Quantum Schrödinger Equation with Octic Potential In Non Commutaive Two-Dimensional Complex space

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Abstract: In this work, The effect of the non commutativity is studied, on the **with Octic** potential (Harmonic, Quadratic, sextic...) $V(r) = ar^2 - br^4 + cr^6 - dr^8 + er^{10}$, by applying the Boopp's shift method to first order in the non-commutativity parameter θ, we shown that the NC Hamiltonian will be represented into two matrixes. The modified of the energies levels at the fundamental state determined only with a numerical solution. One can conclude from this work that the non-commutativity applying on the **Octic** potential, produced two types of interactions between a particle with spin $(1/2)$ and an external magnetic field, the first one represent the ordinary Zeeman effect (anisotropic interaction),and the new interaction represent a cobbling between the total monument and external magnetic field (Isotropic interaction).·

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1-Introduction:

The divergence problem in quantum field theory (QFT), unified of gravitational interaction with standard model,are a fundamental reasons to introduce a new concepts of space-time, known by noncommutative spaces $[1,3,15,19,20,21,25,26]$, the noncommutativity is introduced by many ways, the simple approach, it consider the position and momentum operators obeys to the Heisenberg commutation relation, that is similarly to quantize space-time coordinates $[\theta]$, when the commutator $[\hat{x}_i, \hat{x}_j] \neq 0$, the mathematical formalism of star product, Boopp's shift method and the Seiberg-Witten map are plays a fundamental roles in this new theory. The rich mathematical structure of the noncommutative theory gives a rise to the hop to get a better understanding of physics phenomena at smallness distances and to solve above-mentioned problems. The physics idea of a noncommutative space satisfied by anew mathematical product which replaces the old ordinary product known by star product, noted by $(*)$ [6, 7, 8, 9, 10, 11, 12, 25, 26]:

$$[x_i, x_j]_* = x_i * x_j - x_j * x_i = i\theta_{ij} \quad \text{.........(1)}$$

Throughout this paper the natural unites $(c = \hbar = 1)$ and $\mu = \frac{1}{2}$ are employed. A Boopp's shift method will be used in our paper, Instead of solving the non commutative Schrödinger equation by using stat product procedure:

$$[\hat{x}_i, \hat{x}_j] = \hat{x}_i\hat{x}_j - \hat{x}_j\hat{x}_i = i\theta_{ij} \quad \text{......(2)}$$

We replace the star product with usual product together with a Boopp's shift $[25, 26, 27, 28, 29, 30]$:

$$\hat{x}_i = x_i - \frac{\theta^{ij}}{2}p_j \text{ and } \hat{p}_i = p_i \quad \text{where } i = \overline{1,N} \quad \text{....(3)}$$

The parameter θ^{ij} is an antisymmetric real matrix of dimension square length in the noncommutative canonical-type space. The star product between two arbitrary functions $f(x)$ and $g(x)$, in the first order of θ, as follow [6, 7, 8, 9, 10, 11, 12, 25, 26]:

$$f(x) * g(x) = f(x).g(x) - \frac{i}{2}\theta^{ij}(\partial_i f(x))(\partial_j g(x)) \quad \text{......(4)}$$

We remarked that the result of equation (1) satisfied by applying the notion of the star product represented by eq.(4). Actually there are many attempts to study noncommutative space time in three dimensional space, but a limited physics phenomenon's are studied in two dimensional space, the hydrogen atom considered a ideal model in physics, we want to study this atom in two dimensional (2D) space, by applying the new concepts of space, based on the complex coordinate $\hat{z} = \hat{x} + i\hat{y}$ and corresponding momentums $p_{\hat{z}} = (p_x - ip_y)/2$ [6,17].The aim to this work, is to study an to discover the physical effect of the non-commutativity two dimensional complex space of the Octic potential.

This paper is organized as follows. In section 2 we present the Complex non commutative space**.** In section 3, we present Octic potential in ordinary two dimensional complexes. In section 4 we derive the deformed Hydrogen atom with Octic potential in noncommutative complex space. We solve this equation and obtain the non-commutative modification of the energy levels. Finally, in section 5, we draw our conclusions.

2-Complex non-commutative two dimensional spaces:

In 2D space, the complex coordinates system $(z, \bar{z})$ and their momentums $(p_z, p_{\bar{z}})$, defined by [6,17]:

$$\begin{cases} z = x + iy \quad , \quad \bar{z} = x - iy \\ \text{and} \\ p_z = \frac{1}{2}(p_x - ip_y) \quad , \quad p_{\bar{z}} = \bar{p}_z = \frac{1}{2}(p_x + ip_y) \end{cases} \quad ....(5)$$

The NC two dimensional complex space, formulated by the following operators coordinates and their momentums $\hat{z}$, $\hat{\bar{z}}$, $\hat{p}_z$ and $\hat{p}_{\bar{z}}$, as follows:

$$\begin{cases} \hat{z} = \hat{x} + i\hat{y} = z + i\frac{\theta^{xy}}{2}(p_x + ip_y) = z + i\theta p_{\bar{z}} \\ \hat{\bar{z}} = \hat{x} - i\hat{y} = \bar{z} - i\frac{\theta^{xy}}{2}(p_x - ip_y) = \bar{z} - i\theta^{xy} p_z \\ \hat{p}_z = p_z, \quad \hat{p}_{\bar{z}} = p_{\bar{z}} \end{cases} \quad ....(6)$$

One can show that, the square of the position operator $\hat{r}^2$, in NC 2D complex space, can be determined, from two possibility methods, the first one $\hat{r}^2 = \hat{\bar{z}}\hat{z}$, using the algebra (6), one show that, in the first order of the parameter θ:

$$\hat{\bar{z}}\hat{z} = z\bar{z} - \theta(L_z - 1) \quad ....(7)$$

While the second values of $\hat{r}^2 = \hat{z}\hat{\bar{z}}$, after a straightforward calculation, one get this values, in the first order of the parameter θ:

$$\hat{z}\hat{\bar{z}} = z\bar{z} - \theta(L_z + 1) \quad ...(8)$$

Furthermore, one can show that the two commutators $[\hat{z}, \hat{\bar{z}}]$ and $[\hat{z}, p_z]$ expressed as, after a straightforward calculation:

$$\begin{cases} [\hat{z}, \hat{\bar{z}}] = 2\theta \\ [\hat{z}, p_z] = i \end{cases} \quad ......(9)$$

For a kinetic term $\frac{1}{2m_0}\overset{\leftrightarrow}{p}^2$ term, defined into ordinary 2D Complex space, as follows:

$$\frac{1}{2m_0}(p_x + ip_y)(p_x - ip_y) = \frac{1}{2m_0}(p_x - ip_y)(p_x + ip_y) \quad ......(10)$$

Where m_0 is the rest masses, which take the same values, in NC 2D complex space:

$$\frac{1}{2m_0}\overset{\leftrightarrow}{p}^2 = \frac{2}{m_0}p_{\bar{z}}p_z = \frac{2}{m_0}p_z p_{\bar{z}} \quad ....(11)$$

This results satisfied because the non commutativity of space-time represented by eq.(3) imposed that $\hat{p}_i = p_i$.

3-The Octic potential in ordinary two dimensional complex spaces:

We expressed the Schrödinger equation corresponding the Octic potential $V(r)$ in ordinary two dimensional spaces, in the polar coordinate (r, φ) [23,27]:

$$\left(-\frac{1}{2m}\Delta + V(r)\right)\Psi(\vec{r}) = E\Psi(\vec{r}) \quad ....(12)$$

Where $\Delta = r\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}$ is Laplacien in a polar coordinate and $V(r)$ described Octic potential in 2D ordinary space:

$$V(r) = ar^2 - br^4 + cr^6 - dr^8 + er^{10} \quad ....(13)$$

The parameter $d \prec 0$, the normalized complete wave eq. $\Psi(\vec{r})$ will be writing to the form:

$$\Psi(\vec{r}) = \frac{R_m(r)}{\sqrt{r}}\Phi(\varphi) \quad ....(14)$$

Where $R(r)$ and $\Phi(\varphi)$ are the radial function and the angler function satisfied the following form respectively:

$$\left(\frac{d^2}{dr^2} + E - V(r) - \frac{m^2 - \frac{1}{4}}{r^2}\right)R(r) = 0$$

$$\frac{d^2\Phi_m(\varphi)}{d\varphi^2} + m^2\Phi_m(\varphi) = 0 \quad \ldots\ldots\ldots\ldots(15)$$

Where E is the energy eigenvalues and m is the orbital angular momentum quantum numbers, the standard solution of $\Phi(\varphi)$ given by :

$\Phi(\varphi) = \exp(\pm im\varphi)$ where $m = 0,1,2...$ …………..(16)

The radial function $R_m(r)$ determined for the ground state by [27]:

$R_m(r) = \exp(p_{m0}(r))$ ………………………(17)

Where $p_{m0}(r)$ given by:

$$p_{m0}(r) = \frac{1}{2}\alpha r^2 - \frac{1}{2}\beta r^4 + \frac{1}{6}\sqrt{e}r^6 + k\ln(r) \quad ..(18)$$

Where

$$\beta^2 + 2\alpha\sqrt{e} = c,\ 2\beta\sqrt{e} = d$$

$$\alpha^2 - 2\beta\sqrt{e} - 3\beta = a,\ 5\sqrt{e} + 2\left(k = \pm m + \frac{1}{2}\right)\sqrt{e} - 2\alpha\beta = -b \quad .(19)$$

Then, the radial function $R_0(r)$ and the energy of the fundamental state E_0 are respectively [27] :

$$\begin{cases} R_{m0}(r) = N_0 r^k \exp\left(\frac{1}{2}\alpha r^2 - \frac{1}{4}\beta r^4 + \frac{1}{6}\tau r^6\right) \\ E_0 = -\dfrac{(1+2k)\left(d^2 - 4ce\right)}{8e^{\frac{3}{2}}} \end{cases} \quad \ldots\ldots\ldots(20)$$

Where N_0 is the normalized constant determined from the normalization relation:

$$\int_0^{+\infty} |R_{m0}(r)|^2 dr = 1 \quad \ldots\ldots\ldots\ldots(21)$$

4-The Octic potential in NC two dimensional complex spaces:

The NC Hamiltonian operator associated with Octic potential $\hat{H}_{NC}$ in the NC 2D complex space, determined from the relation:

$$\hat{H}_{NC} = \frac{2}{m_0}p_z p_{\bar{z}} + \hat{V}(\hat{r}) \quad \ldots\ldots..(22)$$

Where $\hat{V}(\hat{r})$ is the operator of Octic potential in NC 2D complex space. The Schrödinger equation n **NC** 2D complex space:

$$\left(-\frac{1}{2m}\Delta + \hat{V}(\hat{r})\right) * \hat{\Psi}(\overleftrightarrow{r}) = E_{NC}\hat{\Psi}(\overleftrightarrow{r}) \quad \ldots\ldots(23)$$

Where $\hat{\Psi}(\overleftrightarrow{r})$ is complete wave function, the kinetic term $\frac{2}{m_0}p_z p_{\bar{z}}$ replaced by $\left(-\frac{1}{2m_0}\Delta\right)$, while E_{NC} is the NC eigenvalues of energy associated with Octic potential. With a Boopp's shift method, the above eq. will be, as follows:

$$\left(-\frac{1}{2m}\Delta + V\left(\hat{x}_i = x_i - \frac{\theta^{ij}}{2}p_j\right)\right)\Psi(\vec{r}) = E_{NC}\Psi(\vec{r}) \text{ and } \Psi(\vec{r}) = \frac{R_m(r)}{\sqrt{r}}\Phi_m(\varphi) \quad \ldots(24)$$

Where $V\left(\hat{x}_i = x_i - \frac{\theta^{ij}}{2}p_j\right)$, is the ordinary potential in 2D space, as a function of $\left(\hat{x}_i = x_i - \frac{\theta^{ij}}{2}p_j\right)$ instead of ordinary position x_i, then, the Octic potential $V(\hat{r})$, will be written to the form:

$$V(\hat{r}) = a\hat{r}^2 - b\hat{r}^4 + c\hat{r}^6 - d\hat{r}^8 + e\hat{r}^{10} \quad \ldots\ldots(25)$$

The radial equation and the angler function in NC 2D complex space satisfied the following form respectively:

$$\begin{cases} \left(\frac{d^2}{dr^2} + E_{NC} - V(r) - \frac{m^2 - \frac{1}{4}}{r^2}\right)R(r) = 0 \\ \frac{d^2\Phi_m(\varphi)}{d\varphi^2} + m^2\Phi_m(\varphi) = 0 \end{cases} \quad \ldots\ldots.(26)$$

we have seen that in section 2 the position operator in two dimensional NC 2D complex space have two possible values, the first one is $\hat{r}^2 = z\bar{z} - \theta(L_z - 1)$, the different terms of Octic potential $V(\hat{r})$ are:

$$\begin{cases} a\hat{r}^2 = ar^2 - a\theta(L_z - 1) \\ -b\hat{r}^4 = -br^4 - 2\frac{b\theta}{r^6}(L_z - 1) \\ c\hat{r}^6 = cr^6 - 3c\theta r^4(L_z - 1) \\ -d\hat{r}^8 = -dr^8 + 4d\theta r^6(L_z - 1) \\ e\hat{r}^{10} = er^{10} - 5e\theta r^8(L_z - 1) \end{cases} \quad \ldots(27)$$

Which follow to write the Octic potential as follows:

$$V(\hat{r}) = V(r) + V_{pert1}(r,\theta) \quad \ldots\ldots\ldots(28)$$

Where $V_{pert1}(r)$ Is given by:

$$V_{pert1}(r) = \theta\left(-a - 2\frac{b}{r^{6}} - 3cr^{4} + 4dr^{6} - 5er^{8}\right)(L_z - 1)$$
…….(29)

We remarked that the term $V_{pert1}(r)$ is proportional to the smallness parameter θ, then we considers as a perturbaive term. Know using the second values of operator $r^2 \rightarrow r^2 - \theta(L_z - 1)$ to get the terms of Octic potential:

$$\begin{cases} ar^{2} \rightarrow ar^{2} - a\theta(L_z - 1) \\ br^{-2} \rightarrow br^{-4} - 2\frac{b}{r^{6}}\theta(L_z - 1) \\ cr^{6} \rightarrow cr^{6} - 3cr^{4}\theta(L_z - 1) \\ dr^{8} \rightarrow dr^{8} + 4dr^{6}\theta(L_z - 1) \\ er^{10} \rightarrow er^{10} - 5er^{8}\theta(L_z - 1) \end{cases}$$
….(30)

Winch follow to write:

$$V(r) \rightarrow V(r) + V_{pert2}(r)$$
……………..……(31)

Where $V_{pert2}(r)$ Is given by:

$$V_{pert2}(r) = \theta\left(-a - 2\frac{b}{r^{6}} - 3cr^{4} + 4dr^{6} - 5er^{8}\right)(L_z + 1)$$
….(32)

Also we remarked that, the term $V_{pert2}(r)$ is consider a perturbaive term. A straightforward calculation leads to get the two radials functions corresponding $V_{pert1}(r)$ and $V_{pert2}(r)$ as follow:

$$\begin{cases} \left(\frac{d^2}{dr^2} + E_{NC} - V(r) - V_{pert1}(r) - \frac{m^2 - \frac{1}{4}}{r^2}\right)R(r) = 0 \\ \text{and} \\ \left(\frac{d^2}{dr^2} + E_{NC} - V(r) - V_{pert2}(r) - \frac{m^2 - \frac{1}{4}}{r^2}\right)R(r) = 0 \end{cases}$$
……

…..(33)

We observed that the two operators $V_{pert1}(r)$ and $V_{pert2}(r)$ are proportional to $\theta(L_z - 1)$ and $\theta(L_z + 1)$ respectively. Its important to notice that $\theta(L_z - 1)$ and $\theta(L_z + 1)$ will be rewrite to the two forms $\theta(L_z - 2(s_z = +\frac{1}{2}))$ and $\theta(L_z - 2(s_z = -\frac{1}{2}))$ which are corresponding a particle with *spin up* and *spin down* respectively, the modification to the energy associate with *spin up* $(E_{NC}\uparrow)$ and *spin down* $(E_{NC}\downarrow)$ at first order of θ, obtained by the applying the perturbation theory:

$$E_{01}\uparrow = \int \psi^{*}(\vec{r}) V_{pert1}(r) \psi(\vec{r}) ds$$

and

$$E_{01}\downarrow = \int \psi^{*}(\vec{r}) V_{pert2}(r) \psi(\vec{r}) ds$$

…..(34)

Where $ds = rdrd\varphi$ instead of $dxdydz$, the new radial function $f(r)$, defined as follows:

$$f(r) = -a - 2\frac{b}{r^{6}} - 3cr^{4} + 4dr^{6} - 5er^{8}$$
……….(35)

The non-commutative modification of the energy levels associated with *spin up* at the first order of θ corresponding the ground state $E_{01}\uparrow$ determined by using equation two eqs. (20) and (34):

$$E_{01}\uparrow = -2\pi(0-1)\theta|N_0|^2 \int_0^{+\infty} r^{2k} \exp\left(\alpha r^2 - \beta r^4 + \frac{1}{3}\tau r^6\right) f(r) dr$$
…….(36)

This can be written as follow:

$$E_{01}\uparrow = -2\pi\theta|N_0|^2\left(-aT^1 - 2bT^2 - 3cT^3 + 4dT^4 - 5eT^5\right)$$
……(37)

Where T^1, T^2, T^3, T^4 and T^5 are given by:

$$\begin{cases} T^1 = \int_0^{+\infty} r^{2k+1} \exp\left(\alpha r^2 - \beta r^4 + \frac{1}{3}\tau r^6\right) dr \\ T^2 = \int_0^{+\infty} r^{2k-5} \exp\left(\alpha r^2 - \beta r^4 + \frac{1}{3}\tau r^6\right) dr \\ T^3 = \int_0^{+\infty} r^{2k+5} \exp\left(\alpha r^2 - \beta r^4 + \frac{1}{3}\tau r^6\right) dr \\ T^4 = \int_0^{+\infty} r^{2k+7} \exp\left(\alpha r^2 - \beta r^4 + \frac{1}{3}\tau r^6\right) dr \\ T^5 = \int_0^{+\infty} r^{2k+9} \exp\left(\alpha r^2 - \beta r^4 + \frac{1}{3}\tau r^6\right) dr \end{cases}$$
……..(38)

Concerning the non-commutative modification of the energy levels associated with *spin down* at the first order of θ corresponding the ground state $E_{01}\downarrow$ determined by using equation two eqs. (20) and (34):

$$E_{10}\downarrow = 4\pi\theta|N_0|^2 \int_0^{+\infty} r^{2k} \exp\left(\alpha r^2 - \beta r^4 + \frac{1}{3}\tau r^6\right) f(r) dr$$

…..(39)

This can be written as follow:

$E_{10}\downarrow = 4\pi\theta|N_0|^2\left(-aT^1 - 2bT^2 - 3cT^3 + 4dT^4 - 5eT^5\right)$

….(40)

It is important to notice that, the terms: T^1, T^2, T^3, T^4 and T^5 represented by equation (38), impossible to determined, because the integral form: $\int_0^{+\infty} r^{2k}\exp\left(\alpha r^2 - \beta r^4 + \frac{1}{3}\tau r^6\right)dr$ is very difficult task, as its observed in page: 6 at reference [23], the only solution to this problem will be numerically. Know we summarize the obtained results of the energy of fundamental state $(E_{NC}\uparrow$ and $E_{NC}\downarrow)$ associated with (*spin up* and *spin down*) particle respectively:

$$E_{NC}\uparrow = E_0 + E_{01}\uparrow$$
$$E_{NC}\downarrow = E_0 + E_{01}\downarrow \quad ……..(41)$$

Winch follows to write:

$$E_{NC}\uparrow = -\frac{(1+2k)(d^2-4ce)}{8e^{\frac{3}{2}}} - 2\pi\theta|N_0|^2\left(-aT^1 - 2bT^2 - 3cT^3 + 4dT^4 - 5eT^5\right)$$
$$E_{NC}\downarrow = -\frac{(1+2k)(d^2-4ce)}{8e^{\frac{3}{2}}} + 2\pi(m+1)\theta|N_0|^2\left(-aT^1 - 2bT^2 - 3cT^3 + 4dT^4 - 5eT^5\right)$$

…..(42)

Regarding the two eqs. (29) and (32), one can show that, the corresponds two NC Hamiltonian operators $\hat{H}_{nc}^{\uparrow}$ and $\hat{H}_{nc}^{\downarrow}$ respectively, determined from the relation:

$$\hat{H}_{nc}^{\uparrow} = \frac{2}{m}p_z p_{\bar z} + V(r) + \theta(r)(L_z - 2s_z)\hat{H} \quad \text{where} \quad s_z = +1/2 \text{ spin down}$$
$$\hat{H}_{nc}^{\downarrow} = \frac{2}{m}p_z p_{\bar z} + V(r) + \theta(r)(L_z - 2s_z)\hat{H} \quad \text{where} \quad s_z = -1/2 \text{ spin down}$$

…..(43)

These results confirmed that, the NC Hamiltonian $\hat{H}_{NC-mag}$, in NC 2D complex space will be represented by a diagonal matrix of order (2×2), as follow:

$$\hat{H}_{NC-mag} = \begin{pmatrix} \hat{H}_{nc}^{\uparrow} & 0 \\ 0 & \hat{H}_{nc}^{\downarrow} \end{pmatrix}$$

………(44)

The explicit form of above diagonal matrix determined from:

$$\hat{H}_{NC-mag} = \begin{pmatrix} \frac{2}{m_0}p_z p_{\bar z} + V(r) + \theta(r)(L_z - 2s_z = +1/2) & 0 \\ 0 & \frac{2}{m_0}p_z p_{\bar z} + V(r) + \theta(r)(L_z - 2s_z = -1/2) \end{pmatrix}$$

….(45)

Regarding to the eq.(42), the eigenvalues of energies are real, then the NC Hamiltonian $\hat{H}_{NC-mag}$ is Hermitian, represented a particle with spin $(1/2)$ in a uniform external magnetic field. The matrix (45) can be rewritten as follows:

$$\hat{H}_{NC-mag} = \hat{H}_{or} + \hat{H}_{mag-}$$

….(46)

Where $\hat{H}_{or}$, is represented by a following matrix, as follows:

$$\hat{H}_{or} = \left[\frac{2}{m_0}p_z p_{\bar z} + V(r)\right]I_{2\times 2}$$

….(47)

Where the united identity matrix is $I_{2\times 2}$ in 2D space, and $\hat{H}_{mag-}$ is represented by a following matrix, as follows:

$$\hat{H}_{mag-} = \theta(r)\begin{pmatrix} (L_z - 2s_z = +1/2) & 0 \\ 0 & (L_z - 2s_z = -1/2) \end{pmatrix}$$

…..(48)

Furthermore, on based to the references [28, 29, 30, 31], one can write:

$$\theta(L_z - 1) = \theta(L_z - 2s_z) = \alpha\overleftrightarrow{J}\overleftrightarrow{B} - 3\alpha\overleftrightarrow{s}\overleftrightarrow{B} \quad \text{if} \quad s_z = +\tfrac{1}{2}$$
$$\theta(L_z + 1) = \theta(L_z - 2s_z) = \alpha\overleftrightarrow{J}\overleftrightarrow{B} - 3\alpha\overleftrightarrow{s}\overleftrightarrow{B} \quad \text{if} \quad s_z = -\tfrac{1}{2}$$

……….(49)

Where $\overleftrightarrow{B} = B\overleftrightarrow{k}$ (the vector of a magnetic field, which oriented with (Oz.) axes), $\theta = \alpha B$ (α is a proportional constant), ($\overleftrightarrow{\mu} = \gamma\overleftrightarrow{s}$ is the magnetic moment, as its remarked in the introduction, $\overleftrightarrow{\mu} = \overleftrightarrow{s}$, then $\gamma = 1$), one can write the Hamiltonian represented the Zeeman effect H_{Zeeman} as follows [30]:

$$H_{Zeeman} = -\overleftrightarrow{\mu}\overleftrightarrow{B} = -\overleftrightarrow{s}\overleftrightarrow{B} \quad ……..(50)$$

Where $\left(\overleftrightarrow{J} = \overleftrightarrow{L} + \overleftrightarrow{s}\right)$, using two eqs. (49) and (50), one can write the matrix $\hat{H}_{mag-}$ as follows:

$$\hat{H}_{mag-} = -\alpha(r)\overrightarrow{J}\overrightarrow{B}I_{2\times 2} - 3\alpha(r)H_{Zeeman}I_{2\times 2}$$

…….(51)

Physically, the matrix (47), represented a particle with spin $(1/2)$ interacted exactly with the **Octic** potential in ordinary 2D space [27], the corresponded wave $\Psi(r,\varphi)$ and the energy are determined from the eq. (20), while the matrix (48) represented two interactions between a particle with spin $(1/2)$ and a external magnetic field, the first one represent the ordinary Zeeman effect and the new interaction represent a cobbling between the total monument $\overleftrightarrow{J}$ and external magnetic field $\overleftrightarrow{B}$. The radial function $f(r)$, traduced the physics particularity of the studied potential. Furthermore, instead to eq.(43) one can write:

$$\hat{H} = \frac{2}{m_0} p_z p_z + V(r) + \theta(L_z - 2s_z) \hat{H} \quad \text{where} \quad s_z = -1/2 \quad \text{spin down}$$
$$\hat{H} = \frac{2}{m_0} p_z p_z + V(r) + \theta(L_z + 2s_z) \hat{H} \quad \text{where} \quad s_z = +1/2 \quad \text{spin up}$$

…..(52)

Which follow to give another form of the second matrix to the NC Hamiltonian operator $\hat{H}_{NC2}$, in NC 2D complex space, as follows:

$$\hat{H}_{NC2} = \begin{pmatrix} \hat{H}_{or} + f(r)\theta(L_z + 2\theta_z = -1/2) & 0 \\ 0 & \hat{H}_{or} + f(r)\theta(L_z + 2\theta_z = +1/2) \end{pmatrix}$$

…(53)

Furthermore, one can write:

$$\theta(L_z - 1) = \theta(L_z + 2s_z) = \alpha\left(\overleftrightarrow{J}\overleftrightarrow{B} + \overleftrightarrow{s}\overleftrightarrow{B}\right) \quad \text{if} \quad s_z = -\tfrac{1}{2}$$
$$\theta(L_z + 1) = \theta(L_z + 2s_z) = \alpha\left(\overleftrightarrow{J}\overleftrightarrow{B} + \overleftrightarrow{s}\overleftrightarrow{B}\right) \quad \text{if} \quad s_z = +\tfrac{1}{2} \ldots$$

…..(54)

The matrix (53) represents the NC Hamiltonian operator $\hat{H}_{NC2}$:

$$\hat{H}_{NC2} = \begin{pmatrix} \hat{H}_{or} + f(r)\left(\alpha\vec{J}\vec{B} + \alpha\left(\vec{s} = -1/2\right)\vec{B}\right) & 0 \\ 0 & \hat{H}_{or} + f(r)\left(\alpha\vec{J}\vec{B} + \alpha\left(\vec{s} = +1/2\right)\vec{B}\right) \end{pmatrix}$$

…(55)

Also, after a straightforward calculation, one can write $\widehat{H}_{mag2}$:

$$\widehat{H}_{mag2} = \alpha f(r) H_{Zeeman} I_{2\times 2} - \alpha f(r) \overrightarrow{J}\overrightarrow{B} I_{2\times 2}$$

….(56)

Which represented a particle fermionic interacted with a magnetic field in two types, the first one is the effect of Zeeman and the second is a new cobbling between the total monument $\overleftrightarrow{J}$ and external magnetic field $\overleftrightarrow{B}$. Furthermore, regarding to the two eqs. (51) and (56), we observed that the first term $(-3\alpha f(r)H_{Zeeman}I_{2\times2})$ in eq. (51) and the first term $(\alpha f(r)H_{Zeeman}I_{2\times2})$ in eq. (56) changed in the values and the direction orientation of direction to projection of spin with $\overleftrightarrow{B}$, while, the second magnetic parts $(-\alpha f(r)\overleftrightarrow{J}\overleftrightarrow{B}I_{2\times2})$ is invariant in two eqs.

Thus, the first part $(-3\alpha f(r)H_{Zeeman}I_{2\times2})$ in eq. (51) and the first part $(\alpha f(r)H_{Zeeman}I_{2\times2})$ in eq. (56) is considered an anisotropic interaction, while the second magnetic parts $(-\alpha f(r)\overleftrightarrow{J}\overleftrightarrow{B}I_{2\times2})$ is considered an isotropic interaction. The global interaction between the external magnetic field modified the energy of ordinary space by E_{NC1} and E_{NC2}:

$$E_{NC1} \uparrow = 2\pi(m-1)\theta|N_0|^2\left(-aT^1 - 2bT^2 - 3cT^3 + 4dT^4 - 5eT^5\right)$$
$$E_{NC2} \downarrow = 2\pi(m+1)\theta|N_0|^2\left(-aT^1 - 2bT^2 - 3cT^3 + 4dT^4 - 5eT^5\right)$$

……(57)

Then, we can talk that, the property of the noncommutativity of space created automatically two types of the interactions between the particle and external magnetic field $\overrightarrow{B}$. Its worth to mention that, when the parameter ($\theta \rightarrow 0$), we get ordinary 2D space results, as follows:

$$\lim_{\theta \to 0} \left(\widehat{H}_{mag1} \text{ and } \widehat{H}_{mag2}\right) = 0$$

….(58)

5-Conclusion:

The effect of the non commutativity is studied, on Octic potential by applying the Boopp's shift method to first order in the non-commutativity parameter θ, we derived the two NC Hamiltonian matrixes $\hat{H}_{NC1}$ and $\hat{H}_{NC2}$, we show that the modified of the energies levels at the fundamental state determined only with a numerical solution. One can conclude from this work that the non-commutativity applying on the Octic potential, produced two types of interactions between a particle with spin $(1/2)$ and an external magnetic field, the first one represent the ordinary Zeeman effect and the new interaction represent a coupling between the total monument $\overleftrightarrow{J}$ and external magnetic field $\overleftrightarrow{B}$. The simple physical limited ($\theta \rightarrow 0$) proved that our correct obtained results.

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