

## Contact Force Calculation of the Machine Operational Point

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**Abstract:** The paper contains the analyses of the road roller with the flexible shell structure operability and calculation of strength and traction coefficient alteration for road surface compacting. Significant complicacy and variety of mechanical properties of contacting bodies (the road roller operating element and deformed ground) have resulted in appearance of different schemes and their interaction. At the same time, analytical solutions concerning the contact of shifting roll with compacted material have not been carried to the logical completion for the opportunity of their practical use by engineers-designers. There is still an urgent problem of recording and functional approximation of shearing (tangent) contact pressures that along with normal stress that significantly impact on drag coefficient and deflected mode of the road surface under the roll rolling conditions. The absence of general of compacting theory methodology stimulates the designers of road-building machines to create different approximate methods of machines calculation; the number of methods now can be comparable with the range of proposed road roller designs.

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### 1. Introduction

Concerning fixed contact, they have developed quite effective methods of qualitative assessment of contact parameters based on the tested experimental-fundamental Hertz-Belyaev solution [1, 2, 3] for circular road rollers, and also for physical linear axisymmetric Shtaerman mathematical model [3] for improving the structure of flexible steel roll [3] with variable geometry of the shell from the circular profile to the elliptic one.

### 2. Calculation methods.

The given paper is devoted to a new universal mechanical-mathematical model of the road roller static contact with the compacted layer that generalizes a particular design theory [3] for movable (driving and driven) elliptic circular rolls taking into account the well-known achievements and recommendations concerning this promising research orientation.

The model basis includes formulated below assumptions and hypotheses confirmed by fundamental-applied and experimental-theoretical works [4]:

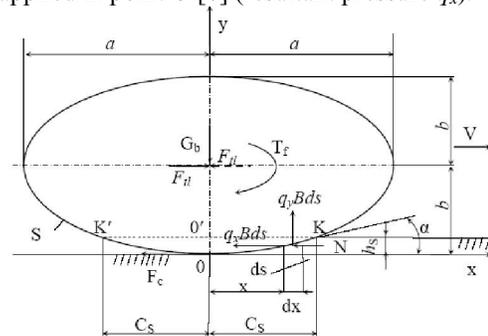
1) The flexible roll of  $B$  width rolls without sliding with constant linear rate  $V=const$  on elastodeformed plane keeping elliptic shape of the cylinder surface with semiaxes  $a$  and  $b$  [4] (Figure 1);

2) The layer of soil or road surface having the given modulus of deformation  $E_k$  [3], is compacted to the termination of residual movements;

and the roll shell is proposed to be absolutely rigid and ideally smooth [5];

3) The rolling driving roll is effected with the following external force factors of static character (Figure 1) [5]:

- vertical force  $G_b$ , equal to the mass of the roll and corresponding frame part;
- the desired horizontal component or tractional load  $F_{it}$ ;
- torsion torque  $T_f$ , transmitted with the road roller roll transmission;
- normal  $q_y$  and shearing  $q_x$  are reactive distributed loads in moving frame of reference  $XOY$  (Figure 1) on contact surfaces of interacting bodies directed parallel to axes  $Y$  and  $X$ ;
- force  $F_c$  of compacted material cohesion with the roll which keeps the road roller from sliding and applied in point  $O$  [6] (resultant pressure  $q_x$ ).



**Figure 1 – The diagram of movable flexible roll contact with the compacted material**

**3. Problem solution.** Torque force  $T_f$  depends upon the power  $W$  of the road roller motor for its moving in the stated operating mode with the speed of  $V=const$  and the number of its revolutions  $n$  per minute. Taking into account the known correlation between the dimensions (1kgf=9.81H; 1 h.p.=0.736 kw; 1 kw=102 kgm/s) [6], we will get:  
or, when  $W$  is measured in horse power (h.p.),

$$Tf = 716.2 \frac{W}{n} (kgm) = 7026 \frac{W}{n} (Hm),$$

Where the parameter  $n$  is connected with the linear speed  $V$  and small semiaxis  $b$  of rotating ellipse (within assumptions1) dependence [6,7]:

$$n = \frac{V}{2\pi b} \tag{1}$$

Formula (1) is derived from the solution of classical kinematics problem about displacement of a point with the constant angular velocity (Figure 2) on the elliptic curve being a guide of the cylinder surface of the roll moving without sliding with the velocity of  $V = const$  [6,7].

$$W = 2\pi n l \tag{2}$$

Calculating the parameter  $W$ , in the applied diagram in Figure 2, let us examine point O, where the road roller touches the compact plane  $y=0$ .

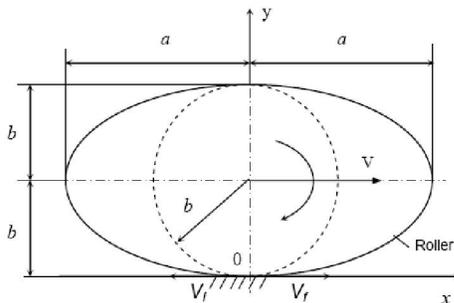


Figure 2 – To the determining of  $N$  roll number of revolutions

Relative linear velocity  $V_l$  of this point is directed to the left on the tangent этой точки  $x$  and is expressed with product [7]

$$V_l = -bW, \tag{3}$$

and frame velocity is

$$V_f = V \tag{4}$$

Since, according to the man-made assumption, the roll rolls down without sliding, the absolute velocity  $V_a$  of point O at the given time moment equals naught:

$$V_A = |V_f + V_l| = 0.$$

Taking into account (3) and (4),

$= V$ , and  $W = \frac{V}{b}$ ; after excluding  $W$  from formula (2), we get mentioned above correlation (1) for calculating  $n$ .

4) The specific pressures  $q_x=q_x(x)$ ,  $q_y=q_y(x)$ , will be considered as constant along forming cylinder surface of the roll and dependable only upon the distance  $x$  to vertical plane coming through the ellipse center (Figure 1) [7]. This precondition complementing condition 2), is the basis to schematize the examined stress state as two-dimensional problem of mechanics of deformable bodies [7].

5) When the road roller moves, the reactive forces  $q_x$  and  $q_y$  from the compacted surface are transformed to the roll rotating surface through the half of the arch K'OK [7,8], that is, operating within the limits  $0 \leq x \leq C_s$ , where  $C_s$  is semichord K'K desired length (in case of roll rolling).

The given precondition implies the convertibility of parameter  $C_s$  and  $h_s$  depth of the road roller sinking into the compacted material layer (K'OK segment height, Figure 1), comparing with the

analog contact characteristics  $C$  and  $h$  for the rigid roll when  $F_{il}=0$  and

$$Tf = 0.$$

In accord with the diagram of Figure 1, and well-known data [8,9], we present (on the basis of I.F. Shtaerman solution [9]), the formulas for calculating the parameters  $C$ ,  $h$ , of standard pressure dependence  $q_y=q=q(x)$  in coordinates XOY (Figure 1) and its maximum value  $q_m=q(0)$ , taking into account that  $q_x(x)=0$  [9]:

$$q = \frac{q_M}{c} F(x) = \frac{q_M}{c} \sqrt{c^2 - x^2} \tag{5}$$

$$q_M = \frac{2G_b}{\pi BC},$$

$$C = 1.0925a \sqrt{\frac{G_b}{E_k b B}}, \tag{5}$$

$$h = 0.5968 \frac{G_b}{E_k B}, \tag{6}$$

where  $\omega_g R_c(0) = \frac{a^2}{b}$  is ellipse radius of curvature in point O (Fig. 1);  $h < b$ ;  $-C \leq x \leq C$ ;  $c < R_c(0) = a/b$ .

$F(x) = \sqrt{c^2 - x^2}$  – is Hertz - Belyaev - Shtaerman function which as pointed in [9], most

exactly reflects the physics of road roller interaction with compacted ground and is appropriate for the experimental data.

6) Because of small sizes  $C, h, C_s, h_s$ , that is, insignificant correction of contact surface (see inequalities (5) – (6), and the similar ones for  $C_s, h_s$  [9,10]) and that was proved by studies [9,10], we replace the arch OK with the chord:

$$O'K = C_s \approx OK, \tag{7}$$

which without harm for achieving the calculated model demanded accuracy, significantly simplifies the following mathematical manipulations as a result of obvious correlations coming out of Figure 1 diagram 1:

$$\sin \alpha \approx 0; \cos \alpha \approx 1; ds - \frac{dx}{\cos \alpha} \approx dx;$$

where  $\alpha$  is rate of change in the arbitrary point of  $N(x,y)$  ellipse.

Taking into account the mentioned above assumptions force power  $q_x, q_y$  are approximately the resulting projection on axes  $x,y$  of pressures  $q_\sigma, q_\tau$ , operating in the directions of normal and tangent to the roll surface (Figure 3) [9,10]:

$$\begin{aligned} q_x &= q_\sigma \sin \alpha + q_\tau \cos \alpha \approx q_\tau, \\ q_y &= q_\sigma \cos \alpha - q_\tau \sin \alpha \approx q_\sigma \end{aligned}$$

Like in published papers [9,10], where they describe symmetric mathematical model of stationary (fixed) contacting, we use generally accepted in strength of materials assumption for bending elements about the invariability of length  $S = const$  of the elliptic guide of the cylinder roll in the process of its transformation in radius  $R$  circumference (Figure 1). In this connection, trying to compare the calculated values at different ellipse semiaxes  $a, b$ , we present the methodology of their calculating according to the given radius  $R$  of the round roll surface and eccentricity [9,10] (elliptic module), at  $0 \leq \xi < 1$  (for the circumference  $\xi=0$ ):

$$\xi = \frac{\sqrt{a^2 - b^2}}{a}, \tag{8}$$

a)  $S$  guide length calculation

$$S = 2\pi R; \tag{9}$$

b) determining the correlation between  $S$  and  $a$  through the complete elliptic integral of the second order  $E(\xi)$  in Legendre form:

$$S = 4a \int_0^{\frac{\pi}{2}} \sqrt{1 - \xi^2 \sin^2 \varphi} d\varphi = 4aE(\xi); \tag{10}$$

c) the solution of combined equations (8), (10) relative to ellipse sizes  $a$  and  $b$  (Figure 1):

$$a = \frac{S}{4E(\xi)}, \quad b = a\sqrt{1 - \xi^2}$$

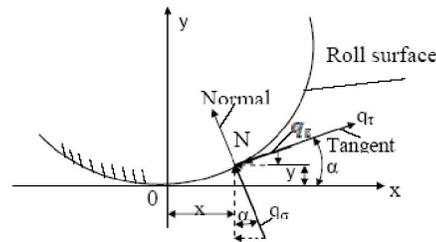


Figure 3 – Contact stresses (pressure) at the road roller rolling.

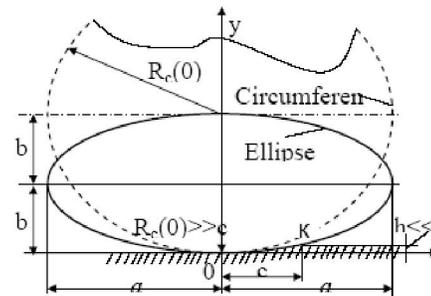


Figure 4 –Design diagram for strength coefficient  $f_r$  formula calculation

The most important physical-mechanical parameter for the road roller moving is the strength coefficient  $f_r$ , which according to the experimental-theoretical study is presented with the following semiempirical dependence for the round shape rolls [10]:

$$f_r = 0.75 \sqrt{\frac{h}{2R}}. \tag{11}$$

We are modifying the formula (11) with regard to the flexible elliptic roll taking into account inequality (5), assumption 6) and replacing  $R$  in a small interval (Figure 1) with the constant value, corresponding to the value of function  $R_c(x)$  of ellipse radius of curvature in point O, when  $x=0$  (Figure 4) [10]:

$$R_c(0) = \frac{a^2}{b} = const, \tag{12}$$

At substitution in (11) of the fixed roller immersion depth  $h$ , according to (6), and radius  $R_c(0)$ , in accord with (12), we get the desired dependence

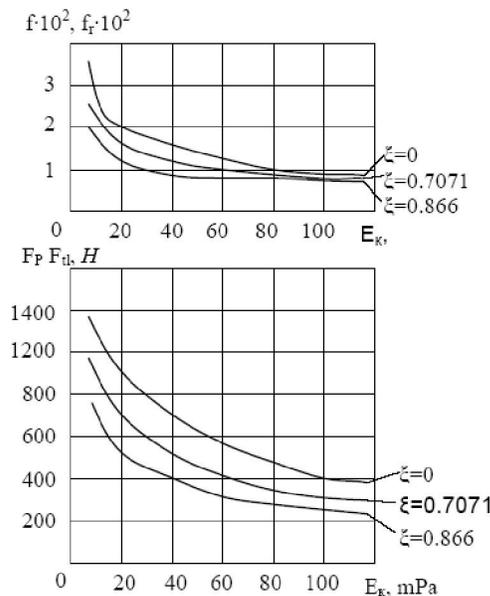
$$f_r = 0.375 \frac{bc}{a^2} = 0.41 \frac{b}{a} \sqrt{\frac{G_b}{bBE_k}}, \tag{13}$$

Which in special case  $a=b=R$ , is converted into the known formula after changing the product  $\alpha E_k$  into  $E_k$  ( $\alpha$  is the correction coefficient [10]):

$$f_r = 0.41 \sqrt{\frac{G_b}{RBE_k}}$$

**Table 1 – values by  $f, f_r, F_P, F_{tl}$**

Eccentricity $\xi$ (the roll form)	Desired parameters	The modulus of deformation $E_k, \text{ mPa (N/mm}^2\text{)}$				
		8	20	30	65	116
$\xi = 0$ (circumference)	$f$	0.0325	0.0206	0.0168	0.0114	0.0085
	$F_P, H$	1381.2	875.5	714.0	484.5	361.2
$\xi = 0.7071$ (ellipse)	$f_r$	0.0254	0.0161	0.0131	0.0089	0.0067
	$F_{tl}, H$	1079.5	684.2	556.8	378.2	284.8
$\xi = 0.866$ (ellipse)	$f_r$	0.0202	0.0128	0.0105	0.0071	0.0053
	$F_{tl}, H$	858.5	544.0	446.2	301.8	225.2



**Figure 5 – Calculated diagrams of dependencies  $f(E_k), f_r(E_k), F_P(E_k), F_{tl}(E_k)$**

Figure 5 and Table 1 illustrate the functions  $f = f(E_k), f_r = f_r(E_k)$  and propelling forces [10]:

$$F_P = F_P(E_k) = f G_b,$$

$$F_{tl} = F_{tl}(E_k) = f_r G_b$$

for the three numeric values  $\xi = 0, \xi = \frac{\sqrt{2}}{2} = 0.7071, \xi = \frac{\sqrt{3}}{2} = 0.866$

for  $B=1400 \text{ Nm}, R=600\text{mm}, G_b=42500\text{H}; E_{k1}=8\text{mPa}; E_{k2}=20\text{mPa}; E_{k3}=30\text{mPa}; E_{k4}=65\text{mPa}; E_{k5}=116\text{mPa}.$

Taking as a guide these original data and the table of elliptic integrals we have according to mentioned above dependencies (9) – (13):

case 1, when  $\xi = 0$  (circumference),  $a=b=R=600 \text{ mm}:$

$$E(\xi) = E(0) = \frac{\pi}{2} \approx 1.5708;$$

$$S = \pi R = 2 \cdot 3.1416 \cdot 600 = 3770 \text{ mm};$$

$$f = f(E_k) = \frac{0.0922}{\sqrt{E_k}}; \tag{14}$$

case 2 – elliptic shell ( $\xi = \frac{\sqrt{2}}{2} = 0.7071$ ):

$$S = 3770 \text{ mm}, E(\xi) = E\left(\frac{\sqrt{2}}{2}\right) = 1.3506;$$

$$a = 697.8 \text{ mm}, \quad b = 493.4 \text{ mm};$$

$$f_r = f_r(E_k) = \frac{0.0719}{\sqrt{E_k}};$$

case 3 – the roll elliptic profile ( $\xi = \frac{\sqrt{3}}{2} = 0.866$ ):

$$S = 3770 \text{ mm}, E(\xi) = E\left(\frac{\sqrt{3}}{2}\right) = 1.2111;$$

$$a=778.2 \text{ mm}; \quad b=389.1\text{mm};$$

$$f_r = f_r(E_k) = \frac{0.0573}{\sqrt{E_k}}$$

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**Conclusions.** Comparing the curves of Figure 5 and analyzing numerical information in Table 1, we may note that with the increasing of ovalization degree of the road roller roll shell, that is, with the increasing of the parameter  $\xi$ , characterizing ellipse oblongness in the direction of x axis (Figure 1), there is the decreasing of drag coefficient  $f_r$  and traction  $F_{tl}$ , comparing with analog values  $f$  and  $F_P$  for the round shape roll. In the studied calculation examples this effect was 21.7 % at  $\xi = 0.7071$  and 38 %, when  $\xi = 0.866$ .

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