Stochastic Modeling for Rainfall-Runoff in Saudi Arabia

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Abstract: The development of water resources systems in arid and semiarid zones suffers from data availability, especially for storm runoff. Measurements of runoff in arid zones are often not available; therefore, there is a need to estimate runoff that is produced from rainfall events. In the current study, a regional stochastic model is developed to assess the correlation between rainfall and runoff in arid and semiarid zones based on recorded data (383 data pairs were collected) at five gauged watersheds in the southwestern part of the Kingdom of Saudi Arabia during the period 1981 -1984. The model is formulated using the bivariate joint log-normal probability density function of both rainfall and runoff. The estimated correlation coefficient is 0.5 which is considered significant particularly in arid and semi-arid zones. The logarithms of rainfall and runoff data were tested for normality via the application of the Q-Q plot of the marginal distributions. The correlation coefficients of the Q-Q plots were 0.97 and 0.993 for rainfall and runoff respectively. The Mahalanobis square distance and Chi^2 distribution quartiles were used to test the normality of the joint distribution. The correlation coefficient was found to be 0.996. A spreadsheet simulation model was constructed and used to generate realizations of the runoff process conditioned on the recorded rainfall data. A Monte Carlo method is adopted to generate 200 realizations of the runoff process and the conditional ensemble mean and the conditional ensemble variance were estimated and compared with the theoretical model. The model results fall within the 95% confidence intervals. This model could be updated in the future by having experimental watersheds in the region to study the impact of climatic changes on the water resources systems.
[Jarbou Bahrawi. **Stochastic Modeling for Rainfall-Runoff in Saudi Arabia.** *Life Sci J* 2014;11(1):183-191]. (ISSN:1097-8135). http://www.lifesciencesite.com. 28

Keywords: Bivariate joint log-normal distribution, Mahalanobis square distance, rainfall-runoff processes, arid and semiarid zones, stochastic model.

1. Introduction

Nearly half the countries of the world are facing problems of aridity. This has been unveiled by UNESCO [1] classification and Pilgrim et al. [2]. Therefore, there is an obvious need for improved understanding of the hydrology of arid and semiarid regions, and for the development of appropriate techniques for modeling rainfall-runoff relationship. Even in those humid regions where a large number of studies have been carried out, hydrological modeling is at best of only moderate accuracy, and involves many assumptions, simplifications and averaging over space and time. However, some aspects of arid zone hydrology require simplified modeling. It is highly probable that applying hydrologic models that has been developed in humid regions to arid regions could produce greater errors and uncertainty will continue to characterize results for arid zones. Recognition of these issues is fundamental to a realistic approach to arid zone modeling, and to a rational interpretation and application of the results obtained [3].

Rainfall-runoff modeling is a major activity among hydrologists around the world. There remains a very practical need for rainfall-runoff modeling for practical problem in water resource assessment. Rainfall-runoff models are amongst the most important tools for the practical solution of flood estimation problems, as well as for theoretical investigations of controls on the flood frequency curve or for analysis of catchment and climate change. Several types of models have been proposed for the rainfall-runoff relationship, based on either deterministic (lumped or distributed) or stochastic approaches (e.g. classical time series analysis). Since, the current study focuses on stochastic approach, therefore review of deterministic models is beyond the scope of this paper. In the stochastic approach, complex hydrologic processes often require knowledge of the joint distribution of several variables and the correlation between them [4]. These hydrologic events must be studied as the joint occurrence of two or more random variables, and the frequency analysis must therefore consider their joint probability distribution. Many bivariate gamma distribution models are difficult to be implemented to solve practical problems, and seldom succeeded in gaining popularity among practitioners in the field of hydrological frequency analysis [5]. Aldama and Ramirez [6] have developed a new approach for estimating the design flood of dams and reservoirs. The method is based on the use of the bivariate extreme-value probability distribution of peak discharge and volume and they developed a model for such relation. Yue [7] has applied the joint probability density function approach to correlate annual maximum storm intensities and total storm amount at specific duration. The Box-Cox transformation technique [8] is used to normalize the data regardless of the original distribution of the data. The theoretical distribution shows a good fit to the observed ones. Zhang and Singh [9] have applied bivariate frequency distributions using Archimedian copulas for rainfall depth, intensity and durations on a data set collected from the Amite River basin in Louisiana, United States. They concluded that the Ali-Mikhal-Haq and Frank copula families can be used to represent both the negatively and positively variables. Grimaldi and Seinaldi [10] have utilized asymmetric Archimedian copulas in multivariate flood frequency analysis. They have correlated flood peak, flood volume and duration using trivariate density function to have clearer picture for flood inundation management.

For the development of water recourses in arid and semiarid zones, there is a need to measure runoff that is produced from rainfall events. Measurements of runoff in arid zones are often not available. Therefore, there is essential to develop a rainfall-runoff relation from gauged watersheds in arid and semi-arid zones. In the current study, a stochastic model is developed to assess the correlation between rainfall and runoff in arid zones based on recorded data at five watersheds in the Kingdom of Saudi Arabia during the period between1981-1984 due to data limitation. To the best of the authors' knowledge, there is no such a stochastic model that has been developed neither in the hydrological literature in general nor in arid and semi-arid zones in particular. The model is formulated using the bivariate log-normal probability density function of both rainfall and runoff [11]. A simulation model has been formulated and used to generate realizations of the runoff process conditioned on the recorded rainfall data. A Monte Carlo approach is adopted to generate 200 realizations of the runoff process and the conditional ensemble mean and the conditional ensemble variance have been estimated and compared with the theoretical model.

Description of The Study Basins

In the period of 1981 to 1984, Saudi Arabia Dams and Moore [12] has developed a detailed study of five selected basins in the southwestern part of Saudi Arabia. The locations of the five study basins are shown in Figure 1. The rain gauges are spread over the basins with the number of rainfall gauges per basin varying from 12 gauges in Wadi Liyyah (456 km^2) to 35 in Wadi Habawnah (4930km^2), while the number of runoff gauges ranges from 2 gauges at Wadi Liyyah to 5 gauges at Wadi Habawnah. The region is dominated by the Asir escarpment which runs parallel to the Red Sea coast with elevations of up to 3000 m. As shown in Figure 1, three of the catchments, Wadi Al Lith, Yiba and Liyyah, drain towards the Red Sea and in these basins altitude generally increases from southwest to northeast. The other two basins, Tabalah and Habawnah, drain from the mountains to the interior, towards the Rub al Khali or "the empty quarter". The region is subject to two major influences with respect to the supply of moisture for precipitation. In the winter period, weather systems from the north and west, generally of Mediterranean origin predominate, while in the summer moist air from the southwest monsoon systems penetrates the region [13]. Local climate is modified by the influence of the Red Sea, the hot interior of Saudi Arabia and the orographic effect of the Asir Mountains. It can therefore be expected that rainfall characteristics will be subject to complex regional and seasonal variability, with topographic effects likely to exert a significant influence on rainfall distribution.

Annual rainfall is strongly related to elevation, with annual totals of the order of 30-100 mm on the Red Sea coastal plain (Tihama) and up to 450mm at elevations larger than 2000m a.s.1..

Data collection

During the study period (1981-1984), data on rainfall and runoff has been collected at these basins from the storm events recorded in this period. The collected rainfall – runoff data has been transferred into a time series of monthly rainfall and runoff as shown in Figure 2. The figure portrayed a typical rainfall and runoff records at station SA 401 in Wadi Al Lith basin. The data is utilized for setting up a stochastic model that relates rainfall and runoff. Figure 3 shows a scatter plot of the rainfall-runoff data. The data shows a sparsely pattern.

2. Methodology

In this study, a logarithmic transformation of the data has been performed, and the statistical tests of normality have been executed to validate the normality assumption.

It is assumed that

$$x = \ln(Rainfall, mm), \quad and \quad y = \ln(Runoff, mm)$$

The joint probability density of a bi-variate normal density function, $f(x,y)$, is given by

$$f(x,y) = K\exp(M) \tag{1}$$

where K is given by,

$$K = \frac{1}{2\pi\sigma_x\sigma_y\left(1-\rho^2\right)^{\frac{1}{2}}} \tag{2}$$

and M is given by,

$$M=\left\{\left[\frac{1}{2\left(1-\rho^{2}\right)}\right]\left[\left(\frac{x-\mu_{x}}{\sigma_{x}}\right)^{2}-2\rho\left(\frac{x-\mu_{x}}{\sigma_{x}}\right)\left(\frac{y-\mu_{y}}{\sigma_{y}}\right)+\left(\frac{y-\mu_{y}}{\sigma_{y}}\right)^{2}\right]\right\} \quad (3)$$

Where

μ_x = mean of the logarithm of the rainfall, where rainfall is measured in mm,

μ_y = mean of the logarithm of the runoff, where runoff is also measured in mm,

σ_x = standard deviation of the logarithm of the rainfall,

σ_y = standard deviation of the logarithm of the runoff, and

ρ = correlation coefficient between logarithm of rainfall and runoff.

The covariance matrix of the joint distribution is given by,

$$C=\begin{pmatrix} \sigma^{2}_{x} & \rho_{\sigma_x \sigma_y} \\ \rho_{\sigma_x \sigma_y} & \sigma^{2}_{y} \end{pmatrix} \quad (4)$$

In order to have a complete description of the joint distribution, the marginal distribution of the X and Y variates should be introduced as,

$$f(x)=\frac{1}{\sigma_{x}\sqrt{2\pi}}Exp-\left\{\frac{\left[x-\mu_{x}\right]^{2}}{2\sigma_{x}^{2}}\right\} \quad (5)$$

$$f(y)=\frac{1}{\sigma_{y}\sqrt{2\pi}}Exp-\left\{\frac{\left[y-\mu_{y}\right]^{2}}{2\sigma_{y}^{2}}\right\} \quad (6)$$

and the conditional density of Y given that $X=x$ for any bivariate distribution, is defined as,

$$f(x/y)=\frac{f(x/y)}{f(x)} \quad (7)$$

The condition density is given by,

$$f(x/y)=\frac{1}{\sqrt{2\pi}\sqrt{\sigma_{y}\left(1-\rho^{2}\right)}}Exp-\left\{\frac{\left[y-\mu_{y}-\rho\left(\frac{\sigma_{y}}{\sigma_{x}}\right)\left(x-\mu_{x}\right)\right]^{2}}{2\sigma_{y}\left(1-\rho^{2}\right)}\right\} \quad (8)$$

Estimation of the Model Parameters

The model parameters have been estimated from the observed data in the gauged watersheds presented in this study. The model parameters (the mean, variance of both variables and the correlation coefficient) are estimated by the method of moments as follows,

$$\mu_{x}=\frac{1}{n}\sum_{i=1}^{n}x_{i}\ , \qquad \mu_{y}=\frac{1}{n}\sum_{i=1}^{n}y_{i} \quad (9)$$

$$\sigma_{x}^{2}=\frac{1}{n}\sum_{i=1}^{n}\left[x_{i}-\mu_{x}\right]^{2}\ , \quad \sigma_{y}^{2}=\frac{1}{n}\sum_{i=1}^{n}\left[y_{i}-\mu_{y}\right]^{2} \quad (10)$$

and the correlation coefficient is calculated by,

$$\rho=\frac{\sum_{k=1}^{n}\left(x_{k}-\mu_{x}\right)\left(y_{k}-\mu_{y}\right)}{n\sigma_{x}\sigma_{y}} \quad (11)$$

Table 1 shows the statistical parameters estimated from the original and the transformed data. The parameters of the original data show high values of skewness and kurtosis for both rainfall and runoff depths, while the degree of skewness and kurtosis for the runoff data are relatively high when compared with the rainfall data. The mean and median of both rainfall and runoff data are far apart. After the logarithmic transformation, the skewness and the kurtosis are reduced. The skewness is moved to the right for rainfall data, while there is almost no skewness for the runoff data that provides some manifestation of normality. On the other hand the kurtosis of the rainfall data is very close to three that provides also some manifestation of normality. The mean and median of both transformed rainfall and runoff data are very close as well that gives some evidence of normality or at least near normality. The correlation coefficient for both original and transformed data is similar, and equal to 0.5 which is significant particularly in arid zones [14]. This value has been supported by the analysis made by Saudi Arabian Dames and Moore [12].

Goodness of fit tests

Marginal Distributions:

A Q–Q plot is used to compare the shapes of distributions, providing a graphical view of how properties such as location, scale, and skewness are similar or different in the two distributions. The method is described in Johnson et al. [15]. Q–Q plots are commonly used to compare a data set to a theoretical model. The Q-Q plot provides an assessment of the "goodness of fit" that is graphical, rather than reducing to a numerical summary. The idea is that if a sample is supposed to follow a normal distribution, a plot the sample quantile versus the quantile of the normal distribution should be made, the points should lie very nearly on a straight line. If the points deviate much from a straight line, normality is suspect. In addition, the pattern of deviation can give some information about the nature of the non-normality. The correlation coefficient for the Q-Q plot is a powerful test of normality.

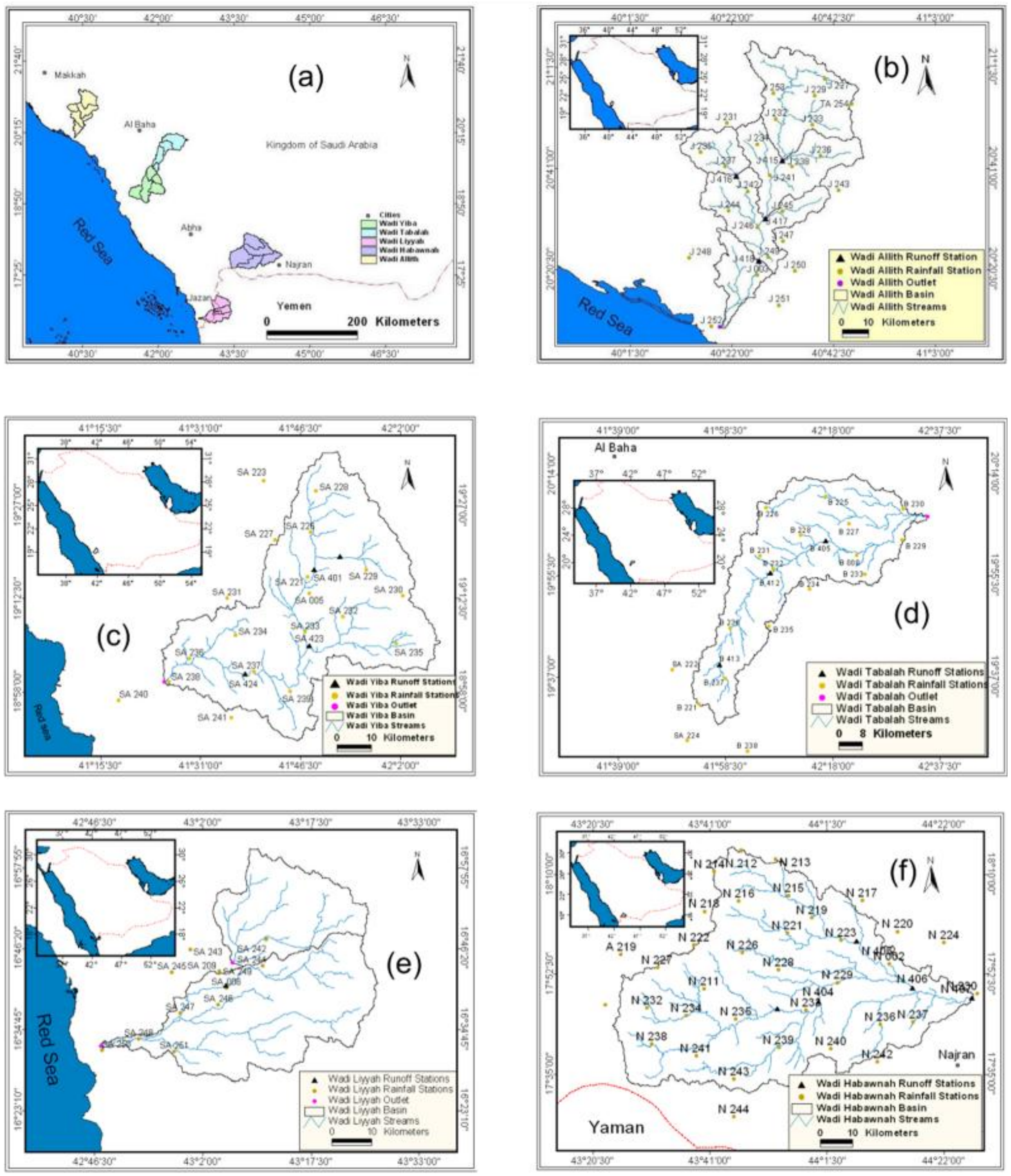
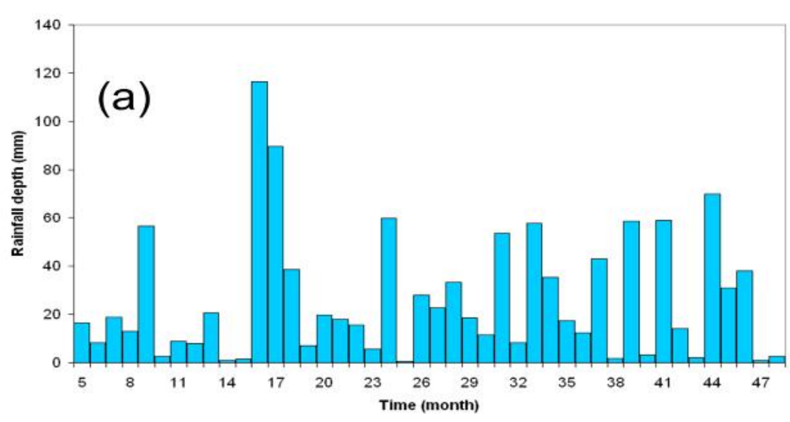


Figure 1. The representative basins: (a) General layout of the locations of the basins, (b) wadi Al-Lith basin, (c) wadi Yiba basin, (d) wadi Tabalah basin, (e) wadi Lyyia basin, and (f) wadi Habawna basin.



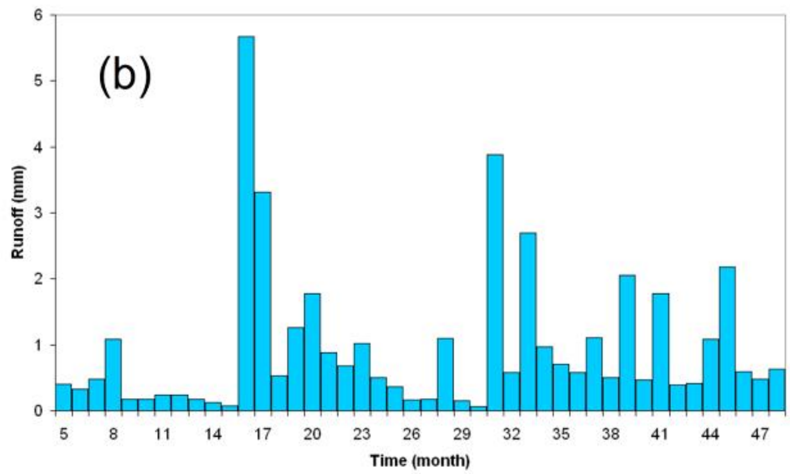


Figure 2. Sample of rainfall data (a) and runoff data (b) at a typical station (SA 401) at a representative basin (Wadi Al Lith), during the period 1981-1984 (44 months).

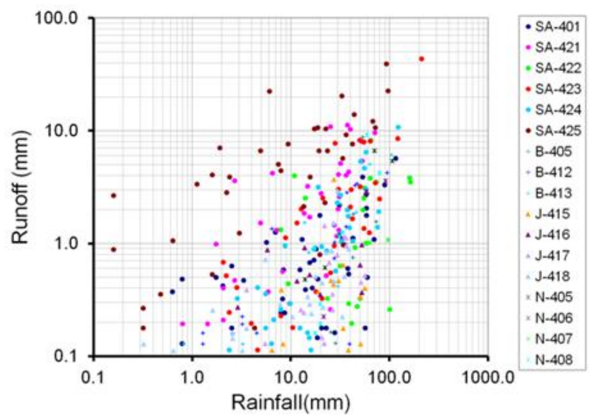


Figure 3. Scatter plot of the rainfall-runoff data from all stations.

Table 1. Statistical parameters of rainfall-runoff data

Statistical parameter	Rainfall	Runoff
Arithmetic mean (mm)	26.48	2.02
Geometric mean (mm)	17.17	0.57
Harmonic mean (mm)	3.82	0.18
Median (mm)	19.70	0.52
Standard deviation (mm)	27.71	4.20
Skewness	2.23	5.60
kurtosis	11.17	46.59
Coefficient of variation (CV)	1.05	2.08
Correlation coefficient	0.504	
Mean of the logarithms	2.59	-0.56
Median of the logarithms	2.98	-0.66
SD of the logarithms	1.41	1.68
Skewness of the logarithms	-0.84	0.08
kurtosis of the logarithms	3.13	2.23
CV of the logarithms	0.54	-2.97
Correlation coefficient	0.501	

Figure 4 shows the Q-Q plot of the data and the corresponding normal distribution. The data shows good agreement with the Normal distribution in the sense that they reasonably fit the line of 45 degrees for both the rainfall and the runoff.

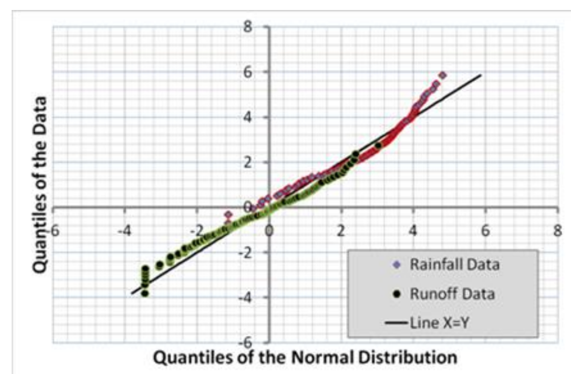


Figure 4. Q–Q plot of rainfall and runoff transformed data with the normal distribution.

The straightness of the Q-Q plot can be measured by calculating the correlation coefficient of the points on the plot. The correlation coefficient, r_Q, for the Q-Q plot is defined by Johnson et al. [15] as,

$$r_Q = \frac{\sum_{j=1}^{n}\left(x_j - \text{«}_x\right)\left(q_j - \text{«}_q\right)}{\sqrt{\sum_{j=1}^{n}\left(x_j - \text{«}_x\right)^2}\sqrt{\sum_{j=1}^{n}\left(q_j - \text{«}_q\right)^2}} \quad (12)$$

Where,

x_j is the quantile *j* of the data,

«_x is the mean of the quantiles of the data,

q_j is the quantile *j* of the theoretical distribution, and

$«_q$ is the mean of the quantiles of the theoretical distribution.

The correlation coefficient between the quantiles of the data and the quantiles of the normal distribution are 0.97 and 0.993 for the rainfall and the runoff data respectively. This shows that the normal distribution represents the data reasonably well confirming the adequacy of the normal distribution.

Joint Bivariate Normal Distribution:

A Q- Q plot for multivariate normality is built as presented in the following steps:

1. Calculate the Mahalanobis square distance, D^2, for multivariate data by,

$$D^2 = (x-«)' \sum{}^{-1} (x-«) \qquad (13)$$

Where,

x is the *p*-vector of the variables,

$«$ is the mean vector, $\sum^{-1}$ is the inverse of the covariance matrix of the variables, and

$(x-«)'$ is the transpose of the vector $(x-«)$.

In the case of bivariate data, the Mahalanobis square distance can be derived from the above equation to give,

$$D^2 = \frac{1}{1-°^2}\left[\left(\frac{x-«_x}{±_x}\right)^2 - 2°\left(\frac{x-«_x}{±_x}\right)\left(\frac{y-«_y}{±_y}\right) + \left(\frac{y-«_y}{±_y}\right)^2\right] \qquad (14)$$

2. Ordering the Mahalanobis square distance from the smallest to the largest across all observations,

3. Calculating the quantiles of each data point,

4. Looking up the quantiles of each data point using Chi^2 distribution (df = # of variables, in our case df = 2),

5. Plotting the data against the value predicted by the theoretical statistical distribution, and

6. Calculating the correlation coefficient between the quantiles of the data and the predicted quantiles from the theoretical Chi^2 distribution. Figure 5 displays the squared distance versus Chi-square Q-Q plot for the bivariate normality test. The correlation coefficient between the quartiles of the squared distance and the quartiles of the Chi-square has been calculated to give a value of 0.996 reflecting almost perfect correlation and manifesting bivariate normality.

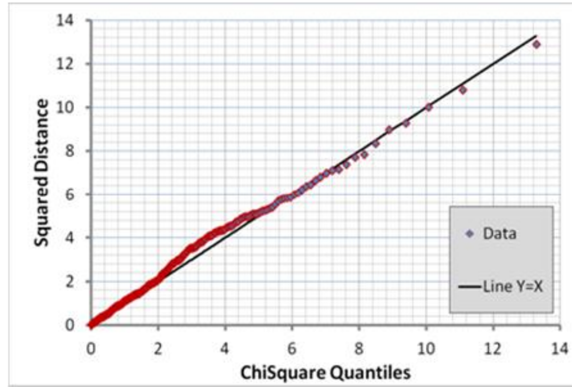


Figure 5. Squared distance versus Chi-square Q-Q plot for bivariate normality test.

Stochastic Generation Model

To set up the model for generating realizations of the joint distribution, the covariance matrix is introduced as,

$$C = \begin{pmatrix} ±^2_x & °_{±_x ±_y} \\ °_{±_x ±_y} & ±^2_y \end{pmatrix} \qquad (15)$$

The covariance matrix is decomposed into lower and upper matrices as,

$$L = \begin{pmatrix} ±_x & 0 \\ °_{±_Y} & ±_y\sqrt{1-°^2} \end{pmatrix}, \quad U = \begin{pmatrix} ±_x & °_{±_Y} \\ 0 & ±_y\sqrt{1-°^2} \end{pmatrix} \qquad (16)$$

The generation model is represented in a matrix form as,

$$\begin{pmatrix} X_i \\ Y_i \end{pmatrix} = \begin{pmatrix} «_x \\ «_y \end{pmatrix} + \begin{pmatrix} ±_X & 0 \\ °_{±_Y} & ±_y\sqrt{1-°^2} \end{pmatrix}\begin{pmatrix} £_i \\ £_{i+1} \end{pmatrix} \qquad (17)$$

Where, X_i is the realization # *i* of the logarithm of rainfall,

Y_i is the realization # *i* of the logarithm of runoff, and

$£_i$ and $£_{i+1}$ are random numbers drawn from a normal distribution with a zero mean and a unit variance.

The generation of realizations of the logarithm of the rainfall-runoff process using unconditional distribution is given by,

$$X_i = «_x + ±_x £_i \qquad (18)$$

$$Y_i = «_y + °±_y £_i + ±_y\sqrt{1-°^2}\,£_{i+1} \qquad (19)$$

Consequently, for the generation of realizations from a conditional distribution given that,

$$X_i = X_j \qquad (20)$$

$$Y_i = \mu_y + \sigma_y \rho \left(\frac{X_i - \mu_x}{\sigma_x} \right) + \sqrt{1-\rho^2}\, \varepsilon_i \sigma_y \quad (21)$$

$X_i =$ is a given conditional value of the logarithm of rainfall to condition on.

The conditional mean of the logarithm of the runoff is given by,

$$\mu_{y/x} = E(Y/\ X = x) = \mu_y + \rho \frac{\sigma_y}{\sigma_x}(x - \mu_x) \quad (22)$$

and the conditional variance is given by,

$$\sigma_{y/x} = Var(Y/\ X = x) = \sigma_y^{\,2}(1-\rho^2) \quad (23)$$

The equations are implemented within a spreadsheet model to perform the statistical parameter estimation and the generation process with a graphical interface. The results are displayed in the following section.

3. Results and Discussion

The conditional stochastic generation model, described in the previous section, has been applied on the rainfall data to make runoff predictions. A Monte Carlo approach is followed to generate realizations of the runoff predictions conditioned on rainfall data. 200 realizations have been generated based on the model parameters estimated from the data namely, the mean, variance of both the logarithm of the rainfall and the runoff data and the correlation coefficient between them.

Figure 6 shows four realizations, out of 200 realizations, generated by the model and compared with the runoff data. The model results are conditioned on the rainfall data. The figure shows reasonable agreement between the generated runoff results and the runoff data. The probability contours of 0.5 and 0.05 have been displayed on the figure together with the centorid of the data.

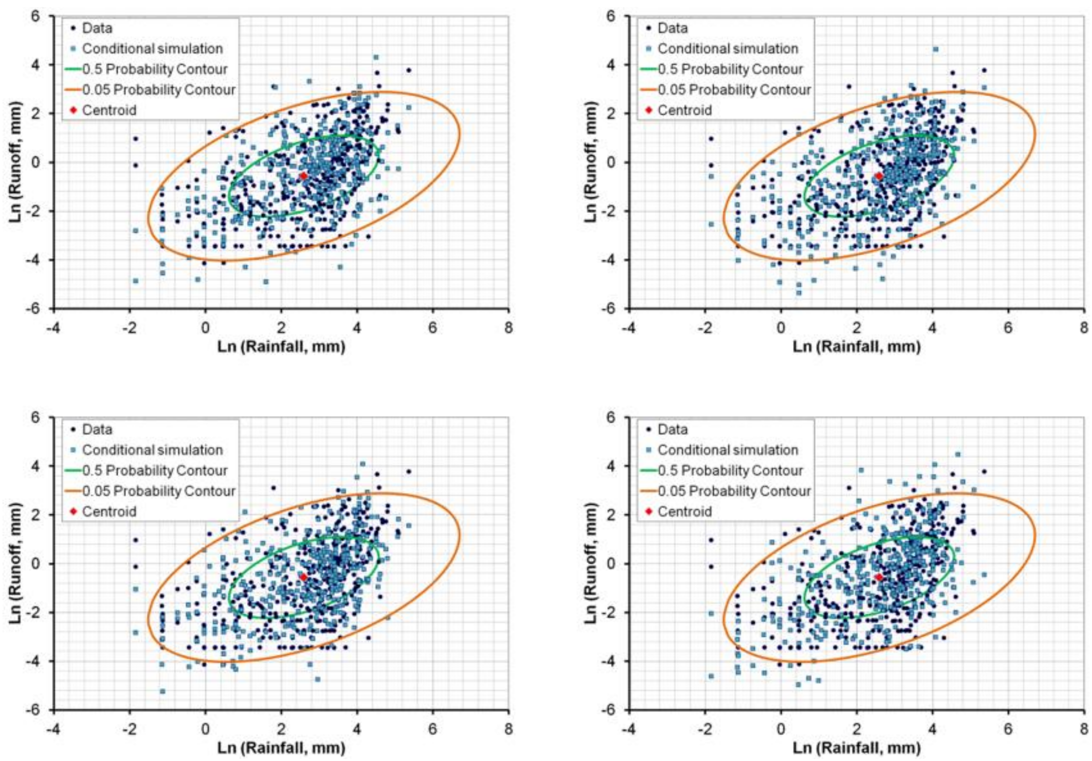


Figure 6. Four different realizations of the generated Ln (Runoff, mm) conditioned on Ln (Rainfall, mm) compared with the data from all stations.

The probability contours of constant probability density for p-dimensional normal distribution are ellipsoids defined by **x** such that [15],

$$D^2 = (x-\mu)' \sum{}^{-1} (x-\mu) \le \chi^2(p) \quad (24)$$

Where, $\chi^2(p)$ is the chi-squared distribution with p-degrees of freedom.

In case of bivarite case, the formula (Eq. 24) leads to,

$$\frac{1}{1-\rho^2}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right) + \left(\frac{y-\mu_y}{\sigma_y}\right)^2\right] = \chi^2(2) \quad (25)$$

Further manipulation of Eq. 25 will lead to the equation of the ellipse in a functional form as,

$$y = \mu_x + \sigma_y \left\{ \rho \left(\frac{x-\mu_x}{\sigma_x} \right) \pm \sqrt{ \left(1-\rho^2\right) \left[\chi^2(2) - \left(\frac{x-\mu_x}{\sigma_x} \right)^2 \right] } \right\} \quad (26)$$

Where, the plus sign in the front of the square root makes the upper part arc of the ellipse and the minus sign makes the lower arc of the ellipse.

Eq. 26 has been evaluated for two values of joint probabilities at 0.5 and 0.05 that correspond to 50% and 95% confidence respectively. It is obvious that the majority of the data points lie within the joint probability elliptic contour of 0.05 that corresponds to 95% confidence.

Figure 7 shows the ensemble average over 200 realizations of the logarithm of the runoff conditioned on the rainfall data values. The ensemble conditional mean of the logarithm of runoff fits well with the theoretical mean model. Also, the ensemble of the upper and the lower 95% confidence limits for the conditional variance are fitting well the theoretical model. The figure shows the data points fall within the 95% confidence limits.

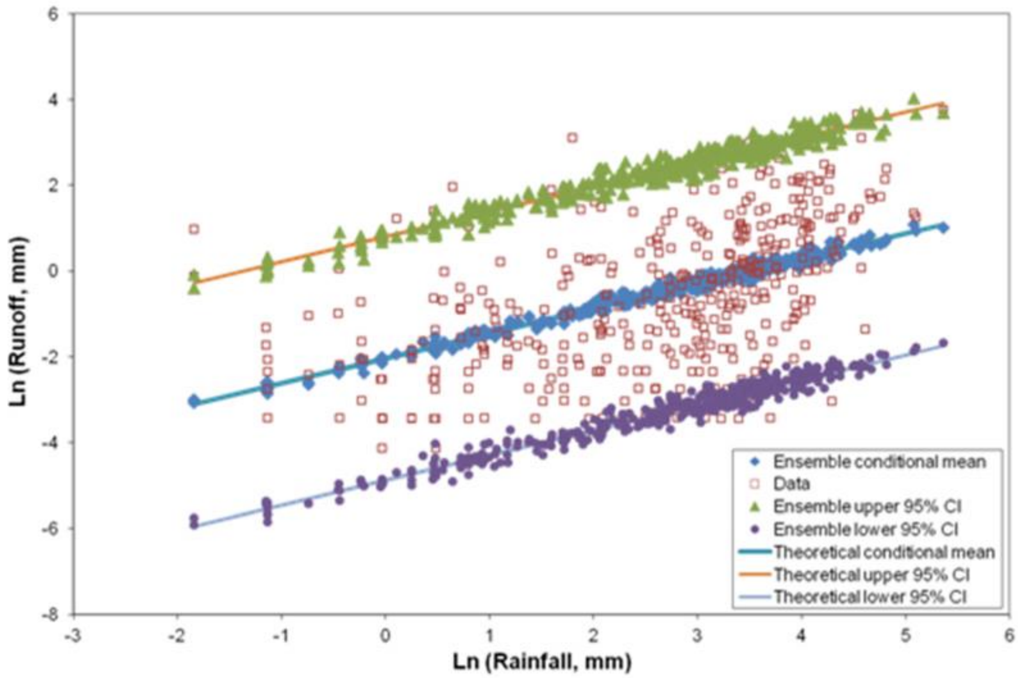


Figure 7. Conditional theoretical mean of the Ln(Runoff, mm) conditioned on Ln(Rainfall, mm) and its theoretical upper and lower 95% confidence intervals compared with the data from all stations with its corresponding ensemble upper and lower 95% confidence intervals estimated from 200 Monte Carlo runs.

Figure 8 shows the conditional distribution function (CDF) calculated FROM the 200 Monte Carlo runs. These conditional distributions are made at three values of the logarithm of the rainfall namely condition on the mean, the mean minus standard deviation (SD), and the mean plus standard deviation. The figure shows the theoretical normal curve and the empirical conditional distribution. According to our aim to do develop a stochastic model based on joint probability distribution for rainfall-runoff, the results show very good agreement between the theory and the numerical simulation experiments.

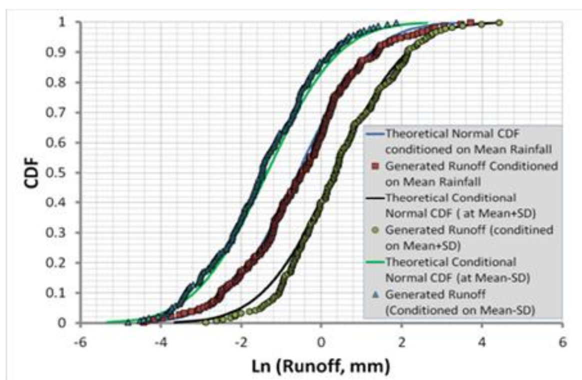


Figure 8. Comparison between conditional theoretical (CDF) of Ln(Runoff, mm) conditioned on Ln(Rainfall, mm) at mean rainfall, mean+SD and mean-SD and the corresponding empirical CDF.

Conclusions

A stochastic model has been formulated based on the bivariate log-normal probability density function of both rainfall and runoff in arid and semiarid basins. The estimated correlation coefficient is 0.5 which is significant particularly in arid and semi-arid zones. Tests for normality for logarithms of rainfall and runoff data has been archived via the application of the Q-Q plot of the marginal distribution and the normal distribution with correlation coefficient of 0.97 and 0.993 for the rainfall and the runoff respectively. Also, the Q-Q plot of the Mahalanobis square distance with Chi^2 distribution for the bivariate normal distribution has been made with correlation coefficient of 0.996. A Monte Carlo approach is adopted to generate realizations of the runoff process conditioned on the rainfall data. The conditional ensemble mean and the conditional ensemble variance have been estimated and compared fairly well with the theoretical model. The runoff data and the runoff predictions fall within the ellipse of 95% confidence interval that gives confidence in the proposed model. This model could be updated in the future by having experimental watersheds in the region to study the impact of climatic changes on the water resources systems.

The finding of the current research is to predict runoff from rainfall with confidence limit. Model implementation is to generate flood as depth over the catchment and can be transferred into volume by multiplying with the area. It can asset in flood estimation as a new tool.

Acknowledgements

This work was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah, under grant No. (6-155-D1432). The authors, therefore, acknowledge with thanks DSR technical and financial support. The authors would also like to thank our technicians Mr. Abdalaziz Al-Bishri for preparing the GIS maps of the basins, Mr. Abdullah Al-Maliki and Mr. Yamen Al-Gahdali for transferring the data from hard copy to a digital form, working on the developed model.

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1/9/2014