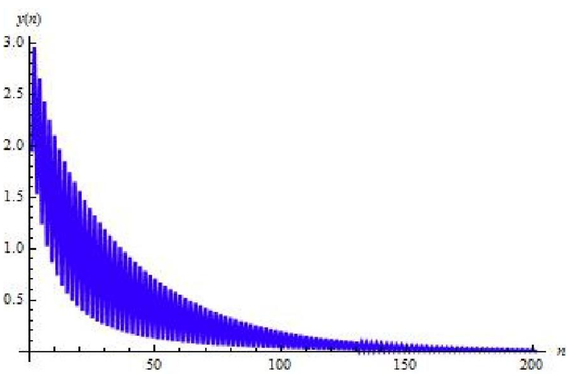
$$x_{n+1} = \frac{115 y_n}{120 + 2.9\, x_n^{1.994}}, \quad y_{n+1} = \frac{116 x_n}{117 + 0.04\, y_n^{1.994}}, \qquad (14)$$

$n = 0, 1, \cdots$, and with initial conditions $x_0 = 2.99$, $y_0 = 1.96$.

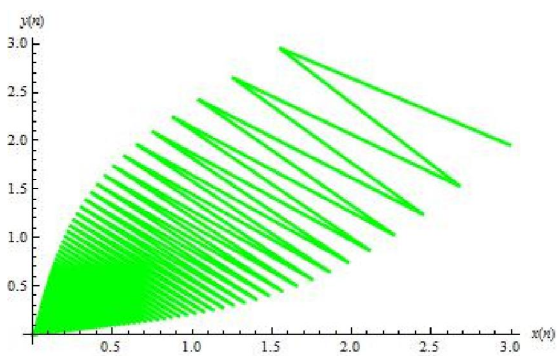
Moreover, in Fig. 1 the plot of $x_n$ is shown in Fig. 1a, the plot of $y_n$ is shown in Fig. 1b and an attractor of the system (14) is shown in Fig. 1c.




(a) Plot of $x_n$ for the system (14)




(b) Plot of $y_n$ for the system (14)




(c) An attractor of the system (14)

**Figure 1: Plots for the system (14)**

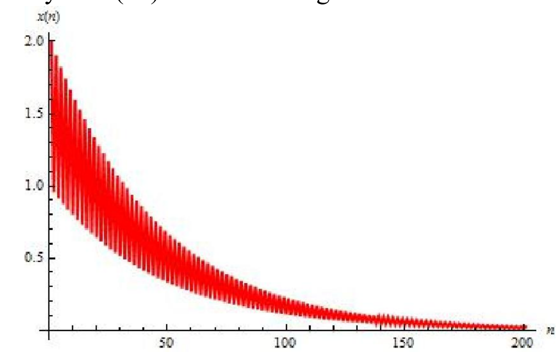
**Example 2.** Consider the system (1) with initial conditions $x_0 = 1.99, y_0 = 0.99$. Moreover, choosing the parameters $\alpha = 155, \beta = 160.6, \gamma = 0.0089$, $\alpha_1 = 169.5, \beta_1 = 170.95, \gamma_1 = 0.0095, r = 0.94$. Then, the system (1) can be written as

$$x_{n+1} = \frac{155 y_n}{160.6 + 0.0089\, x_n^{0.94}}, \quad y_{n+1} = \frac{169.5 x_n}{170.95 + 0.0095\, y_n^{0.94}}$$

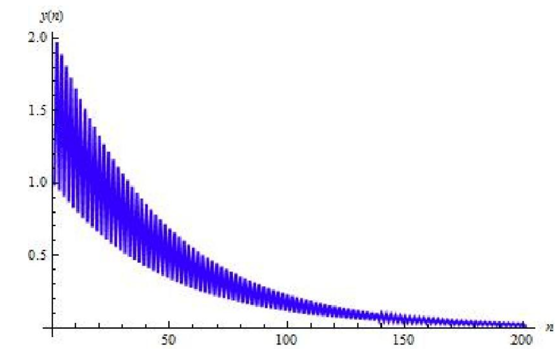
(15)

$n = 0, 1, \cdots$, and with initial conditions $x_0 = 1.99, y_0 = 0.99$.

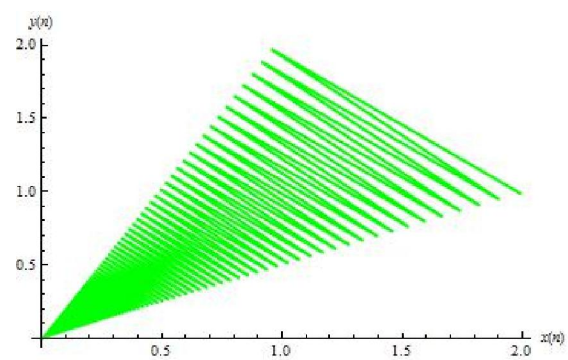
Moreover, in Fig. 2 the plot of $x_n$ is shown in Fig. 2a, the plot of $y_n$ is shown in Fig. 2b and an attractor of the system (15) is shown in Fig. 2c.




(a) Plot of $x_n$ for the system (15)




(b) Plot of $y_n$ for the system (15)




(c) An attractor for the system (15)

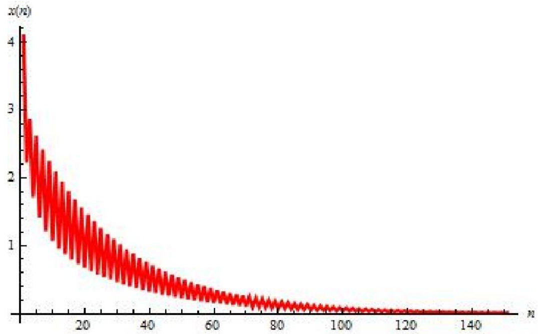
**Figure 2: Plots for the system (15)**

**Example 3.** Consider the system (1) with initial conditions $x_0 = 4.09, y_0 = 3.9$. Moreover, choosing the parameters $\alpha = 13.9, \beta = 14, \gamma = 0.009$, $\alpha_1 = 15, \beta_1 = 16, \gamma_1 = 0.005, r = 5.004$. Then, the system (1) can be written as

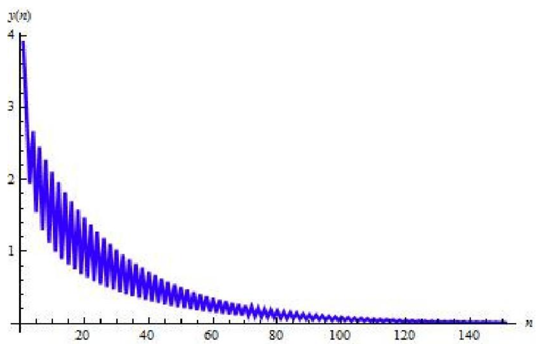
$$x_{n+1} = \frac{13.9 y_n}{14 + 0.009\, x_n^{5.004}}, \quad y_{n+1} = \frac{15 x_n}{16 + 0.005\, y_n^{5.004}} \qquad (16)$$

$n = 0,1,\cdots,$ and with initial conditions $x_0 = 4.09, y_0 = 3.9$.

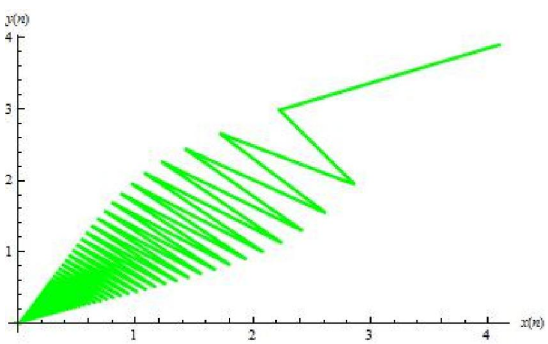
Moreover, in Fig. 3 the plot of $x_n$ is shown in Fig. 3a, the plot of $y_n$ is shown in Fig. 3b and an attractor of the system (16) is shown in Fig. 3c.




(a) Plot of $x_n$ for the system (16)




(b) Plot of $y_n$ for the system (16)




(c) An attractor for the system (16)

**Figure 3: Plots for the system (16)**

## 6. Conclusions

In the paper, we study the asymptotic behavior of an anti-competitive system of rational difference equations. The system has only one equilibrium point (0,0) which is stable under some restrictions on parameters. The linearization method is used to show that equilibrium point (0,0) is locally asymptotically stable. The most important finding here is that the unique equilibrium point (0,0) can be a global asymptotic attractor for the systems (1). Moreover, we have determined the rate of convergence of a solution that converges to unique equilibrium point (0,0). Some numerical examples are provided to support our theoretical results. These examples are experimental verifications of theoretical discussions.

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