Coefficient Of The Differential Equation Of Transverse Vibration Of The Conveyor Belt

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Abstract: Conveyer belts with pretension are a striking example of machines containing blocks (components) with initial stress. Pretension is necessary for passing pulling power by drive pulley, and also for restricting of belt sagging between the roller carriages. The problems about vibrations of the conveyer belt under the influence of longitudinal and transverse forces have been studied. Information on determination of critical speed of belt movement and frequencies of its transverse vibration [1] has been stated; Influence on the values of the above stated parameters of geometrical and physical characteristics of the belt used in numerical methods of the amplitude-frequency characteristic of vibration of the conveyer belt has been considered. Determination of flexural stress in a conveyer belt is one of the major problems which solving is necessary for determination of reliability and durability of a conveyor belt.

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1.Introduction

A physical model of a conveyer belt represents a continuous strip passing on two blocks between two guides without friction with a constant axial transport velocity of υ.

The blocks report the initial static tension to the strip. The tension depends on the speed under normal acceleration on the blocks.

Change of increase in tension is a function from the used system of supporting of blocks. For the fixed blocks the tension is constant as deformation is continuous. For blocks under a constant static tension change of the tension is $+\rho A\upsilon^2$. For spring system of supporting the change is $+\eta\rho A\upsilon^2$ where $0<\eta<1$. The part of the model which is of interest is between two guides.

Methodology and theoretical part.

At researches for periodic solutions of the system of two connected equations of motion the perturbation method is used.

With Hamilton principle we receive the equations of transverse and longitudinal movement of the part of the belt located between two guides without rubbing. The speed of transverse movement of a point in this area is defined with a ratio

$$\frac{\partial V}{\partial t} = \frac{\partial \upsilon}{\partial t} + v\,\frac{\partial \upsilon}{\partial} \qquad (1.1)$$

The speed of longitudinal movement of the point of the belt consists of two parts – the speed of axial transfer of the belt and the local speed caused by the changes of longitudinal movement:

$$: \quad \frac{\partial м}{\partial t} = v + \frac{\partial U}{\partial t} + v\frac{\partial U}{\partial x} \qquad (1.2)$$

The total kinetic energy (T) of the part of the strip located between the guides at any moment is expressed through

$$T=\tfrac{1}{2}\rho A$$

$$\int_0^L\left[\left(\tfrac{\partial \upsilon}{\partial t}+\vartheta\tfrac{\partial \vartheta}{\partial x}\right)^2+\left(\tfrac{\partial U}{\partial t}+\vartheta\left(1+\tfrac{\partial U}{\partial x}\right)\right)^2\right]\partial x \qquad (1.3)$$

Potential energy of deformation (П) equals to the work of external forces.

The contribution to this work from axial loading equals to

$$\Pi_R = \tfrac{1}{2}\int_0^L R\,\varepsilon dx \quad , \qquad (1.4)$$

Where R is resultant tension at any point x. The element with an initial length Δx is deformed into the element with a length

$\Delta x \left[\left(1+\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial \vartheta}{\partial x}\right)^2\right]^{\frac{1}{2}}$ as a result of transverse and axial movements.

Thus, full deformation is defined with the expression

$$\varepsilon = \tfrac{R_0}{EA} + \left[\left(1+\tfrac{\partial U}{\partial x}\right)^2 + \left(\tfrac{\partial \vartheta}{\partial x}\right)^2\right]^{\frac{1}{2}} - 1 \quad , \qquad (1.5)$$

And tension with the expression

R=

$$R_0 \pm EA\left\{\left[\left(1+\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial \vartheta}{\partial x}\right)^2\right]^{\frac{1}{2}}-1\right\}, \quad (1.6)$$

Stuffing of (1.5) and (1.6) into (1.4) gives the contribution to the potential energy defined by the tension П=

$$\frac{EA}{2}\int_0^L\left\{\frac{R_0}{EA}+\left[\left[\left(1+\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial \vartheta}{\partial x}\right)^2\right]^{\frac{1}{2}}-1\right]\right\}^2 dx, \quad (1.7)$$

The potential energy of the bend equals to

П=

$$\frac{1}{2EJ}\int_0^L M^2\,dx, \quad (1.8)$$

The linear ratio between the moment and curvature

$$M = EJ\frac{\partial^2 \vartheta}{\partial x^2}, \quad (1.9)$$

is quite accurate.

Thus, for the part of the belt located between the supports,

Ą=

$$\frac{1}{2}\int_{t_1}^{t_2}\int_0^L\left\{\begin{array}{l}\rho A\left(\frac{\partial \vartheta}{\partial t}+\upsilon\frac{\partial \upsilon}{\partial x}\right)^2+\rho \\ A\left(\frac{\partial U}{\partial t}+\vartheta\right)^2- \\ \left(1+\frac{\partial U}{\partial x}\right) \\ \left[R_0+EA\left[\left(1+\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial}{\partial x}\right)^2\right]^{\frac{1}{2}}\right] \\ {}^2-EJ\frac{\partial^2 \upsilon}{\partial x^2}\end{array}\right\}dxd, \quad (1.10)$$

The equation of motion determined by the two first terms of the expression (1.11) has the following form:

$$\rho A\frac{\partial^2 \upsilon}{\partial t^2}+2\rho A\upsilon\frac{\partial 2U}{\partial x\partial t}+\left(\rho A\upsilon^2-EA\right)\frac{\partial^2 U}{\partial x^2}-$$

$$\left(EA-R_0\right)\frac{\left(1+\frac{\partial U}{\partial x}\right)\frac{\partial \upsilon}{\partial x}\frac{\partial^2 \upsilon}{\partial x^2}-\left(\frac{\partial \upsilon}{\partial x}\right)^2\frac{\partial^2 U}{\partial x^2}}{\left[\left(1+\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial \upsilon}{\partial x}\right)^2\right]^{\frac{3}{2}}}=0. \quad (1.12б)$$

$$\rho A\frac{\partial^2 \upsilon}{\partial t^2}+2\rho A\upsilon\frac{\partial^2 \upsilon}{\partial x\partial t}+\left(\rho A\upsilon^2-EA\right)\frac{\partial^2 \upsilon}{\partial x^2}+EJ\frac{\partial^4 \upsilon}{\partial x^4}+\left(EA-R_0\right)\frac{\left(1+\frac{\partial U}{\partial x}\right)^2\frac{\partial^2 \upsilon}{\partial x^2}-\left(1+\frac{\partial U}{\partial x}\right)\frac{\partial \upsilon}{\partial x}\frac{\partial^2 U}{\partial x^2}}{\left[\left(1+\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial \upsilon}{\partial x}\right)^2\right]^{\frac{3}{2}}}=0. \quad (1.12a)$$

The other terms of the expression (1.11) are defined with natural boundary conditions. The meaning of the third term from (1.11) is either the resultant force of vertical shift equals to zero, or on two supports cross transverse movements are set. The fourth term means that either the bending moment equals to zero, or on the ends the inclination is set. The fifth term requires either the tension on two ends to be the same, or axial motion to be set. The last two terms turn into zero as at the point of time t_1 и t_2 t2 the variations equal to zero by definition (axiomatically).

To bring the equations of motion to dimensionless form we will make the following substitutions:

$$V=\frac{\upsilon}{Д},\quad X=\frac{ч}{Д}.\quad \alpha^2=\frac{T_0}{\rho A}.$$

$$U=\frac{u}{L}.\ \tau=\frac{\alpha t}{L}.\ \beta=\frac{\vartheta}{\alpha}.\ P_0^2=\frac{EJ}{\sigma_0 L^2}.\ P_1^2=\frac{EA}{\sigma_0} \quad (1.13)$$

The tension depending on the speed is expressed with the following formula:

$$R_0=\sigma_0+\eta\rho A\upsilon^2, \quad (1.14)$$

where σ_0- tension applied to the belt at the state of rest $(\upsilon=0)$, and $\eta-$ the constant depending on the system of belt support $(0\le\eta\le I)$. Supposing $\kappa=I-\eta$ and making research of orders of magnitude under the assumption that $V^{4^2}<<V^2$, we will get the system of equations of motion more convenient for solving which still describes this phenomenon:

$$\frac{\partial^2 V}{\partial\tau^2}+2\beta\frac{\partial^2 V}{\partial X\partial\tau}-\left(1-\kappa\beta^2\right)\frac{\partial^2 V}{\partial X^2}+P_0^2\frac{\partial^4 V}{\partial X^4}=$$
$$\left(P_1^2-1+\eta\beta^2\right)\left(\frac{3}{2}\left(\frac{\partial V}{\partial X}\right)^2\frac{\partial^2 V}{\partial X^2}+\frac{\partial U}{\partial X}\frac{\partial^2 V}{\partial X^2}+\frac{\partial V}{\partial X}\frac{\partial^2 U}{\partial X^2}\right)$$
$$.(2.15)$$

$$\frac{\partial^2 U}{\partial\tau^2}+2\beta\frac{\partial^2 U}{\partial X\partial\tau}-\left(P_1^2-\beta^2\right)$$
$$\frac{\partial^2 U}{\partial X^2}=\left(P_1^2-1-\eta\beta^2\right)\frac{\partial V}{\partial X}\frac{\partial^2 V}{\partial X^2}\quad,(1.16)$$

Boundary conditions of free supporting have the following view:

$V(0.\tau)= V(1,\tau) = 0,$

$$\frac{\partial^2 M}{\partial Ч}\left(0,\tau\right)=\frac{\partial^2 M}{\partial Ч^2}\left(1,\tau\right)=0,\qquad(1.17)$$

$U(0,\tau) = U(1,\tau) = 0.$

At such formulation the solution of the uniform equation corresponding to the equation (1.16) is not taken into account.

Therefore, satisfaction of boundary conditions for U in (1.17) is guaranteed by fulfillment of boundary conditions for V.

Any element of the belt is affected by four forces having cross direction as it is visible from (1.15). The first force represents inertial loading and is expressed by $\frac{\partial^2 V}{\partial\tau^2}+2\beta\frac{\partial V}{\partial X\partial\tau}+\beta^2\frac{\partial^2 V}{\partial X^2}$. The value $\frac{\partial^2 V}{\partial\tau^2}$ is connected with local cross acceleration. The value $2\beta\frac{\partial^2 V}{\partial X\partial\tau}$ is the result of Coriolis acceleration, which arises because the material moves with the axial speed β and comes back with the angular speed $\frac{\partial^2 M}{\partial Ч\partial\tau}$. The value $\beta^2\frac{\partial^2 M}{\partial Ч^2}$ is connected with the change of direction β, caused by curvature of the strip $\frac{\partial^2 2V}{\partial X^2}$. The second force - $\left(1+\eta\beta^2\right)\frac{\partial^2 V}{\partial X^2}$ represents the cross shearing force determined by the influence of axial tension. The third force $P_0^2\frac{\partial^4 V}{\partial X^4}$ is usual distributed loading connected with a bend. And the fourth force determined by the right part of the expression (1.15) results from the additional cross shift caused by change of tension at moving of the belt. Similar interpretation can be given to the terms of the equation of longitudinal motion.

The results of the research

The research of the order of values shows that the main equations (1.15) and (1.16) are poorly nonlinear for not too big amplitudes V, as nonlinear terms in (1.15) have the order V3. Therefore for determination of the value of the vibration period we use the perturbation method.

We will substitute the following expansion:

$U=U_0+\mu U_1+\mu^2 U_2+\ldots,$

$V=V_0+\mu V_1+\mu^2 V_2+\ldots,\qquad(1.18)$

$$\tau=\frac{\sigma_0}{\varpi_0}(1+\mu h T_1+\mu^2 T_2+\ldots).$$

into the equations (1.15) and (1.16). Here μ<1 is a small indeterminate parameter, ω_0 is the frequency of linear vibrations, h_j are the constants which are subject to detection. Equating the coefficients at the identical degrees μ to the zero, we receive the following system of equations:

$L_0V_0=0.\qquad(1.19)$

$$L_1U_0=\left(P_1^2-1-\eta\beta^2\right)\frac{\partial V_0}{\partial X}\frac{\partial^2 V_0}{\partial X^2}.\qquad(1.20)$$

$L_1U_0=F_1-$

$$2h\left[\beta\omega_0\frac{\partial^2 V_0}{\partial X\partial T}-\left(1-k\beta^2\right)\frac{\partial^2 V_0}{\partial X^2}+P_0^2\frac{\partial^4 V_0}{\partial X^4}\right].\qquad(1.21)$$

$$L_1U_1=\left(P_1^2-1-\mu\beta^2\right)\left(\frac{\partial V_1}{\partial X}\frac{\partial^2 V_0}{\partial X^2}+\frac{\partial V_0}{\partial X}\frac{\partial^2 V_1}{\partial X^2}\right)$$
$$-2\,{}_1\left[\beta\omega_0\frac{\partial^2 U_0}{\partial X\partial T}-\left(P_1^2-\beta^2\right)\frac{\partial^2 U_0}{\partial X^2}\right]-$$
$$-\left(P_1^2-1-\eta\beta^2\right)\frac{\partial V_0}{\partial X}\frac{\partial^2 V_0}{\partial X^2}\qquad(1.22)$$

$$L_0V_2=F_2-2\,{}_2\left[\begin{array}{l}\beta\omega_0\frac{\partial^2 V_0}{\partial x\partial T}-\left(1-k\beta^2\right)\frac{\partial^2 V_0}{\partial X^2}+\\ P_0^2\frac{\partial^4 V_0}{\partial X^4}\end{array}\right]$$
$$-\,{}_1^2\left[-\left(1-k\beta^2\right)\frac{\partial^2 V_0}{\partial X^2}+P_0^2\frac{\partial^4 V_0}{\partial X^4}\right]-$$
$$-2\,{}_1\left[\beta\omega_0\frac{\partial^2 V_1}{\partial X\partial T}-\left(1-k\beta^2\right)\frac{\partial^2 V_1}{\partial X^2}+P_0^2\frac{\partial^4 V_1}{\partial X^4}-F_1\right]$$
(1.23)

$$L_0=\varpi_0^2\frac{\partial^2}{\partial T^2}+2\beta\varpi_0\frac{\partial^2}{\partial X\partial T}-$$
$$\left(1-k\beta^2\right)\frac{\partial^2}{\partial X^2}+P_0^2\frac{\partial^4}{\partial X^4}$$
$$.\qquad(1.24)$$

$$L_1=\omega_0^2\frac{\partial^2}{\partial T^2}+2\beta\omega_0\frac{\partial^2}{\partial X\partial T}-$$
$$\left(P_1^2-\beta^2\right)\frac{\partial^2}{\partial X^2}\quad.\qquad(1.25)$$

$$F_1=\frac{1}{\mu}\left(P_1^2-1-\eta\beta^2\right)$$
$$\left[\frac{3}{2}\left(\frac{\partial V_0}{\partial X}\right)^2\frac{\partial^2 V_0}{\partial X^2}+\frac{\partial U_0}{\partial X}\frac{\partial^2 V_0}{\partial X^2}+\frac{\partial V_0}{\partial X}\frac{\partial^2 U}{\partial X^2}\right]\qquad(1.26)$$

$$F_2 = \frac{1}{U}\left(P_1^2 - 1 - \eta\beta^2\right)$$
$$\left[\frac{3}{2}\left(\frac{\partial V_0}{\partial X}\right)^2 \frac{\partial^2 V_1}{\partial X^2} + 3\frac{\partial V_1}{\partial X}\frac{\partial V_0}{\partial X}\frac{\partial^2 V_0}{\partial X^2} + \frac{\partial U_0}{\partial X}\frac{\partial^2 V_1}{\partial X^2} + \frac{\partial V_1}{\partial X}\frac{\partial^2 U_0}{\partial X^2} + \frac{\partial U_1}{\partial X}\frac{\partial^2 V_0}{\partial X^2} + \frac{\partial V_0}{\partial X}\frac{\partial^2 V_1}{\partial X^2}\right]. \quad (1.27)$$

Coefficient $\hbar_1$ equals to

$$\hbar_1 = \frac{\int_0^1 \int_0^{2\pi} V_0 F_1 dT dX}{2\int_0^1 \int_0^{2\pi} V_0 \left[\beta_0 \omega_0 \frac{\partial^2 V_0}{\partial X \partial T} - \left(1 - k\beta^2\right)\frac{\partial^2 V_0}{\partial X^2} + P_0^2 \frac{\partial^4 V_0}{\partial X^4}\right] dx dt} \quad (1.28)$$

Then the first approximation for the linear problem has the following view:

$$V=V_0.\ U=U_0.\quad \omega = \frac{\omega_0}{(1+\mu\hbar_1)}. \quad (1.29)$$

$$\hbar_2 = \frac{\int_0^1\int_0^{2\pi} V_0 F_2 dT dX}{2\int_0^1 \int_0^{2\pi} V_0 \left[\beta\omega_0 \frac{\partial^2 V_0}{\partial X \partial T} - \left(1 - k\beta^2\right)\frac{\partial^2 V_0}{\partial X^2} + P_0^2 \frac{\partial^2 V_0}{}\right] dT dX} + \hbar_1^2 \left\{2 - \frac{\int_0^1 \int_0^{2\pi} M_0 \left[- \left(1 - \eta\beta^2\right)\frac{\partial^2 M_0}{\partial HE}\right] + P_0^2 \frac{\partial^4 V_0}{\partial X^4}] dx dT}{2\int_0^1\int_0^{2\pi} V_0 \left[\beta\omega_0 \frac{\partial^2 V_0}{\partial X \partial T} - \left(1 - k\beta^2\right)\frac{\partial^2 V_0}{\partial X^2} + P_0^2 \frac{\partial^4 V_0}{\partial X^4}\right] dX dT}\right\} \quad (1.30)$$

We will notice that the first approximation does not require determination of V1, U1, the following, or the second approximation is determined in parallel (simultaneously), though in more difficult way. We determine the particular solution V1 of the equation (1.21) and reject (give up) small terms in the solution for the first harmonic when they are negligibly small in comparison with the lagging terms. When the values V0, V1, U0 and h1 are known, the particular solution U1 of the equation (1.22) is determined. In the same way from the equation (1.27) F2 is found, and then the right part of the equation (1.23) becomes orthogonal on average to the fundamental solution V_0 by choosing corresponding coefficient h_2. The coefficient h_2 is determined by the following expression.

Then the second approximation for solving of nonlinear equations has the following view:

$$V=V_0 + \mu V_1. \quad U=U_0 + \mu U_1.$$

$$\omega = \frac{\omega_0}{\left(1 + \mu\hbar_1 + \mu^2 \hbar_2^2\right)}. \quad (1.31)$$

Two approximations stated above were used for receiving evaluation of the period of nonlinear vibrations and the speed of convergence of approximate solutions. In the field of low amplitudes of transport velocity the first approximation gives very good results.

Mathematical modeling of oscillation process of a belt with an initial stress.

In conveyer belts initial stress is created artificially with the purpose of ensuring their working capacity, and it has all features of the environment with initial stress. It should be noted that initial stress caused by constructive needs of machines substantially influences strength properties of their junctions. Artificial initial stress being in the static loaded condition in the working process of the machines, turns into dynamic condition that promotes occurrence of complex wave effects in the belt. In this connection research of the nature of interference of static fields of initial stress and perturbed state of the conveyer belt during its operation is of great practical interest. The solution of this problem requires attraction of the theory of environments with initial stress /2, 3/.

Let stress state of the conveyer belt in static state be determined by a tensor of initial stress σ^0_{ij} in perturbed state (wave vibration processes), hereupon an additional tensor of stress S_{ij} appears.

In accordance with /1/ we assume correctness of the principle of superimposed stresses.

Then action of external forces will be determined by stress tensor which components are the following:

$$\sigma_{ij} = \sigma^0_{ij} + S_{ij} \quad (2.1).$$

Then, respectively, the values σ_{xx}, σ_{xy}, σ_{yy} will be determined by dependences of the following type:

$\sigma_{xx}= \sigma_{xx}+ S_{xx}$
$\sigma_{xy}= \sigma_{xy}+ S_{yy}$ (2.2)
$\sigma_{yy}= \sigma_{yy}+ S_{yy}$

similarly

$\sigma_{11}= \sigma^0_{xx}+ S_{11}$
$\sigma_{22}= \sigma^0_{yy} + S_{22}$ (2.3)
$\sigma_{12}= \sigma^0_{xy} + S_{12}$

Between (2.2) and (2.3) there is a usual connection of the following type:

$\sigma_{11}= \sigma^0_{xx} \cos^2\alpha + \sigma^0_{yy}\sin^2\alpha + \sigma^0_{xy}\sin 2\alpha$
$\sigma_{22}=\sigma^0_{xx} \sin^2\alpha + \sigma^0_{yy}\cos^2\alpha + \sigma^0_{xy}\sin 2\alpha$ (2.4)
$\sigma_{12}= ½(\sigma^0_{yy} + \sigma^0_{xx}) \sin 2\alpha + \sigma^0_{xy}\sin 2\alpha$

Substituting in (2.4) ratios (2.2) and (2.3) and supposing that

$\mathrm{Cos}\alpha \approx \cos 2\alpha \approx 1$

$\sin 2\alpha \approx \frac{1}{2} \sin 2\alpha \approx \alpha$ (2.5)

i.e. are correct at small α (α≈w), we will receive Biot formulas/1/

$S_{xx}= S_{11}-2\sigma^0_{yy}w,$
$S_{yy}= S_{22}+2\sigma^0_{xy}w,$ (2.6)
$S_{xy}= S_{12}+(\sigma^0_{xx}+\sigma^0_{yy})\, w.$

In the three-dimensional case $\sigma^*_{ij}= \sigma^0_{ij}+ S_{ij}+ \sigma^0_{\mu j} w_{i\mu}$ (2.7)

where, as above

$$\vartheta_{ij}= \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right) \quad (2.8)$$

$$w_{ij}= \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right) \quad (2.9)$$

Substituting (2.7) into a classical equation of the elasticity theory in the deformed state /40,46/and considering lengthening and shifting to be small in comparison with a unity, we have the following system of equations of motions for the environments with initial stress:

$$\frac{\partial s_{ij}}{\partial x_j}+\sigma^0_{jk}\left(\frac{\partial w_{ir}}{\partial x_j}\right)+\sigma^0_{ik}\left(\frac{\partial w_{ik}}{\partial x_i}\right)-e_{jk}\left(\frac{\partial \sigma^0_{jk}}{\partial xj}\right)=\rho\frac{\partial^2 u}{\partial t^2} \quad (2.10)$$

$\Delta f_i=(S_{ij}+\sigma^0_{jk}w_{ik}+e_{kk}\sigma^0_{ij}-\sigma^0_{ik}e_{jk})n_j$ (2.11)

The received systems of the equations (2.10) and (.11) are the main equations of motion and boundary conditions of the environments with initial stress. The advantage of the systems (2.10) and (2.11) is that initial stress is included in them in a differential form, and the boundary conditions are written in Euler coordinates. Thus, the system (2.10) with boundary conditions (2.11) can be successfully used for analysis of nonlinear oscillatory processes in the belts with initial stress.

Conveyer belts with preliminary tension are a striking example of cars containing knots with initial tension. Pretension is necessary for transfer of pulling power by rubbing with a drive pulley and also for restriction of sagging of the belt between the roller carriages. It is characterized by tensile force and the speed of the movable pulley. Vibrations of the flexible belt with initial stress in one-dimensional case are described with nonlinear equation. We will note that in one-dimensional case only longitudinal wave which speed is determined is taking into account.

For perturbed state of the conveyer belt the connection between stresses and deformations is set in the following way:

$$S_{\xi\xi} = B_{11} e_{xx} + B_{12} e_{yy} + B_{13} e_{zz},$$
$$S_{\eta\eta} = B_{21} e_{xx} + B_{22} e_{yy} + B_{23} e_{zz},$$
$$S_{\zeta\zeta} = B_{31} e_{xx} + B_{32} e_{yy} + B_{33} e_{zz}. \quad (2.12)$$

$$S_{\eta\xi} = 2\theta_2 e_{yz},$$
$$S_{\xi\zeta} = 2\theta_2 e_{zx},$$
$$S_{\zeta\eta} = 2\theta_3 e_{xy}. \quad (2.13)$$

Thus shear coefficients θ_i are determined with the following ratio:

$$\theta_1 = \frac{1}{2}\left(\sigma^0_{22} - \sigma^0_{33}\right)\frac{\lambda_2^2+\lambda_1^2}{\lambda_3^2-\lambda_1^2},$$
$$\theta_2 = \frac{1}{2}\left(\sigma^0_{33} - \sigma^0_{11}\right)\frac{\lambda_3^2+\lambda_1^2}{\lambda_3^2-\lambda_1^2},$$
$$\theta_3 = \frac{1}{2}\left(\sigma^0_{11} - \sigma^0_{22}\right)\frac{\lambda_1^2+\lambda_2^2}{\lambda_1^2-\lambda_2^2} \quad (2.14)$$

Vibrations of the flexible belt with initial stress in one-dimensional case can be described with nonlinear equations of the following type:

$$\frac{\partial^2 u}{\partial t^2}+\frac{1}{\alpha}\frac{\partial \sigma^0_{xx}}{\partial_{xx}}\frac{\partial u}{\partial x}=\frac{1}{a_1^2}\frac{\partial^2 u}{\partial t^2} \quad (2.15)$$

where

$$a_1 = \sqrt{\frac{1}{\rho}E\left(1+e_{xx}\right)\pm\frac{\sigma^0_{xx}}{\rho}}, \quad (2.16)$$

a_1 is the velocity of elastic wave propagation. We will note that in one-dimensional case only the longitudinal wave which speed is determined by dependence (2.16) is taken into account. Depending on the simplification systems we get various formulas for determination of velocity of the longitudinal wave, for example:

$$a_2 = \sqrt{\frac{1}{\rho} E\left(1 + e_{xx}\right) \pm \frac{\sigma_{xx}^0}{\rho}}, \quad (2.17)$$

Thus it was found out that in the belts with initial stress longitudinal waves arise before appearance of transverse vibrations. The velocity of distribution of these waves depends on the tensor of initial stress σ^0_{ij}. At determination of durability of the belts it is necessary to use dynamic coefficient of Young modulus. In the case when lengthening and shifts are small in comparison with a unity it is possible to linearize the main nonlinear equation (2.15) that considerably simplifies its research and receiving analytical solution /4/.

Conclusions

Conveyer belts with pretension are a striking example of machines containing junctions with initial stress. Pretension is necessary for transfer of pulling power by rubbing with a drive pulley and also for restriction of sagging of the belt between the roller carriages. It is characterized by tensile force and the speed of the movable pulley. Vibrations of the flexible belt with initial stress in one-dimensional case are described with nonlinear equation. We will note that in one-dimensional case only the longitudinal wave which speed is determined is taking into account.

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