Making up of the logarithmic equations by complication method

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Abstract: In materials of article methods of making up of tasks are considered by complication. This method promotes formation of mathematical thinking and development of powers of thinking being trained. By independent drawing up of tasks logic regularities are used, new communications between the mathematical facts that promotes a creative approach during the studying mathematics reveal. According to a complication method by making up of the logarithmic equations properties of logarithmic function and laws of mathematical transformations are staticized, and it develops ability of management of own thinking activity. We understand transition as complication from simple expressions to the difficult structures containing big information loading. Complication – process return to process of reduction which introduces more in process of formation and knowledge development. In article ways of application of this method and its role in the course of gradual development of cogitative activity are considered. Our purpose also consists in it. Thus, in the publication requirements for performance of conditions of application of a method of complication in the course of making up of tasks and also a way of realization and methods of formation of cogitative activity of the being trained are considered.

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Introduction

Let's notice that mathematics teachers as a whole rather well are able to state ready knowledge, achieving thus quite good results in assimilation by school students of system of special knowledge, that is process of teaching of mathematics is carried out now successfully. However other component of process of training – the doctrine of school students is shown in practice more not enough. The analysis of researches of Pulfer, J. D and Whitehead, M. A. [1, p.693], Abylkasymova A.E. [2, p.12], Esmukhan M. E. [3, p.115], George L. Trigg [4, p.33], Page Starr and Vladimir Rokhlin [5, p.1117], Cheryl A. Lubinski and Albert D. Otto [6, p.336], Peter D. Miller, Nicholas M. Ercolani, Igor M. Krichever and C. David Levermore [7, p.1369], Kellah Edens and Ellen Potter [8, p.184], Kai Velten [9, p.46], David K. Pugalee [10, p.236], Sakenov, D. Zh. [11, 1431], Karl Wesley Kosko and Anderson Norton [12, p.340] allows us to note that educational activities for assimilation of bases of sciences have still unilateral character: in it assimilation and storing of ready knowledge and absolutely insufficient place prevails independent creative work of pupils borrows – them knowledge a little learns to extract, to analyze them, to apply in various situations.

George L. Trigg [4, p.33], Page Starr and Vladimir Rokhlin [5, p.1117], Cheryl A. Lubinski and Albert D. Otto [6, p.336], Peter D. Miller, Nicholas M. Ercolani, Igor M. Krichever and C. David Levermore [7, p.1369] watching development of cogitative activity of trainees, came to a conclusion that, solving problems the pupil repeats the programmed algorithms. Certainly, and in this process there is a formation of knowledge, but it goes slowly. When the person himself makes a task, it meets various situations which tries to overcome, comparing and comparing objective structures to structure of known formulas, the expressions containing in theorems and others mathematical offers. In other words, he thinks, looks for the decision. Process of drawing up of a task develops creative activity, orders thinking [4, 5, 6]. In this process own logic of thinking is formed. Whom you will become in the future, i.e. what profession you will get, nobody knows. The mathematics cannot give ready recipes of the solution of those tasks which you will meet. She can only logically teach to think.

Material and methods

For the solution of the set objectives and verification of initial assumptions the following research methods were used: theoretical - analysis of the studied problem in scientific literature, empirical - observation, conversations, questioning, discussions, interviewing, analysis of the best pedagogical practices, analysis of creative works of students, studying of high school documentation (state standards of education, curricula, standard programs, educational-methodical complexes of pedagogical disciplines) experiments, modeling.

Main part

According to Peter D. Miller, Nicholas M. Ercolani, Igor M. Krichever and C. David Levermore[7, p.1369], Kellah Edens and Ellen Potter[8, p.184], Kai Velten[9, p.46], David K.

Pugalee[10, p.236] in the course of the solution of tasks the difference of identical expressions is replaced with zero, and their relation – unit. Thus there is a simplification. If in this expression we will add coordinated with its structure *0* we will increase it by unit, the received expression will become complicated, and at the same time will allow consistently, step by step to build algorithm of the decision.

Task 1. $\log_3(2x+1)=2 \Rightarrow \log_3 f(x)=\log_3\varphi(x).$

Decision. The structure of the requirement contains the following information: 1) The bases of logarithms should be such what has a logarithm containing in a condition; 2) Expressions of logarithms can be any. It is information it is taken from f and φ. On its basis we make the decision on addition in equation structures $\log_3(2x+1)=2$ zero $(\log_3 5x-\log_3 5x)$, it is possible to present that equation in a look

$$\log_3(2x+1)+\log_3 5x-\log_3 5x=2,$$
$$\log_3 5x(2x+1)=2+\log_3 5x,$$
$$\log_3 5x(2x+1)=\log_3 9+\log_3 5x,$$
$$\log_3 5x(2x+1)=\log_3(9\cdot 5x),$$
$$\log_3 5x(2x+1)=\log_3 45x.$$

So, the complicated equation is received, the task is solved.

$$\log_3(2x+1)=2 \Rightarrow \log_3(10x^2+5x)=\log_3 45x.$$

Let's notice that the complicated equation $\log_3(10x^2+5x)=\log_3 45x$ has the same decision, as the standard equation for in the course of the decision added a difference of identical expressions.

Task 2. $\log_3(2x+1)=2 \Rightarrow \log_3 f(x)=\log_c\varphi(x).$

Decision. It is necessary to make change to structure of the requirement so that the basis of a logarithm was another. Let's take for the logarithm basis number 9 and we will add a difference $\log_9 5x-\log_9 5x$, thereby complicating the equations. Thus process of the decision will look as follows:

$$\log_3(2x+1)+\log_9 5x-\log_9 5x=2,$$
$$\log_3(2x+1)+\log_9 5x-\log_9 5x=2,$$
$$\log_3(2x+1)+\frac{\log_3 5x}{\log_3 9}=2+\log_9 5x,$$
$$\log_3(2x+1)+\frac{\log_3 5x}{2}=\log_9 81+\log_9 5x,$$
$$\log_3\sqrt{5x}(2x+1)=\log_9(81\cdot 5x),$$
$$\log_3\sqrt{5x}(2x+1)=\log_9 405x.$$

Solution of a task: $\log_3(2x+1)=2 \Rightarrow \log_3\sqrt{5x}(2x+1)=\log_9 405x.$

Task 3. $\log_3(2x+1)=2 \Rightarrow (2x+1)^{\log_3\varphi(x)}=f(x).$

Decision. By equation drawing up as initial object we take the equation, the left part of a sign of following«$\Rightarrow$», and in the right part – for a reference point. So,

$H:\ \log_3(2x+1)=2$, $O:\ (2x+1)^{\log_3\varphi(x)}=f(x).$

In *About* such information contains: it is necessary to pass to indicative function with the basis 2x+1, and with preservation of logarithmic function $\log_3(\bullet)$, only having changed its expression. This requirement we can execute when we will use that

$$\log_3(\bullet)\,\log_3(2x+1)=\log_3(2x+1)^{\log_3(\bullet)}.$$

So, partially nature of change of structure already was defined. But we should keep an equal-sign. This condition will be executed when both parts H we will increase $\log_3(\bullet)/\log_3(\bullet)$.

In structure of a reference point concrete functions are not given. Means instead of a point it is possible to take any function, for example 7x. Then

$$(\log_3 7x/\log_3 7x)\,\log_3(2x+1)=2,$$
$$\log_3 7x\cdot\frac{\log_3(2x+1)}{\log_3 7x}=2,$$
$$\frac{\log_3(2x+1)^{\log_3 7x}}{\log_3 7x}=2,$$
$$3^{\log_3(2x+1)^{\log_3 7x}}=3^{\log_3 49x^2},$$
$$(2x+1)^{\log_3 7x}=49x^2.$$

So, the complicated equation is received:

$$\log_3(2x+1)=2 \Rightarrow (2x+1)^{\log_3 7x}=49x^2.$$

For ensuring understanding of process of transformation of this equation considered separately a case of addition of zero and multiplication to unit. In general, introduction of zero and unit can be carried out in any place of transformation. Now we will consider other ways of complication of the equation.

Task 4. On value of an independent variable x =-4,5 to work out the look equation $\log_{1/a} f(x) = k.$

The maintenance of a task we will translate into mathematical language.

$x = -4{,}5 \quad \Rightarrow \quad O: \quad \log_{1/a} f(x) = k. \ (a > 0; k \in Z).$

Decision. According to the requirement should take the logarithm a statement of the problem. However we can not fulfill this requirement, as value x – a negative number. Therefore

$x = -4{,}5 \ \ /\cdot(-1) \ \Rightarrow \ -x = 4{,}5.$

Now as initial object we will take this equality, and for a reference point we will take the task requirement.

$H: \ x = -4{,}5, \quad O: \ \log_{1/a} f(x) = k. \ (a > 0; k \in Z).$

1) *N* срав. with *About* $\Rightarrow$ $H_1: \ -2x = 9$ $\Rightarrow$ 1) H_1 to compare with *About*

2) $P: 4{,}5 \notin Z$ 2) P: There is no ravine

3) $C: 4{,}5 \cdot 2$ 3) $C: \log_{1/a} H_1$

$\Rightarrow \ \log_{1/a}(-2x) = \log_{1/a} 9.$

Let's give to parameter and different values and we will simplify equation structure. We have:

1) Let a=3. Then

$$\log_{1/3}(-2x) = \log_{1/3} 9,$$

$$\log_{1/3}(-2x) = \log_{1/3}(3^2),$$

$$\log_{1/3}(-2x) = \log_{1/3}(3^{-1})^{-2},$$

$$\log_{1/3}(-2x) = \log_{1/3}(1/3)^{-2},$$

$$\log_{1/3}(-2x) = -2\log_{1/3}(1/3),$$

$$\log_{1/3}(-2x) = -2.$$

Worked out logarithmic the equations satisfies conditions O.

So, the task is solved: $x = -4{,}5 \quad \Rightarrow \quad \log_{1/3}(-2x) = -2.$

Task 5. $x = -1 \quad \Rightarrow \quad \log_a(x+5) = 2\log_a(1-x).$

Decision

Making up of the non-standard equation we will begin with a statement of the problem H: $x + 1 = 0$, and the requirement of a task we will take for a reference point. For ensuring information clearness we will transform a reference point. We have

$O: \ \log_a(x+5) = \log_a(1-x)^2.$

$O_1: \quad x + 5 = (1-x)^2, \qquad O_2: \ x + 5 = 1 - 2x + x^2,$

$O_3: \quad x^2 - 3x - 4 = 0, \qquad O_4: \ (x+1)(x-4) = 0.$

In a reference point O_4 the way of transformation of initial object therefore passing consistently to structures of the following intermediate reference points is concluded, it is possible to solve an objective. So, sequence

$O \to O_1 \to O_2 \to O_3 \to O_4$ is the solution of the non-standard logarithmic equation, and the sequence upside-down is equation drawing up. Thus there is a set condition $x = -1$ is the decision of a quadratic, $x = 4$ also is its decision, but it is not the solution of the logarithmic equation, as $\log_a(1-4)^2 \neq 2\log_a(1-4).$

So, according to Kai Velten[9, p.46], David K. Pugalee[10, p.236], Sakenov, D. Zh.[11, 1431], Karl Wesley Kosko and Anderson Norton[12, p.340] solution of the equation and making up of the equation are mutually return processes. In this process there is independent a logarithm basis. Giving it various values, it is possible to make some concrete equations.

The statement of the problem is not connected with the second root of a quadratic. Changing values of the second root of a quadratic, it is possible to make also a set of the logarithmic equations. For example, proceeding from equality $(x+1)(x-6) = 0$ and having repeated above-mentioned processes upside-down, it is possible to work out the equation:

$$\log_a(x^2 - x) = \log_a(2x+3) + \log_a 2.$$

At and = 2 we will receive $\log_2(x^2 - x) = \log_2(2x+3)+1.$

Let's generalize processes of drawing up of the equation. Let x = x1 will be an entry condition of a task. It is required to work out the look equation:

$$\log_a(x^2 - x_1 x + \varphi(x)) = \log_a(xx_2 - x_1 x_2 + \varphi(x)).$$

Task 6. $x = x_1$ $\Rightarrow$ $\log_a(x^2 - x_1 x + \varphi(x)) = \log_a(xx_2 - x_1 x_2 + \varphi(x)).$

Decision. $H: \quad x - x_1 = 0 \quad \Rightarrow \quad H_1: \; (x - x_1)(x - x_2) = 0$

$$x^2 - (x_1 + x_2)x + x_1 \cdot x_2 = 0,$$

$$x^2 - x \cdot x_1 + x_1 \cdot x_2 = 0,$$

$$x^2 - x \cdot x_1 = x \cdot x_2 - x_1 \cdot x_2,$$

$$x^2 - x \cdot x_1 + \varphi(x) = x \cdot x_2 - x_1 \cdot x_2 + \varphi(x).$$

Now it is possible to take a logarithm on the basis *and.* Then

$$\log_a(x^2 - x \cdot x_1 + \varphi(x)) = \log_a(x \cdot x_2 - x_1 \cdot x_2 + \varphi(x))$$

So, the task is solved. $\varphi(x)$ - any function, satisfying to a condition

$$x \cdot x_2 - x_1 \cdot x_2 + \varphi(x) > 0.$$

Let's put

$$\varphi(x) = \frac{x_1^2}{4} = \log_a\left(x^2 - 2 \cdot \frac{x_1}{2} x + \frac{x_1^2}{4}\right) = \log_a\left(\frac{x_1^2}{4} + x_2 x - x_2 x_1\right)$$

$$\log_a\left(x - \frac{x_1}{2}\right)^2 = \log_a\left(x_2 x - \frac{x_1^2 - 4x_1 x_2}{4}\right)$$

$$2\log_a\left(x - \frac{x_1}{2}\right) = \log_a\left(x_2 x - \frac{x_1^2 - 4x_1 x_2}{4}\right)$$

We received the equation which can be duplicated.

If $\varphi(x) = 2x_1 x$, that $\log_a(x^2 + x \cdot x_1) = \log_a(x \cdot (2x_1 + x_2) - x_2 \cdot x_1).$

$$\varphi(x) = \frac{x_1^2}{4} = \log_a\left(x^2 - 2 \cdot \frac{x_1}{2} x + \frac{x_1^2}{4}\right) = \log_a\left(\frac{x_1^2}{4} + x_2 x - x_2 x_1\right),$$

$$\log_a\left(x - \frac{x_1}{2}\right)^2 = \log_a\left(x_2 x - \frac{x_1^2 - 4x_1 x_2}{4}\right),$$

$$2\log_a\left(x - \frac{x_1}{2}\right) = \log_a\left(x_2 x - \frac{x_1^2 - 4x_1 x_2}{4}\right)$$

From here it is possible to receive specific objectives. For example,

$$x = 4 \quad \Rightarrow \quad 2\log_a(x-2) = \log_a(6x-20)$$

$$\Rightarrow |x_2 = 6, a = 2|$$

$$\log_2(x-2) = \frac{1}{2}\log_2 2(3x-10),$$

$$\log_2(x-2) = \frac{1}{2}\log_2 2 + \log_2(3x-10),$$

$$\log_2(x-2) = \frac{1}{2} + \log_2(3x-10).$$

Task 7. $x = 7 \quad \Rightarrow \quad O: \; \log_x a\sqrt[k]{b} = c(m),$ $(a, b, k, c, m \in N)$

Decision. According to information containing in structure of a reference point, the independent variable needs to be entered into the logarithm basis. Therefore a statement of the problem we take the logarithm on the basis x.

$$\log_x x = \log_x 7 \quad \Rightarrow \quad H_1: \; \log_x 7 = 1.$$

Following requirement of a reference point: the number standing in the right part of equality should be fractional, and in a denominator there should be simple number 3. In numerator there can be any number at which division the whole part would be a digit. There are some such numbers. We will take $\frac{14}{3}$.

$$H_2: \; \frac{14}{3}\log_x 7 = \frac{14}{3}.$$

Having compared H_2 with O, we come to a conclusion that: 1) with $\frac{14}{3}$ it is necessary to enter under a logarithm sign; 2) it is necessary to present this number located to the right of an equal-sign in the form of decimal fraction. We have

$$H_3: \; \log_x 7^{\frac{14}{3}} = 4,(6).$$

Further, by simplification we will create the structure similar to structure O. We have

$$\log_x 7^{4+\frac{2}{3}} = 4,(6) \quad \Rightarrow$$

$$\log_x 7^4 \cdot 7^{\frac{2}{3}} = 4,(6)$$

It is worked out logarithmic the equations: $\log_x(2401 \cdot \sqrt[3]{49} = 4,(6).$ The task is solved.

Answer: $\log_x(2401 \cdot \sqrt[3]{49} = 4,(6).$

Task 8. $x=12 \Rightarrow O: \; a^{\log_b f(x)} = \log_k c$ $(a,b,k,c \in N)$.

Decision. At the solution of the equations it is possible to use the following methods: logarithmical both parts the equation, but this method does not satisfy the task requirement. Therefore given the equations we will transform

$$H: \; x=12 \Rightarrow H_1: \; x-7=5.$$

Now it is found function f and the logarithmical shown in a reference point on the basis of 5 is defined.

$$\log_5(x-7)=\log_5 5. \quad \Rightarrow \quad H_2: \; \log_5(x-7)=1.$$

$$H_3: \; 3^{\log_5(x-7)}=3^1,$$

$$H_4: \quad 3^{\log_5(x-7)}=3\cdot\log_4 4, \quad \Rightarrow \quad 3^{\log_5(x-7)}=\log_4 4^3,$$

$$3^{\log_5(x-7)}=\log_4 64.$$

So, the complicated equation is received, it satisfies conditions O.

Answer: $3^{\log_5(x-7)}=\log_4 64.$

Conclusion

Let's notice that by equation drawing up information is taken from structure of a reference point and the relevant decisions are made. If there are some options, the best option gets out. In other words, information containing in structure O, operates thoughts of the pupil, directing it on the solution of a task. Solving problems on equation making up, the pupil forms in itself ability to argue and think. In other words, he forms abstract thinking. It that is necessary.

Now, more than ever, becomes clear that the mathematics is not only set of the facts stated in the form of theorems, but first of all – the arsenal of methods and even that is still before language for the description of the facts and methods of the most different areas of a science and practical activities.

Mathematics as any other science is in continuous development. Rapid development of mathematics makes a great impact on other sciences, including on pedagogics and a mathematics technique.

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