Solution of the problem of transient process in the circuit, whose nonlinear element I-V characteristic is a polynomial

Assel Anuarkyzy Yerzhan, Zautbek Kuralbaevish Kuralbayev, Gulzada Dauletbekovna Musapirova

Almaty University of Power Engineering & Telecommunications, Baytusynova Str. 126, Almaty, 050013, Kazakhstan

Abstract. For the analysis and design of electronic circuits, a mathematical model of possible processes that can occur in a particular circuit under consideration, is developed. As the result of this modeling differential and algebraic equations will be obtained. However, the current-voltage characteristics of nonlinear elements that make up the circuit and determine the relationship between the current and the voltage may not be set in the form of an analytical dependence. For solution of differential equations describing the processes in the circuit, it would be useful to have such dependence. Therefore, the determination of the current-voltage characteristics of non-linear elements in the form of an analytical formula is the actual problem. This article is devoted to the determination of the current-voltage characteristic of the nonlinear element (varistor) in the circuit, and solving the problem of the transient process in it, arising under external influence.

[Yerzhan A. A., Kuralbayev Z.K., Musapirova G.D. **Solution of the problem of transient process in the circuit, whose nonlinear element I-V characteristic is a polynomial.** *Life Sci J* 2014;11(5s):176-182]. (ISSN:1097-8135). http://www.lifesciencesite.com. 34

Keywords: varistor, electronic circuits, mathematical model, current-voltage characteristics, non-linear elements.

Introduction

It is known that for theoretical research and practical calculations of nonlinear electric circuits it is advisable to have an analytical representation of the current-voltage characteristics (CVC) of the nonlinear elements. Due to this fact, it is an actual problem of determining the analytical formulas that describe with some accuracy the relationship between voltage and current in a nonlinear circuit element [1-3]. As a general rule, to establish such relationships, experiments are conducted and on the basis of experimental data a function that defines the analytical relationship between current and voltage in a nonlinear element is determined. This function is called an approximating function. It solves two problems, called problems of mathematical statistics [4-6]:

- Choosing the general form of the approximating function, a general view of the selected function can contain a number of unknown parameters.
- Determination of the specific form of the function, i.e., determining the values of the unknown parameters.

In many studies various functions are used as approximating functions [7-9]. Analysis of the use of different approximation functions to describe the current-voltage characteristics of non-linear circuit elements, the determination of their parameters from experimental data were considered in the paper [10-13]. In contrast to [14], this article suggests using a polynomial of fifth degree as an approximation function to determine the functional relationship between current and voltage in a non-linear circuit element. Selection of a polynomial of fifth degree is dictated by the following factors: first, to achieve sufficiently high determination accuracy of the required dependence between current and voltage in a non-linear element, and second, to show the possibility of solving the problem by using a polynomial approximation of a high degree.

Problem statement

Let's consider an electric circuit (Figure 1), which has a non-linear element (NE). Two types of varistors are chosen as non-linear elements. It is known that the varistors represent semiconductor resistors with symmetric current-voltage characteristic. They are used to stabilize and protect the electronic equipment and overvoltage. This is ensured by their feature, a pronounced dependence of the resistance from the voltage applied on them. To study the circuit, wherein a nonlinear element such as a varistor is contained, it is important to have analytical relationship between the current and voltage. For this purpose, it is required to solve two problems of mathematical statistics stated above. After receiving the analytical relationships between current and voltage in a non-linear element, it is necessary to consider the problem of the transient process which occurs in the given circuit (Figure 1).

To solve these statistical problems it is required to measure currents and voltages in this non-linear element and on the basis of the results of these measurements to obtain an approximating function in the form of a polynomial of fifth degree.

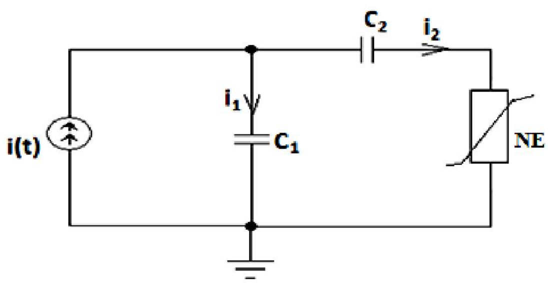

Figure 1 – Electric circuit with a nonlinear element

Description of the experiment

The aim of the experiment is to determine the values of output current and voltage when the input voltage and current change. A simple electric circuit (Figure 1) is used for the experiment. Two sets of experiments were conducted: for the two types of varistors. Following disc zinc oxide varistors were chosen as the non-linear element: TVR05180/CNR05D180 and TVR05220/CNR05D220. In the future, for brevity, the first varistor is named VAR18, and the second - VAR22. In addition to the varistor, the circuit was comprised of 3.5 uF and 2 uF capacitors. The input and output voltage and current (in varistor) were measured.

The experimental results are presented in the form of two tables. Table 1 shows the results for the first VAR18 varistor, and Table 2 - the second varistor VAR22. Here: $I_1, U_1 -$ input current and voltage, $I_2, U_2 -$ output current and voltage.

Table 1 - Results of the experiment for varistor TVR05180/CNR05D180 (VAR18)

$I_1(mkA)$	5	5	$6\cdot10^2$	$3{,}5\cdot 10^3$	$5\cdot10^3$	$8\cdot10^3$
$U_1(B)$	6,36	6,03	5,68	5,32	5,17	5,27
$I_2(mkA)$	0,07	0,074	0,38	2,47	4,5	6,38
$U_2(B)$	2	3	4	5	6	7

Table 1 continued

$I_1(mkA)$	$9\cdot10^3$	$10{,}2\cdot10^3$	$10{,}2\cdot10^3$	$13\cdot10^3$	$14\cdot10^3$	$15{,}5\cdot10^3$
$U_1(B)$	5,54	6	6,7	7,36	7,92	8,58
$I_2(mkA)$	8,04	9,5	10,97	12,03	13,04	14,3
$U_2(B)$	8	9	10	11	12	13

Table 1 continued

$I_1(mkA)$	$16{,}5\cdot10^3$	$17{,}5\cdot10^3$	$19\cdot10^3$	$20{,}5\cdot10^3$	$20{,}5\cdot10^3$	$22{,}5\cdot10^3$	$24\cdot10^3$
$U_1(B)$	9,21	9,88	10,46	11,13	11,62	12,2	12,91
$I_2(kA)$	15,53	16,75	17,9	19,26	20,4	21,65	22,92
$U_2(B)$	14	15	16	17	18	19	20

Table 2 - Results of the experiment for varistor TVR05220/CNR05D220 (VAR22)

$I_1(mA)$	2,5	2,5	2,5	2,5	2,5	6	7,5
$U_1(B)$	4,64	8	8,9	9,74	10,37	5,1	5
$I_2(mkA)$	0,074	0,076	0,076	0,081	0,074	4,95	6
$U_2(B)$	1	2	3	4	5	6	7

Table 2 continued

$I_1(mA)$	12	12	12,8	13	14,5	15,5	16,5	18
$U_1(B)$	5,11	5,41	5,89	6,42	7,01	7,63	8,23	8,87
$I_2(mkA)$	7,86	9,33	10,71	12,85	13,35	14,47	15,78	16,93
$U_2(B)$	8	9	10	11	12	13	14	15

Table 2 continued

$I_1(mk)$	19	20	21,5	22,5	23,5	25	27	27
$U_1(B)$	9,46	10,13	10,71	11,27	11,92	12,57	13,25	13,78
$I_2(mkA)$	17,88	19,16	20,32	21,36	22,59	23,67	24,94	26,03
$U_2(B)$	16	17	18	19	20	21	22	23

The approximation of the experimental results

For approximation of the experimental results the voltage U_2 is taken as the argument and the current I_2 - as a function. For convenience the following notation is introduced: $x = U_2, \quad y = I_2.$ In the future, with this notation in mind, we consider the function $y = f(x),$ which determines the current dependence on the voltage. Therefore, based on the results of experimental data processing the function $y = f(x).$ will be determined

As noted above, the fifth degree polynomial was taken as the approximating function:

$$y = b_1x^5 + b_2x^4 + b_3x^3 + b_4x^2 + b_5x + b_6 \qquad (1)$$

where $b_i \quad (i = 1, 2, \ldots, 6)$ - the unknown coefficients of the polynomial.

To determine the unknown coefficients of the polynomial (1) a well-known method of least squares was used. Under this method, the minimum condition of following function is used:

$$U = \sum_{k=1}^{n}[y_k - (b_1 \cdot x_k^5 + b_2 \cdot x_k^4 + b_3 \cdot x^3 + b_4 \cdot x^2 + b_5 \cdot x + b_6)]^2 \qquad (2)$$

The function $U = U(b_1, b_2, b_3, b_4, b_5, b_6)$ is a function of six parameters. The arguments to this function are the coefficients of the polynomial (1) selected for fitting the experimental data.

It is clear that a necessary and sufficient condition for the minimum of function (2) is the equality of its first partial derivatives with respect to the arguments b_i to zero:

$$\frac{\partial U}{\partial b_i} = 0, \quad i = 1,2,...,6. \tag{3}$$

For the given minimum condition (3) of the function U first we determine the first partial derivatives, which we equate to zero, this leads to a system of six linear algebraic equations with six unknown parameters $b_1, b_2, ..., b_6$:

$$\begin{cases} a_{11} \cdot b_1 + a_{12} \cdot b_2 + a_{13} \cdot b_3 + a_{14} \cdot b_4 + a_{15} \cdot b_5 + a_{16} \cdot b_6 = a_{17} \\ a_{21} \cdot b_1 + a_{22} \cdot b_2 + a_{23} \cdot b_3 + a_{24} \cdot b_4 + a_{25} \cdot b_5 + a_{26} \cdot b_6 = a_{27} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{61} \cdot b_1 + a_{62} \cdot b_2 + a_{63} \cdot b_3 + a_{64} \cdot b_4 + a_{65} \cdot b_5 + a_{66} \cdot b_6 = a_{67} \end{cases}$$

The coefficients of this system are defined by the following sums:

$$\begin{cases} a_{11} = \sum_{k=1}^{n} x_k^{10}; \quad a_{12} = a_{21} = \sum_{k=1}^{n} x_k^{9}; \\ a_{13} = a_{31} = a_{22} = \sum_{k=1}^{n} x_k^{8}; \quad a_{14} = a_{23} = a_{32} = a_{41} = \sum_{k=1}^{n} x_k^{7}; \\ a_{15} = a_{25} = a_{33} = a_{42} = a_{51} = \sum_{k=1}^{n} x_k^{6}; \\ a_{16} = a_{25} = a_{34} = a_{43} = a_{52} = a_{61} = \sum_{k=1}^{n} x_k^{5}; \\ a_{26} = a_{35} = a_{44} = a_{53} = a_{62} = \sum_{k=1}^{n} x_k^{4}; \\ a_{36} = a_{45} = a_{54} = a_{63} = \sum_{k=1}^{n} x_k^{3}; \\ a_{46} = a_{55} = a_{64} = \sum_{k=1}^{n} x_k^{2}; \quad a_{56} = a_{65} = \sum_{k=1}^{n} x_k; \quad a_{66} = n. \end{cases} \tag{4}$$

Solution of the system of equations (4)

The system of equations (4) is solved for the two cases of the experimental results presented in tables1 and 2. To do this, the Gauss-Jordan method is used. The process of solving this system of equations by Gauss-Jordan method consists of six steps. After the last step is performed solution to the system is obtained. The algorithm for solving the system of equations (4), using this method consists of the following steps:

1^0. Enter the values of the argument x_i and the function y_i for $i = 1,2,...,6.$

2^0. Loop through the parameter i for calculating the coefficients of the system of algebraic equations by formulas (4).

3^0. Beginning of the loop to solve the system of equations by Gauss-Jordan method $k = 1,$ $p = a_{kk}$. (Here p-auxiliary variable).

4^0. Loop through the parameter j for calculating the coefficients of k-th equation by the formula $a_{kj} = \frac{a_{kj}}{p}, \ j = 1,2,...,7.$

5^0. Beginning of the loop on the parameter i (the row number of the main matrix of the system of equations) $i = 1.$

6^0. If $i = k,$ the parameter value is incremented $i = i + 1$, i.e. transfer to next equation is performed (Equation number matching the resolution row number).

7^0. If $i \geq 7,$ the transition to step 10^0. (Completion of calculations).

8^0. Loop through the parameter j for calculating the coefficients of i-th equation by the formula of Gauss-Jordan transformations.

9^0. The end of the loop on parameter k. If $k \leq 6,$ go to item 4^0.

10^0. Output of results.

According to this algorithm, a computer program for the solution of algebraic equations was developed. As a result of running this program results, that are allowed to record the approximating functions as polynomials of fifth degree, were obtained.

For varistor VAR18 approximating function has the following form:

$$y = -0{,}00011 \cdot x^5 + 0{,}006703 \cdot x^4 - 0{,}151866 \cdot x^3 + \\ + 1{,}542637 \cdot x^2 - 5{,}297486 \cdot x + 5{,}05751. \tag{5}$$

Graph of the function (5) and the corresponding experimental data for VAR18 are shown in Figure 2.

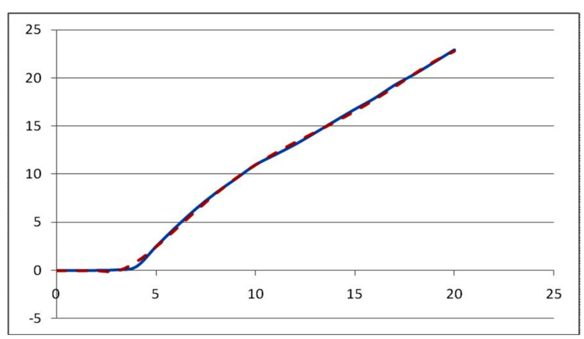

Figure 2 – Graph of the polynomial (5) in comparison with experimental data for VAR18

For varistor VAR22 the same function is obtained in the following form:

$$y = -0{,}000053 \cdot x^5 + 0{,}003668 \cdot x^4 - 0{,}093541 \cdot x^3 + \\ + 1{,}053142 \cdot x^2 - 3{,}514659 \cdot x + 2{,}961554. \quad (6)$$

The graph of this function (6) and the corresponding experimental data for VAR22 are presented in Figure 3.

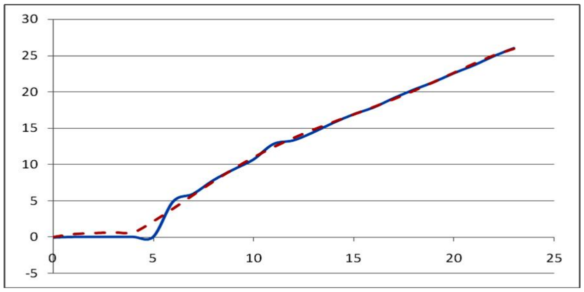

Figure 3 – Graph of the polynomial (6) in comparison with experimental data for VAR22

Mathematical model and formulation of the mathematical problem.

After determining the analytical relationship between current and voltage in a non-linear element we can consider the problem of the transient process which occurs in a circuit under external influence. The solution to this problem is primarily due to the development of a mathematical model and on formulation on its basis of a mathematical statement of the problem.

To produce a mathematical model of the given electrical circuit (Figure 1) the following designations are introduced: $i_1, i_2 -$ currents, $u_1, u_2, u_{NE} -$ voltages, $C_1, C_2 -$ capacitances, $\tau -$ time. Here, C_1 and C_2 are considered constant values.

According to Kirchhoff's law [1] for the scheme under consideration the following formulas hold: $i_1 + i_2 = i(\tau),\ u_1 = u_2 + u_{NE}$. In series connection of the capacitor C_2 and NE the current is equal stays the same, i.e. $i_2 = i_{NE}$.

The equations defining the dependence of the current and voltage for the capacitors are written as the following formulas: a) for the first capacitor $i_1 = C_1 \cdot \dfrac{du_1}{d\tau}$; b) for the second capacitor $i_2 = C_2 \cdot \dfrac{du_2}{d\tau}$.

Suppose that for the approximation of the current-voltage characteristics of the nonlinear element (NE), the following expression is used $i_{NE} = \dfrac{U_0}{R} \cdot f(x)$, where $x = \dfrac{u_{NE}}{U_0}$ - dimensionless voltage $f(x)$ - approximating function of the relationship between the current and the voltage in the non-linear element. In contrast to the problem discussed in [4], instead of using the linear element with a constant resistance *R,* this problem uses a non-linear element. Moreover, the voltage in the non-linear element is defined by the formula $u_{NE} = x \cdot U_0$.

For convenience in calculations it is appropriate to use dimensionless parameters. For this purpose, the following characteristic values are used: U_0 - voltage, $\dfrac{U_0}{R}$ - current. The following substitutions of variables are performed:

$$i_2 = y_2 \cdot \frac{U_0}{R};\ \ i = z \cdot \frac{U_0}{R};\ \ i_1 = y_1 \cdot \frac{U_0}{R};\ \ x_1 = \frac{u_1}{U_0};\ \ x_2 = \frac{u_2}{U_0};\ \ x = \frac{u_{NE}}{U_0};\ \ t = \frac{\tau}{T}.$$

Here, $x, x_1, x_2, y_1, y_2, t -$ dimensionless quantities.

So we obtain the following system of differential equations with respect to the unknown functions $x(t), x_1(t), x_2(t)$:

$$\begin{cases} \dfrac{dx}{dt} + \dfrac{[alpha\ ]_1 + [alpha\ ]_2}{[alpha\ ]_1 \cdot [alpha\ ]_2} \cdot f(x) = \dfrac{1}{[alpha\ ]_1} \cdot z(t); \\ \dfrac{dx_1}{dt} = \dfrac{1}{[alpha\ ]_1} \cdot [z(t) - f(x)]; \\ \dfrac{dx_2}{dt} = \dfrac{1}{[alpha\ ]_2} \cdot f(x) \end{cases} \quad (7)$$

Here, constants $[alpha]_1 = \frac{RC_1}{T}$ and $[alpha]_2 = \frac{RC_2}{T}$ are dimensionless values; RC_1 and RC_2 - time constants.

For electronic circuit considered here it is assumed that at the initial time there was no current (voltage), so for solution to the system of differential equations (7), the following initial conditions were assumed:

$$x_1(0) = 0; \quad x_2(0) = 0; \quad x(0) = 0; \tag{8}$$

Now we can formulate the following statement of the mathematical problem: it is required to find such values of the unknown functions $x_1(t), x_2(t), x(t),$ that satisfy the system of differential equations (7) and the initial conditions (8). The solution of this system is being searched for on the interval $t \in [0,1]$. Due to the fact that there exists a formula $x = x_1 - x_2$ relating these three functions, it is possible that initially a solution for two differential equations, the second and third equations of the system (7), might suffice.

If the values of the dimensionless functions $x_1(t), x_2(t), x(t),$ defining the voltage are found, then the dimensionless values that determine the currents $y_1(t)$ and $y_2(t),$ will be found from the following formulas: $y_2 = f(x), \quad y_1 = z - y_2.$

Numerical solution of the mathematical problem

The problem (7) - (8) is the Cauchy problem for a system of nonlinear differential equations of the first order, resolved with respect to derivatives. To solve this problem one cannot use existing analytical methods because of the presence in the equations of nonlinear function, therefore a numerical method for solving the problem is used. Euler method can be selected as the numerical method.

According to this method, first a step by the independent variable *t*: $[delta] = 0.0001,$ is selected and then substitution of derivatives by finite-difference equations is performed:

$$\frac{dx_1}{dt} \approx \frac{x_{1i+1} - x_{1i}}{[delta]}; \quad \frac{dx_2}{dt} \approx \frac{x_{2l+1} - x_{2i}}{[delta]}. \tag{9}$$

Here $x_{1i} = x_1(t_i), \quad x_{2i} = x_2(t_i), \ t_i = [delta] \cdot i,$ are a number of steps by the independent variable *t*

Using the substitution (9) from the second and third equations (7), we can obtain the following formula to determine the discrete values of the unknown functions $x_1(t)$ and $x_2(t)$:

$$x_{1i+1} = \frac{[delta]}{\alpha_1} \cdot [f(x_i) - z(t_i)],$$

$$x_{2i+1} = \frac{[delta]}{\alpha_2} \cdot f(x_i), \tag{10}$$

where, $x_i = x(t_i) -$ the value of the function $x(t)$ at $t = t_i$. These formulas are valid for the values of the parameter $i = 0, 1, 2, \ldots, n-1.$

From the initial conditions (2) it follows that

$$t_0 = 0, \quad x_{10} = 0, \quad x_{20} = 0. \tag{11}$$

This solution algorithm is designed for any type of functions $f(x)$ and $z(t).$ Cases, when varistors, whose current-voltage characteristics are as defined above by formulas (5) and (6), are considered as a nonlinear element in the, will be discussed below.

Here the current source is considered a variable and the change in the current is set in the form of a sine wave: $z(t) = \sin(2[pi] f t),$ with the frequency $f = 50$ Hz. The following values of the constant parameters were assumed: $C_1 = 3{,}5$ uF, $C_2 =$ 2uF, $R = 10$ kOhm, $T = 0{,}1$ sec., $[alpha]_1 = 0{,}35; \ [alpha]_2 = 0{,}20.$

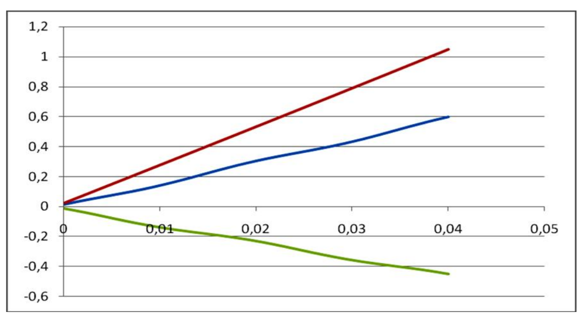

Figure 4 a – Graphs of functions

$x_1(t), x_2(t), x_n(t)$ for 18 Volt

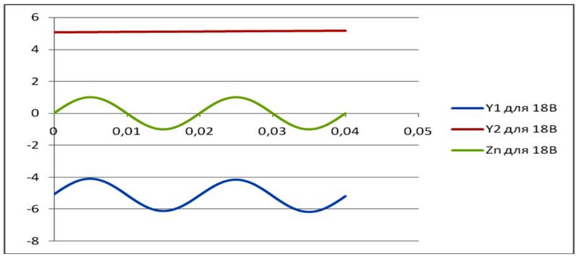

Figure 4b – Graphs of functions

For $y\ (t), y_2(t), z(t)$ 18 Volt

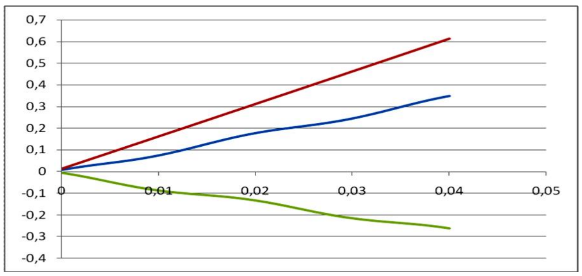

Figure 5 a – Graphs of functions

$x_1(t), x_2(t), x_n(t)$ 22 Volt

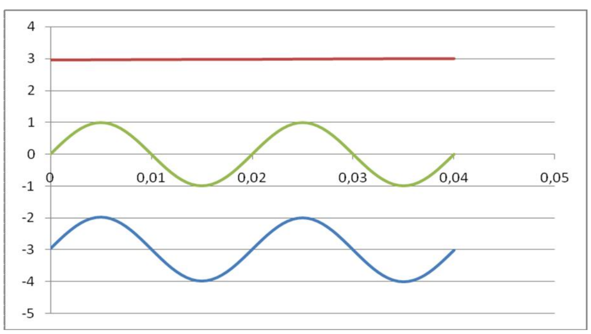

Figure 5b – Graphs of functions

$y\ (t), y_2(t), z(t)$ 22 Volt

Conclusion

The results of the numerical solution of the problem are presented in the form of graphs (Figures 4a and 4b) for the case, when varistor VAR18 is considered as a non-linear element, and (Figures 5a and 5b) for VAR22.

Figures 4a and 5a show the changes in the voltages of the circuit, and Figures 4b and 5b show the changes in the currents of the circuit. From the analysis of these figures it follows that in the varistor voltage takes negative values, and on the capacitors - positive values. Current in the varistor has the same frequency and the same amplitude as the current source. Current in the varistor remains constant.

Findings

Using a high degree polynomial (in this case, the fifth power) for approximation allows determining with a sufficiently high accuracy the dependence of current on voltage in the non-linear element.

The presence of the analytical relationship between the current and the voltage in the non-linear element allows formulating and solving a mathematical problem, which describes the transient process that occurs in an electrical circuit with a nonlinear element.

Corresponding Author:
Dr. Assel Anuarkyzy Yerzhan
Almaty University of Power Engineering & Telecommunications
Baytusynova Str. 126, Almaty, 050013, Kazakhstan

References

1. Bakalov, V.P. and V.F. Dmitracov, 2000. Kruk B.E. Moscow: Radio and communication, pp: 22-132.
2. Fedorova, I.B., 2004. Information technology in radio systems. Moscow: Radio and communication, pp: 37-63.
3. Yakovlev, A.N., 2010. Signal conversion in nonlinear radio circuits. Novosibirsk, pp: 190-192.
4. Akulov, O.A. and N.V. Medvedev, 2005. Computer science: basic course. Moscow, pp: 552-556.
5. Haganov, V.I., 2005. Radio Circuits and Signals. Moscow: Forum, pp: 432-437.
6. Thyagarajan, S., S. Pavan and P. Shancar, 2011. Active-RC Filters Using the Gm-Assisted OTA-RCTecnique. IEEE Journal of Solid-State Circuits, 46(7): 1522-1533.
7. Sotner, R., S. Seccik and T. Dostal, 2011. Multifunctional adjustable biquadratic active RC filters: design approach by modification of corresponding signal flow graphs. Preglad Electrotechniczny, 87(2): 225-229.
8. Troster, G. and W. Langheinrich, 1985. An optimal design of active distributed RC networks for the MOS technology. Int. Symp. Circuits and Syst. Proc., Kyoto, 3: 1431-1434.
9. Ogata, M., Y. Okabe and T. Nishi, 2004. Simple RC models of distributed RC lines in consideration with the delay time. Circuit and

Systems. Proceedings of the 2004 International Symposium (issue 4), ISCAS'04., pp: 649-652.
10. Nanayakkara, Y., H. Moon and D. Armstrong, 2010. A Tunable lonic Liquid Based RC Filter Using Electrowetting: A New Concept. ACS Applied Materials & Interfaces, 2(7): 1785-1787.
11. Suljanovic, N., A. Muj·cic, M. Zajc, and J. F. Tasi·c, 2004. Approximate computation of high frequency characteristics for power line with horizontal disposition and middle – phase to ground coupling. Electric Power Systems Research, 69: 17-24.
12. Kuralbayev, Z.K. and A.A. Yerzhan, 2012. Vestnik ENU im. L.N. Gumilyeva, 6(91): 183-188.
13. Kuralbayev, Z.K. and A.A. Yerzhan, 2013. Vestnik ENU im. L.N. Gumilyeva, 1(287): 183-188.
14. Yerzhan, A.A. and Z.K. Kuralbayev, 2013. Electronic Circuit Responsiveness Determination. World Applied Sciences Journal, 26 (Issue of Technical). Date Views 29.11.2013 www.idosi.org/wasj/wasj26 (8)13/4.pdf

3/23/2014