Feedback Control for Anti-Synchronization Chaotic Dynamical Systems

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Abstract: In this paper we studied feedback control for anti-synchronization chaotic dynamical system. Active control schemes applied on two identical Genisio systems and then on two different Liu-Rossler systems based on the Lyapunov stability theorem. Numerical simulation by using Maple software are used to show effectiveness of the proposed schemes.

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1. Introduction

Chaos synchronization is an important topic both theoretically and practically that has been studied in the recent decades because of its potential applications in secure communications, chemical reactions, information science, biological systems, plasma technologies, etc [1-7]. Since Pecora and Carrol [8] introduced a method to synchronize two identical systems with different initial conditions, a variety of approaches have been proposed for the synchronization of chaotic systems which include complete synchronization [9,10], phase synchronization [11],[12],[13],14], lag synchronization [15], projective synchronization [16,17], etc. In the anti-synchronization it is aim to see opponent behaviour from master and slave system so the sum of two system will converge to zero. Anti-Synchronization is a prevailing phenomenon in symmetrical oscillators. It is well known that the first observation of anti-synchronization of two oscillators by Huygens in the seventeenth century was, in fact, AS between two pendulum clocks. So far, some progresses have been made in the researches of Synchronization and also AS. Kim et al. [18] have found an based on a suitable separation of systems, Zhang and Sun [19] have presented some simple but generic criteria for synchronization and anti-synchronization for chaotic systems. Recently, using different control methods, the Synchronization for some typical chaotic systems has been discussed [20-22].In this paper we will use nonlinear active control method by using Lyapunov stability theorem. The organization of this paper is as follows. Section 2 briefly gives the definition of anti-Synchronization, describes Liu system and Genisio system and then will introduce the active control schemes. Anti-synchronization by using active scheme will apply for those systems in Sections 3 ,4 and 5. Conclusion is obtained in the final section.

2. Definitions

For definition of synchronization consider a class of chaotic systems that described by

$$\dot{x} = f(t,x) \quad (2.1)$$
$$\dot{y} = g(t,y) + u(t,x,y) \quad (2.2)$$

where $x, y \in R^n$ are the state vectors and $f, g : R^n \to R^n$ are continuous functions. Eq. (2.1) is the drive system, Eq. (2.2) is the response system and $u(t,x,y)$ is the imputed control function. It is said that the system (2.1) and system (2.2) have the property of anti-synchronization between $x(t)$ and $y(t)$, if there exists a synchronous manifold $M = (x(t), y(t)) : x(t) = y(t)$ such that all trajectories $(x(t), y(t))$ approach M as time goes to infinity, that is to say,

$$\lim_{t\to\infty}\|e\| = \lim\|y + x\| = 0 \quad (2.3)$$

where $\| \ \|$ is the Euclidean norm.

The chaotic Genisio system is described by

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = -ax_1 - bx_2 - cx_3 + x_1^2 \quad (2.4)$$

where the state variables, and a, b and c are the real constants. When $a = 6, b = 2.92$ and $c = 1.2$ system (2.4) has positive Lyapunov exponents, and shows chaotic behavior with initial conditions $(x_1(0), x_2(0), x_3(0)) = (.1,.3,.2)$.

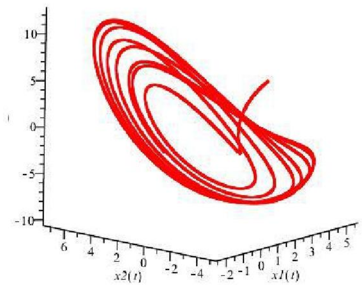


Figure 1: Genisio system

Liu system describe as:

$$\dot{x}_1 = a(x_2 - x_1)$$
$$\dot{x}_2 = bx_1 - x_1x_3$$
$$\dot{x}_3 = -cx_3 + dx_1^2 \qquad (2.5)$$

where $x_i, i = 1,...,3$ are the state variables, and a, b, c and d are real parameters. The system has positive Lyapunov exponents over a wide parameter region. When $a = 10, b = 40, c = 2.5$ and $d = 4$ system (2.5) is chaotic figure, (2.1) shows the system chaotic behaviour with initial conditions $(x_1(0), x_2(0), x_3(0)) = (2,3,3$.

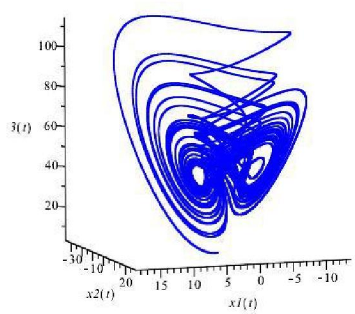


Figure 2: Liu system

The chaotic Rossler system [24] is described by

$$\dot{x}_1 = -x_2 - x_3$$
$$\dot{x}_2 = x_1 + ax_2$$
$$\dot{x}_3 = b + x_3(x_1 - c) \qquad (2.6)$$

where $x_i, i = 1,...,3$ are the state variables, and a, b and c are the real constants. When $a = .2, b = .2$ and $c = 5.7$ system (2.6) has positive Lyapunov exponents, and shows chaotic behavior with initial conditions $(x_1(0), x_2(0), x_3(0)) = (5,-4,2)$.

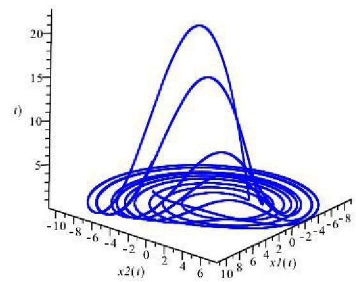


Figure 3: rossler system

3. Active Anti-synchronization of two identical Genisio system

In this section, nonlinear control method will use to synchronize two identical Genisio chaotic system. We assume that (2.4) as a drive system and system (3.1) as slave one as follow:

$$\dot{y}_1 = y_2 + u_1$$
$$\dot{y}_2 = y_3 + u_2$$
$$\dot{y}_3 = -ay_1 - by_2 - cy_3 + y_1^2 + u_3 \qquad (3.1)$$

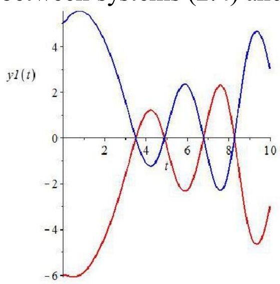
where $u_i = u_i(x_i, y_i, e_i), i = 1,2,3$ are the control functions. In order to determine the control functions such that the synchronization between systems (2.4) and (3.1) are realized, we sum (2.4) from (3.1) and so the error system is:

$$\dot{e}_1 = y_2 + x_2 + u_1$$
$$\dot{e}_2 = y_3 + x_3 + u_2$$
$$\dot{e}_3 = -ay_1 - by_2 - cy_3 + y_1^2 - ax_1 - bx_2 - cx_3 + x_1^2 + u_3 \qquad (3.2)$$

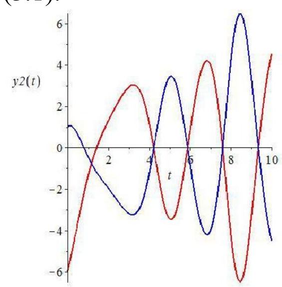
For any initial conditions, if active controllers chosen as

$$u_1 = -e_2 - e_1$$
$$u_2 = -e_3 - e_2$$
$$u_3 = ae_1 + be_2 - x_1^2 - y_1^2 - e_3 \qquad (3.3)$$

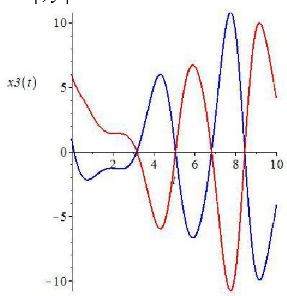
then system (2.4) synchronizes system (3.1). It is obvious that eigenvalues of error matrix A are negative Therefor, system (3.2) is asymptotically stable, which implies that system (2.4) synchronizes system (3.1). In the numerical simulations, figures 4 and 5, the fourth-order Runge-Kutta method is used to solve the system. In the time step size 0.001, the parameters of the drive system are chosen to be $a = 10, b = 40, c = 2.5$ $d = 4$ so that it exhibits chaotic behaviour. The initial values of the drive system and the response system, $x_1, y_1; x_2, y_2$ and x_3, y_3 are chosen as $(x_1(0), x_2(0), x_3(0)) = (5,1,1$ and $(y_1(0), y_2(0), y_3(0)) = (-6,-6,6)$ respectively. Hence, the error system has the initial values $(e_1(0), e_2(0), e_3(0)) = (-1,-5,7)$ the simulation results are shown in Figures 4 and 5. Figure 4 shows the state trajectories of the drive system and the response system. Figure 5 displays the synchronization errors between systems (2.4) and (3.1).



(a): x_1, y_1

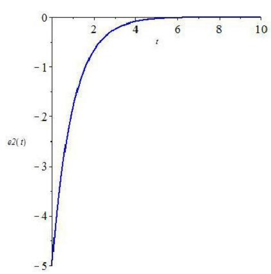


(b): x_2, y_2

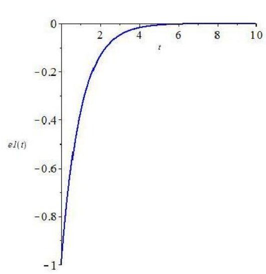


(c): x_3, y_3

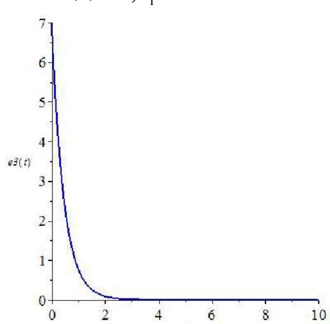
Figure 4: Time evaluation identical Genisio system



(a): t, e_1



(b): t, e_2



(c): t, e_3

Figure 5: Time evaluation error system

4. Active Anti-synchronization between two different Liu and Rossler systems

In this section, we apply the active anti-synchronization on two different Liu and Rossler chaotic systems based on the Lyapunov stability theorem. We assume that liu system (2.5) is the drive system and the controlled Rossler system (4.1) is the response system,

$$\begin{aligned} \dot{y}_1 &= y_2 - y_3 + u_1 \\ \dot{y}_2 &= y_1 + a_2 y_2 + u_2 \\ \dot{y}_3 &= b_2 + y_3(y_1 - c_2) + u_3 \end{aligned} \qquad (4.1)$$

which parameters u_1, u_2, u_3 are control inputs and a_2, b_2, c_2 are known constants. The error dynamical system between drive system (2.5) and response system (4.1) is described by

$$\begin{aligned} \dot{e}_1 &= -y_3 - y_2 + a_1(x_2 - x_1) + u_1 \\ \dot{e}_2 &= y_1 + a_2 y_2 + b_1 x_1 - x_1 x_3 + u_2 \\ \dot{e}_3 &= b_2 + y_3 y_1 - c_2 y_3 - c_1 x_3 + d_1 x_1^2 + u_3 \end{aligned} \qquad (4.2)$$

For any initial conditions, the synchronization between systems (2.5) and (4.1) will be obtained if the controller will designed as below:

$$\begin{aligned} u_1 &= y_3 + y_2 - a_1 x_2 + a_1 x_1 - k e_1 \\ u_2 &= -y_1 + a_2 x_2 - b_1 x_1 + x_1 x_3 - k e_2 \\ u_3 &= -b_2 - y_1 y_3 - c_2 x_3 + c_1 x_3 - d_1 x_1^2 - k e_3 \end{aligned} \qquad (4.3)$$

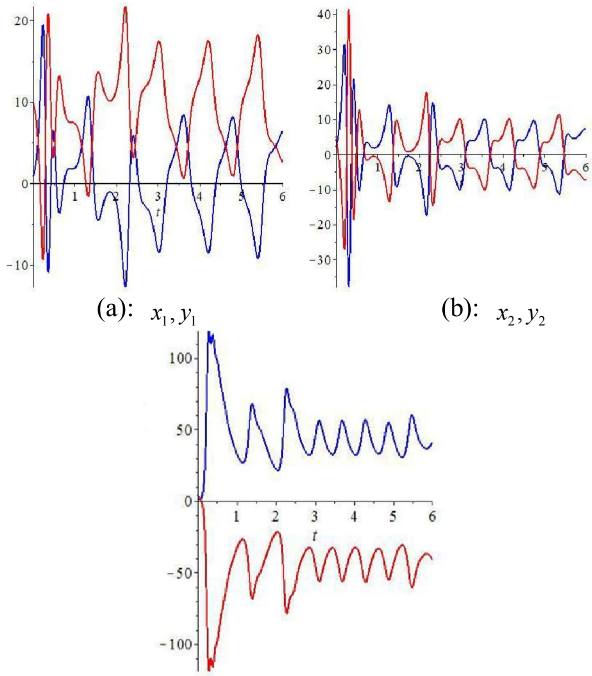
Consider that the following Lyapunov function:

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2)$$

The time derivation of V along the trajectory of the systems is

$$\begin{aligned} \dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 \\ &= e_1(-ke_1) + e_2(a_2 e_2 - ke_2) + e_3(ce_3 - ke_3) < 0 \end{aligned}$$

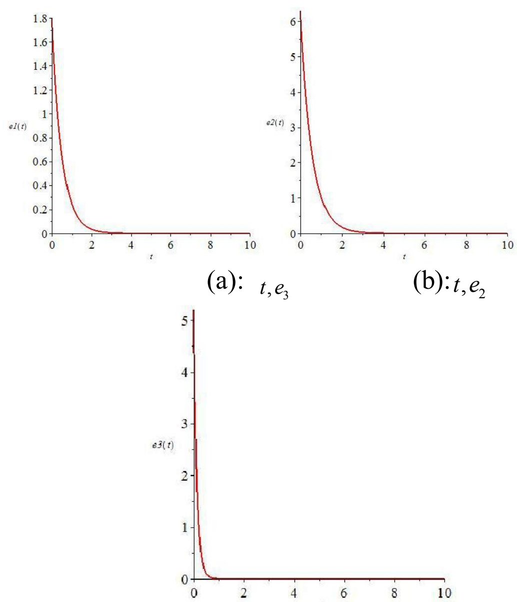
that obviously is negative, since V is positive definite and $\dot{V}$ is negative definite in the neighborhood of zero , according to the Lyapunov stability theorem, the error system (4.2) can converge to the origin asymptotically, which implies that the synchronization of systems (2.5) and (4.1) is achieved. To verify the effectiveness and feasibility of (2.5) and (4.1), we simulate the dynamics of the drive system and the response system. In the simulation, the parameters are chosen as $a_1 = 10, b_1 = 40, c_1 = 2.5, d_1 = 4, a_2 = .2, b_2 = .2, c_2 = 5.7$ and the initial condition of the drive system and the response system is $(x_1(0), x_2(0), x_3(0)) = (1,4,7)$ and $(y_1(0), y_2(0), y_3(0)) = (10,3,5)$ respectively. Therefor, the error system has the initial values $(e_1(0), e_2(0), e_3(0)) = (9,-1,-2)$.



(a): x_1, y_1 (b): x_2, y_2

(c): x_3, y_3

Figure 6: Time evaluation drive and master system



(a): t, e_3 (b): t, e_2

(c): t, e_3

Figure 7: time evaluation error system

5. Conclusions

Active control is applied when parameters are known. In this paper, Active control schemes for anti-synchronization between different and identical chaotic dynamical systems with known parameters is demonstrated. based on the Lyapunov stability theorem, controllers are designed and numerical simulations are used to verify the effectiveness of the proposed control techniques.

References

[1]T. L. Carroll and L. M. Pecora, Synchronizing chaotic circuits, IEEE Trans. Circ. Syst. I, 38 (1991) 453-456.

[2]L. Kacarev and U. Parlitz, General approach for chaotic synchronization, with application to communication, Phys. Rev. Lett., 74 (1996) 5028-5031.

[3]Y. N. Li, L. Chen, Z. S. Cai and X. Z. Zhao, Experimental study of chaos synchronization in the Belousov- Zhabotinsky chemical system, Chaos Solitons Fractals, 22 (2004) 767-771.

[4]N.J. Corron and D.W. Hahs, A new approach to communications using chaotic signals, IEEE Trans. Circ. Syst., 44 (1997) 373-382.

[5]R.A. Ims and H.P. Andreassen, Spatial synchronization of vole population dynamics by predatory birds, Nature, 408 (2000) 194-196.

[6]J. Cao, H.X. Li and Daniel W.C. Ho, Synchronization criteria of Lurâ€™e systems with time-delay feedback control, Chaos Solitons Fractals, 23 (2005) 1285-1298.

[7]J. Liang, J. Cao and J. Lam, Convergence of discrete-time recurrent neural networks with variable delay, Int. J. Bifurc. Chaos, 15 (2005) 581-595.

[8]L. M. Pecora and T. L. Carroll, Synchronization in chaotic systems, Phys. Rev. Lett., 64 (1990) 821-824.

[9]K. M. Cuomo, A .V. Oppenheim and S. H. Strogatz, Synchronization of Lorenz-based chaotic circuits with applications to communications, IEEE Trans. Circ. Syst. I, 40 (1993) 626-633.

[10]H. N. Agiza, Chaos synchronization of Lu dynamical system, Nonlinear Anal., 58 (2004) 11-20.

[11]M. G. Rosenblum, A. S. Pikovsky and J. Kurths, Phase synchronization of chaotic oscillators, Phys. Rev. Lett., 76 (1996) 1804-1807.

[12]C. G. Li and G. R. Chen, Phase synchronization in small-world networks of chaotic oscillators, Phys. Lett. A, 341 (2004) 73-79.

[13]J. Hu, S. H. Chen and L. Chen, Adaptive control for anti-synchronization of ChuaâAZs chaotic system, Phys. Lett. A, 339 (2005) 455-460.

[14]Z. Wang, Anti-synchronization in two non-identical hyperchaotic systems with known or unknown parameters, Commun. Nonlinear Sci. Numer. Simul., 14 (2009) 2366-2372.

[15]E. M. Shahverdiev, S. Sivaprakasam and K. A. Shore, Lag synchro- nization in time-delayed systems, Phys. Lett. A, 292 (2002) 320-324.

[16]G. H. Li, Modiï¬ ed projective synchronization of chaotic system, Chaos Solitons Fractals, 32 (2007)

1786-1790.
[17]W. Li and X. Chen, Projective synchronization of ChuaâAZs chaotic system with dead-zone in the control input, Commun. Nonlinear Sci. Numer. Simul., 14 (2009) 3100-3107.
[18]C. M. Kim, S.H. Rim, W.H. Kye, J.W. Ryu and Y.J. Park, PAnti- synchronization of chaotic oscillators, Phys. Lett. A, 320 (2003) 39-46.
[19]Y.P.Zhang and J.T. Sun, Chaotic synchronization and anti- synchronization based on suitable separation, Phys. Lett. A, 330 (2004) 442-447.
[20]M. Mossa Al-Sawalha and M.S.M. Noorani, Anti-synchronization of two hyperchaotic systems via nonlinear control, Commun. Nonlinear Sci. Numer. Simulat., 14 (2009) 3402-3411.
[21]X. R. Shi and Z. L. Wang, Adaptive added-order anti-synchronization of chaotic systems with fully unknown parameters, Appl. Math. and Comput., 215 (2009) 1711-1717.
[22]M. Mossa Al-Sawalha and M. S. M. Noorani, Adaptive anti- synchronization of two identical and different hyperchaotic systems with un-certain parameters, Commun. Nonlinear Sci. Numer. Simulat., 15 (2010) 1036- 1047.
[23]G. Qi, M. A. Wyk, B. J. Wyk and G. Chen, A new hyperchaotic system and its circuit implementation, Chaos Solitons Fractals, 40 (2009) 2544-2549.
[24]Y. Li, K. S. Tang Wallace and G. Chen, Hyperchaos evolved from gener- alized Lorenz equation, Int. J. Circ. Theory Appl., 33 (2005) 235-251.

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