Abstract: The paper contains the analyses of the road roller with the flexible shell structure operability and calculation of strength and traction coefficient alteration for road surface compacting. Significant complicacy and variety of mechanical properties of contacting bodies (the road roller operating element and deformed ground) have resulted in appearance of different schemes and their interaction. At the same time, analytical solutions concerning the contact of shifting roll with compacted material have not been carried to the logical completion for the opportunity of their practical use by engineers-designers. There is still an urgent problem of recording and functional approximation of shearing (tangent) contact pressures that along with normal stress that significantly impact on drag coefficient and deflected mode of the road surface under the roll rolling conditions. The absence of general of compacting theory methodology stimulates the designers of road-building machines to create different approximate methods of machines calculation; the number of methods now can be comparable with the range of proposed road roller designs.

Keywords: Road roller, flexible roll, contact pressure.

1. Introduction

Issues of deflected mode of road rollers working elements and compacted materials interaction have been constantly studied. Oriented on solving this urgent problem we present new approximate mathematic model of the original-applied theory of elasticity problem being not described in scientific-technical literature.

Known [1] general case of flat contact deformation when the touch of compacted bodies is on the straight line perpendicular to the plane x0y (Figure 1.а), and functions $f_1(x)$ and $f_2(x)$, determining configuration of cylinder surfaces have continuous first and second derivatives in the point area $x = y = 0$.

Directing the axis Ox on the general tangent to the curves $f_1(x)$ and $f_2(x)$, limiting flexible bodies, we will have:

$$f_1(0) = f_2(0) = 0$$  (1)

Approximate.

The sum of second derivatives $f_1''(0) + f_2''(0)$ we consider as being different from zero and, introducing the assumption about infinitesimally of elastic travel, approximately present $f_1 + f_2$ as follows [1]:

$$f_1'(x) + f_2'(x) = [f_1(0) + f_2(0)] \frac{x^2}{2}$$  (2)

Relative to the distributed contact forces $q(x)$ we introduce the assumption that their resultant (force) $P$ (H/m), perpendicular to the axis $0x$, is directed to the point $0$ of the beginning of interacting bodies touch, that is to origin of coordinates.

As initial gap between the contacting bodies is symmetric relative to the axis $0x$, the pressure $q$ on the cylindrical surfaces will be axially symmetrical elliptic function on the argument $x$ [2], being as

$$q = q(x) = \frac{2 \cdot P}{\pi \cdot c^2} \cdot \sqrt{c^2 - x^2} = \frac{4 \cdot q_c}{\pi} \cdot \sqrt{c^2 - x^2}$$

, where $q_m$=max, $q_c$ – correspondingly the maximum and average value of functional relationship $q(x)$ (Figure 1.а), determined by the formula:

$$q_m = \frac{2 \cdot P}{\pi \cdot c} = \frac{4 \cdot q_c}{\pi}$$  (3)

c is contact area half-width [2].
\[ c = \frac{2 \cdot P \cdot (y_1 + y_2)}{\sqrt{f_1''(0) + f_2''(0)}} \]

where \( y_1, y_2 \) are physical-mechanical constants of interacting materials depending upon the modulus's of elasticity \( E_1 \) and \( E_2 \) and Poisson ratio \( \mu_1, \mu_2 \):

\[ y_1 = \frac{P}{\pi E_1} (1 - \mu_1^2), \]

\[ y_2 = \frac{2}{\pi E_2} (1 - \mu_2^2) \]

Force \( P \) is connected with reaction pressure \( q \) by the integral relation:

\[ P = \int_{-c}^{c} q(x)dx = 2 \int_{0}^{c} q(x)dx \]  

**Solution.**

Concerning the solving applied mechanico-mathematical problem we modify mentioned above I.F. Shtaerman formulas when the fixed steel roller modeled with absolutely rigid and smooth cylinder stamp of elliptic profile, \( f_1(x) = 0, E_1>>E_2 \) or \( E_2 = \infty \), makes static pressure on elastodeformed half plane \( f_2(x) = 0 \Rightarrow f_2''(0) = 0 \), presenting compacted up to the residual movement of soil layer or road coat having the average value of Poisson ratio \( \mu_2 = 0.25 \) (0.2…0.3) and deformation module of \( E_2 = E_x \) [3] (Figure 1.b). We will mark that the parameter \( \mu_2 \) has a comparative little impact on the deflected mode of road coats [3].

To transform and adapt the mentioned above fundamental dependencies we present the necessary analytic correlations (Figure 2) [3]:

- Functions of lower part of the roll cylinder surface \( f_1(x) \) and its second derivative \( f_1''(x) \) from the ellipse equation (Figure 1.b):

\[ f_1(x) = y(x) = -b \cdot \sqrt{1 - \frac{x^2}{a^2} - 1}, 0 \leq y \leq b \]  

\[ f_1''(x) = \frac{d^2y}{dx^2} = \frac{b}{a} \cdot (1 - \frac{x^2}{a^2})^{-\frac{3}{2}}, -a \leq x \leq a \]

- Value \( f_1''(0) \) at \( x=0 \)

\[ f_1''(0) = \frac{b}{a^2} \]  

- Radii of curvature \( R=R(x) \) and \( R(0) \) of the cylinder elliptical guide:

\[ R = R(x) = \left[ 1 + (y')^2 \right]^{\frac{3}{2}} \]

\[ R(0) = \left[ 1 - \frac{x^2}{a^2} \cdot (1 - \frac{b^2}{a^2}) \right] \cdot \frac{a^2}{b} \]

- Dependence of specific linear force \( P \) upon the roll \( B \) width and vertical load \( G \) applied to its center:

\[ P = \frac{G}{B} \]

- Depth \( h \) of the road roller sinking into the material compacted layer (the segment \( kO'k' \) hight, Figure 2), which we find from the ellipse equation (Figure 1.b), when \( x = \pm c \) and \( y = \pm h \):

\[ \frac{c^2}{a^2} + \frac{(h-b)^2}{b^2} = 1 \]

or, at \( h-b < 0 \) (Figure 2),

\[ h = b \cdot \left( 1 - \sqrt{1 - \frac{c^2}{a^2}} \right) \]

Where \( c \) is the value of the curve \( kO'k' \) semi-khord.

![Image](http://www.lifesciencesite.com)

**Figure 2 – Scheme of the roll contact with compacted material.**

After substitutions \( E_2 = E_x \), \( f_2(x) = 0 \), \( f_2''(0) = 0 \), \( y_1 = 0 \), \( \mu_2 = 0.25 \) pressure \( q \) on cylinder surfaces:
q = q(x) = \frac{2 \cdot G_s}{\pi \cdot B \cdot c} \cdot \sqrt{c^2 - x^2} = \frac{g_{1u}}{c} \cdot \sqrt{c^2 - x^2} = 4 \cdot \frac{g_{1u}}{\pi \cdot c} \cdot \sqrt{c^2 - x^2}

q_{1u} = \frac{2 \cdot G_s}{\pi \cdot B \cdot c} ; \quad q_c = \frac{G_s}{2 \cdot B \cdot c} ; \quad q_{1u} = \frac{4}{\pi} \cdot q_c

Y_z = \frac{2}{\pi \cdot E_k} \cdot (1 - \mu_z^2) = \frac{1.875}{\pi \cdot E_k}

C = \frac{2G_s \cdot Y_z}{B \cdot f_1(0)} = \frac{3.75 \cdot G_s}{\pi \cdot E_k \cdot b \cdot B}

h = b \cdot \left[1 - \sqrt{1 - \frac{3.75 \cdot G_s}{\pi \cdot E_k \cdot b \cdot B}}\right]

From the physical - mathematical point of view the correctness of shown above formulas based on classical correlations [3.4], it follows first of all that from the axis symmetry of calculated schemes of Figure 1 and primary bodies contact on the axis z \perp x0y, goes through the point x=y=0 (Figure 1.b). And the model of flat deformed mode is adequate applying to the given structure (non-classical) problem [4] for the areas of the roll cylinder surface contact with the compacted material within the limits of the condition observation [4]:

\[ C << R(0) = \frac{a^2}{\theta} \]

That is at \(-c < x \leq c\):

\[ f_1(x) = \theta \left[1 - \sqrt{1 - \frac{x^2}{a^2}}\right] = \frac{\theta}{2} \cdot \frac{x^2}{a^2} \]

\[ f_1(x) + f_2(x) = \left[f_1'(0) + f_2'(0)\right] \cdot \frac{x^2}{2} = \frac{a}{a^2} \cdot \frac{x^2}{2} \]

where \( f_2(x) = 0 \Rightarrow f_2'(0) = 0 \) (Figures 1, b and 2).

The practical realization of obtained analytical dependencies demands the introduction of additional precondition about invariable (dimensional stability longitudinal) of S length of the elliptic cylinder generatrix in the process of its transformation into the circumference with the radius R=const. In this connection, having the goal of comparability the following calculated values at different ellipse semi-axes a, b, we present the methodology of calculating S and the linear size 2l of contact arch to k0k (Figure 2).

For mathematical formulation of the given procedure it is more convenient to write down the canonical equation of the same ellipse (Figure 1, b and 2)

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

**Figure 3 - To determining the length of elliptic semi-arch of the roll shell**

In the cross-bar system \( x_0y_0z \) and in parametric form [4.5]:

\[ x_3 = a \cdot \sin \varphi, \quad y_3 = b \cdot \cos \varphi \]

Geometrical meaning of parameter \( \varphi \) is understood from Picture 3, where ANA'- is the semi-circumference of radius a an point N, taken on the same vertical with the ellipse M point on the same side of the axis AA. Directly in the being solved problem the angle \( \varphi \) has two numerical values:

1) For calculating of one forth of parameter \( S \), when \( x_3 = a \) and then

\[ \varphi = \varphi_S = \frac{\pi}{2} = 90^\circ \]

2) To determining the length for the elliptic semi-arch \( l \) at \( x_3 = c \) (Figure 3):

\[ \varphi = \varphi_l = \arcsin \frac{c}{a} = \arcsin \sqrt{\frac{3.75 \cdot G_s}{\pi \cdot E_k \cdot b \cdot B}} \]

Differential \( dS \) of arch \( S \) (Figure 3) has the view [5]:

\[ dS = a \cdot \sqrt{1 - \xi^2 \cdot \sin^2 \varphi} \cdot d\varphi \]

Where \( \xi \) is ellipse [5.6] eccentricity with bigger semi-axis \( a \geq b \) \( (0 \leq \xi \leq 1)\):

\[ \xi = \sqrt{\frac{a^2 - b^2}{a}} \]

For circumference \((a=b)\), being a particular type of ellipse, \( \xi = 0. \]
We present desired sizes \( l \) and \( S \) with elliptic integrals \( E(\varphi, \xi), \quad E\left(\frac{\pi}{2}, \xi\right) = E(\xi) \) of the second ordain the form of Legendre second order [6] (incomplete and complete correspondingly):

\[
l = a \cdot \int_{0}^{\xi} \sqrt{1 - \xi^2 \cdot \sin^2 \varphi} \quad d\varphi = a \cdot E(\varphi, \xi) \quad (25)
\]

\[
S = 4 \cdot a \cdot \int_{0}^{\xi} \sqrt{1 - \xi^2 \cdot \sin^2 \varphi} \quad d\varphi = 4 \cdot a \cdot E(\xi) \quad (26)
\]

Which as known [5,6], are not expressed through the elementary functions and in their final view are not taken, but lookup tables were composed [6].

In the generally taken (standard) designations \( E(\varphi, \xi) \) and \( E(\xi) \) eccentricity \( \xi \) is called as elliptic module, and \( \varphi \) is called the amplitude [6].

For \( \xi \) different values (in general case \( 0 \leq \xi \leq 1 \)) function \( E(\varphi, \xi) \) changes from \( E(\varphi, \xi) = \varphi \) at \( \xi = 0 \) (for the circumference, Figure 3) to \( E(\varphi, \xi) = \sin \varphi \), if \( \xi = 1 \). When \( \varphi = 0 \) we have

\[
E(0, \xi) = 0 \quad \Rightarrow \quad \text{at} \quad \varphi = \varphi_S = \frac{\pi}{2}
\]

get complete elliptic integral \( E(\xi) \).

The proposed theoretical model can be used for the rolls of round shape having the external radius \( R \), after changing corresponding letters:

\[
c \Rightarrow c_o, \quad h \Rightarrow h_o, \quad q_M \Rightarrow q_{MO}, \quad q_c \Rightarrow q_{co}, \quad a = a = R, \quad \xi = 0, \quad \varphi_i \Rightarrow \varphi_{io}, \quad S = S_O, \quad l = l_o
\]

(27)

Which are transformed to the following:

\[
q_{CO} = \frac{G_o}{2B \cdot C_o}; \quad q_{MO} = \frac{4}{\pi} \cdot q_{CO}
\]

\[
c_o = \frac{3.75 \cdot R \cdot G_o}{\pi \cdot E_k \cdot B}, \quad c_o \ll R \quad (28)
\]

\[
h_o = R \left(1 - \frac{2 \cdot q_{o}^2}{2} \right) = R \left(1 - \frac{3.75 \cdot G_o}{\pi \cdot E_k \cdot R \cdot B}\right) \quad (29)
\]

\[
\varphi_{io} = \arcsin \frac{3.75 \cdot G_o}{\pi \cdot E_k \cdot R \cdot B} = \arcsin \frac{c_o}{R} \quad (30)
\]

\[
S_O = 2\pi R, \quad l_o = R \cdot \varphi_o
\]

As an example of numerical example we present the algorithm of determining major contact characteristics \( c, \quad c_o, \quad h, \quad h_o, \quad q_M, \quad q_{MO}, \quad q_C, \quad q_{CO}, \quad l, \quad l_o \) at \( B = 140 \text{ cm}, \quad R = 60 \text{ sm}, \quad \xi = \frac{\sqrt{2}}{2} = 0.70711, \quad G_b = 42.5 \text{ kH}, \quad E_k = 65 \text{ MPa} \) (asphalt concrete of mark «A» in the compacted state with the coefficient \( K_c = 1 [7] \)) at observing the stability conditions:

\[
S = S_O = 2 \cdot \pi \cdot R
\]

1) the correlation between \( S \) and \( a \), using tables [7], is like: \( S = 5.4024 \cdot a \);

2) the dependence of ellipse \( a \) and \( b \) semi-axes sizes, roll \( S \) perimeter and radius of curvature \( R(0) \) is determined by the system:

\[
\left\{\begin{array}{l}
\frac{a^2}{b^2} = 1 - \xi^2 = 0.5 \\
S_O = 2 \cdot \pi \cdot 60 = 377 \text{ sm} = 5.4024 \cdot a = S
\end{array}\right.
\]

(31)

From which:

\[
a = 69.78 \text{ sm}; \quad a = \frac{\sqrt{2}}{2} \approx 49.34 \text{ sm};
\]

\[
S = S_O = 377 \text{ sm}
\]

\[
R(0) = \frac{a^2}{\theta} = 98.69 \text{ sm}
\]

3) \( c, \quad c_o \) semi-khords length with checking the conditions \( c < < R(0) \) or \( c_o < < R(0) \):

\[
\left\{\begin{array}{l}
c = 2.35 \text{ sm} << 98.69 \text{ sm} \\
c_o = 1.83 \text{ sm} << 60 \text{ sm}
\end{array}\right.
\]

(32)

4) Reaction contact pressures \( q_M, \quad q_{MO}, \quad q_C, \quad q_{CO} \):

\[
\begin{align*}
q_C &= 64.59 \cdot 10^4 \; \text{Pa} \\
q_M &= 4 \cdot \frac{q_C}{\pi} = 82.24 \cdot 10^4 \; \text{Pa} \\
q_{CO} &= 82.94 \cdot 10^4 \; \text{Pa} \\
q_{MO} &= 4 \cdot \frac{q_C}{\pi} = 105.6 \cdot 10^4 \; \text{Pa}
\end{align*}
\]

(33)

(34)

5) Parameters \( h \) and \( h_o \), characterizing the value of the road roller sinking or general dependence at \( a = \theta = R \) for calculating \( h_o \):

\[
h = 49.34 \cdot \sqrt{1 - \left(1 - \frac{2.35}{69.78}\right)^2} = 0.028 \text{ sm}
\]

(35)

\[
h = 60 \cdot \sqrt{1 - \left(1 - \frac{1.83}{60}\right)^2} = 0.028 \text{ sm} = h
\]

That is, accurate within one thousandth of a centimeter \( h = h_o = 0.0028 \text{ sm} \)

6) Amplitude or angles \( \varphi_i, \varphi_o \):
\[
\varphi = \arcsin \frac{2.35}{69.78} = 1.93^\circ = 0.0337 \text{ (rad)} \\
\varphi_O = \arcsin \frac{1.83}{60} = 1.75^\circ = 0.0305 \text{ (rad)}
\]

(36)

7) Linear sizes of \( l, l_O \) semi-arches (Figures 2 and 3), using Tables of incomplete integrals [8], equal:

\[
l = a \cdot E(\varphi, \xi) = 69.78 \cdot E(1.93^\circ, \frac{\sqrt{2}}{2}) = 69.78 \cdot 0.0337 = 2.35 \text{ (sm) \approx c} \\
l_O = R \cdot \varphi_O = 60 \cdot 0.0305 = 1.83 \text{ (sm) = } c_O
\]

Equalities \( l \approx c \) and \( l_O \approx c_O \) in that example were got because of very small angles \( \varphi, \varphi_O \) and corresponding semi-khords \( c, c_O \), which is the result of relatively large rigidity of the compacted layer of mark “A” asphalt concrete which has the maximum module of deformation \( E = 65 \text{ MPa} \) [8].

The done calculation is illustrated in (Figure 4a) and in the table which contains two more types of the road coats with \( E_K < 65 \text{ MPa} \) [8.9] and the same original geometric characteristics of the rollers \( B=140 \text{ sm, } R=60 \text{ sm, } \xi = \frac{\sqrt{2}}{2} = 0.70711; \xi = 0 \), and also the road roller having: \( S=S_O=377 \text{ sm,} \)

eccentricity (module) \( \xi = \frac{\sqrt{3}}{2} = 0.86603 \) and corresponding value of the elliptic integral

\[
E(\xi) = 1.2111 \text{ [10]. The got results significantly differ from the calculating data of the first example where, as it was noted, } E= 65 \text{ MPa = max.}
\]

The same (figure 4b) presents the shapes of the examined rollers surfaces (in sm), which have \( \xi = 0.70711; \xi = 0 \).

Picture 5 shows the dependence of semi-khord length (a) and contact pressure (b) upon the module of deformation, and Picture 6 shows the dependence of contact pressure upon the roller curvature radius.

Compacting parameters dependence upon the module of deformation for different geometrical shape roller of the road roller is presented in Table 1.

![Figure 4 – Pressure calculated diagrams for the roller different shape at E= 65 MPa.](http://www.lifesciencesite.com)

**Table 1 - Compacting parameters dependence upon the module of deformation.**

<table>
<thead>
<tr>
<th>Roller shape</th>
<th>Eccentricity ( \xi )</th>
<th>Unknown parameters (in letters)</th>
<th>Uncompensated asphalt</th>
<th>Asphalt binder + asph.</th>
<th>Coarse soil</th>
<th>Solid gravel</th>
<th>Increased islanded ground</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ellipse</td>
<td>( \xi = 0.70711 )</td>
<td>( c_O )</td>
<td>( 60 \text{ sm} )</td>
<td>( 60 \text{ sm} )</td>
<td>( 60 \text{ sm} )</td>
<td>( 60 \text{ sm} )</td>
<td>( 60 \text{ sm} )</td>
</tr>
<tr>
<td>Circle</td>
<td>( \xi = 0.86603 )</td>
<td>( c_O )</td>
<td>( 60 \text{ sm} )</td>
<td>( 60 \text{ sm} )</td>
<td>( 60 \text{ sm} )</td>
<td>( 60 \text{ sm} )</td>
<td>( 60 \text{ sm} )</td>
</tr>
</tbody>
</table>

![Figure 5 – Dependence of semi-khord (a) length and contact pressure (b) upon the module of deformation ( – – – Ellipse 0.70711; ----- Ellipse 0.86603; – – – Circle)](http://www.lifesciencesite.com)

![Figure 6 - The dependence of the contact pressure of the radius of curvature of the drum.](http://www.lifesciencesite.com)

836
Conclusions.

The done calculations of the developed analytical dependences confirm the assumption done by A.F. Zubkov [11] that the arch “c” length equals to the khord subtending it l (c=l Table 1). The comparison of experimental and theoretical results allows to conclude that the accepted methodology of calculating the roll flexible shell is quite accurate.

Deduced formulas and developed calculating theory prove the opportunity of control (optimization) [11] of the parameters of the road roller contact interaction with compacted layer by varying under operating conditions of designed semi-axes sizes of ellipse-shape surface of the roll flexible shell of the road roller, thus, increasing the quality of the road coat.

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