Modeling Integrative and Derivative Systems in Bond Graph Modeling and using it in Adaptive Control

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Abstract: In this paper a method for modeling integrator and derivative of effort and flow variables in a bond graph model is offered. In many dynamical systems there is a need to get the value of integral and derivative of a variable, for example in PID controller in a closed-loop controlled system. Here the way of implementing individual integrator and derivative elements using bond graph possibilities is presented. In this work the variables that their integral or derivative values are taken, are presented in the form of effort or flow variables as two fundamental variables of the system. This will allow us to model many systems by bond graph that they couldn't be modeled before. In continue, its usage in some examples is presented. For examples classic closed-loop controlled systems by PID controller are used. Then the state equations, ruled over the full closed-loop system, are extracted from the model. It can be seen that the PID gains are appeared in the state equations. This specification is used in adaptive control by gain scheduling approach, and by these equations; the PID gains, required for attaining the desired system behavior along the time, are obtained. Using this method the PID gains for gain scheduling approach for a nonlinear plant can be obtained with no need to linearizing it. Furthermore this approach has the advantage of capability of modeling hybrid systems consist of various domains such as mechanical, electrical, hydraulic, magnetic and …, and designing controller for them. It can be seen that in this paper there are some novelties consist of modeling integrator of effort and flow by bond graph, modeling closed-loop controlled systems entirely by bond graph and also investigating adaptive control by bond graph.

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1. Introduction

In the 1950s, Henry Paynter, the professor of MIT mechanical engineering department, designed a topic of modeling, based on the efficient representation of the relation between two ports by just one line that he called it 'bond'. This so-called 'bond graph' topic was completed later when he finally introduced the concept of the junction in 1959 in a lecture. Junctions in addition to make the bond graph a powerful tool, have the advantage that they are rather abstract concepts that require another paradigm shift. Once this shift is made, it often induces over-enthusiasm and over-expectations that usually lead to disappointment and also unnecessarily scare off experienced engineers and scientists, because they have learned to accept the limitations of modeling.

The Bond Graph works by using of a pair of variables named 'power variables' or "*effort*" and "*flow*" that in each domain of energy are different and special for that domain. These models are made of some bonds which connect together through single port, double port and multi port elements. Each bond has it effort and flow values whose product is the instantaneous power of the bond. For example, the bonds in an electrical system represent the flow of electrical energy and these two variables are electrical voltage and current, whose product is power. Examples of effort include force, torque, voltage, or pressure; while flow examples include velocity, current, and volumetric flow.

In the sixties, the topic, e.g. the half arrow to represent positive orientation and insightful node labeling, was further developed with more details by his students, in particular Dean C. Karnopp, later professor at UC Davis (Ca), and Roland C. Rosenberg, later professor at Michigan State University (Mich.), [Rosenberg, 1968, 1974, 1990]. Rosenberg also designed the first software tool (ENPORT) that supported simulation of bond graph models [Rosenberg, 1965, 1974]. In the early seventies Jan J. Van Dixhoorn [1972, Evans et al., 1974], professor at the University of Twente, NL and Jean U. Thoma [1975] professor at the University of

Waterloo introduced bond graphs in Europe for the first time.

These pioneers in bond graph topic and their students have been spreading its ideas worldwide [Karnopp et al., 1979]. Jan Van Dixhoorn used an early prototype of the block-diagram-based software TUTSIM to input simple causal bond graphs. This work about a decade later, resulted in a PC-based tool [Beukeboom et al., 1985]. This endeavors finally resulted to the development of truly port-based software tool 20-sim at the University of Twente [Broenink and Breedveld, 1988; www.20sim.com]. Breedveld also performed research in modeling more complex physical systems, in particular thermofluid systems [Breedveld, 1979].

Modeling by bond graph has advantage of being able to modeling nonlinear systems. It can also model many domains of dynamical systems. Because of these features, nowadays it is used as a useful tool for modeling various types of dynamical systems such as electrical, solid mechanical, hydraulic, pneumatic and so others. It can also model hybrid systems containing various subsystems [Borutzky, 2007], specially it has been used in mechatronics systems ([Gawthrop, 1991], [Amerongen, Breedveld, 2003], [Breedveld, 2004], [Dauphin, 2008]) and in the last two decades it have been a topic of research or are being used in research at many universities worldwide and part of (engineering) curricula at a steadily growing number of universities. In the last decade, their industrial use also has become more and more important. Although it is mainly used in physical systems, but it also can be used in non physical dynamical systems. The details of modeling dynamical systems may be found in some resources like [Karnopp, 2006].

Dynamical systems that can be modeled by bond graph are a concept in mathematics where a fixed rule describes the time dependence of a point in a geometrical space. At any given time a dynamical system has a *state* given by a set of real numbers. The *evolution rule* of the dynamical system describes what future states from the current state will be. Further details may be found in other resources like [Katok, 1996]. In the most of these systems we are dealing to differential equations. Thus, it is usual to need the integral and derivative value of state variables. Additionally when these systems are modeled using bond graph, often the effort and flow variables play the role of state variables, or the state variables are dependent to them. So it is not unusual to need the integral or derivative value of effort or flow variables. For example one of its very common usages is the PID controller. A proportional–integral–derivative controller (PID controller) is a generic control loop feedback mechanism (controller) widely used in industrial control systems (refer to [Friedland, 2005]).

By what that mentioned, it is sensed that there is a need to have integrator and derivative elements in bond graph modeling. These elements should catch the value of effort or flow variables and give them in the case of effort or flow again, for reusing in the bond graph model. Peter J. Gawthrop, the professor of Glasgow University in [Gawthrop, 1994], presented a way for modeling PID controller using bond graph. But there is a lack of modeling individual integrator and derivative elements in bond graph modeling.

In this work, first the method of modeling integrator and derivative by bond graph is presented in section 2. The bond graph model of PID controller is shown in section 3. Then some examples of closed loop controlled systems, involving PID controller are presented in sections 4 and 5 and the state equations of the entire closed loop system are extracted. It will be seen that the PID gains are appeared in state equations. Thus this feature is used in the case of adaptive control, and it will be shown that using these equations, a suitable set of PID gains could be produced that produce the desired behavior for our system.

2. Modeling integrator and derivative of effort and flow

In this section as the first step, the equivalent bond graph models for the integrator and derivative of effort and flow are shown. For this purpose the inertia and capacity elements should be used. The conversions of the values of effort and flow variables in these elements follow a differential equation. Thus, the desired values of effort and flow can be obtained, using them. In this topic there are four different options, which are listed and explained as below:

2.1 Integral of effort

For obtaining the integral value of effort variable, an inertia element is used. Its schema is shown in Figure 1. It can be seen that there is a one junction with only one input and one output bonds. So the values of effort and flow in both bonds become the same and equal to e_1 and *f*. The ruled equation over it becomes as below:

$$e_1 = I\frac{df}{dt} \Rightarrow \int e_1 dt = If \overset{I=1}{\Longrightarrow} \int e_1 dt = f = e_2 \qquad (1)$$

It can be seen that by taking integral from both sides of the differential equation of the inertia element, the term of the integral of effort (e_1) is obtained. For simplicity the value of the parameter of *I* is set to 1. It is observed that the flow of the junction (*f*) keeps the value of the integral in itself. Thus, if a source be used that its output be equal to this fallow's value; it can be reused in the bond graph model. As it is seen in Figure 1 there are two ways for it: if the integral of effort is wanted in the form of effort, a source of effort is used (Figure1.a), and if it is wanted in the form of flow a source of flow is used (Figure 1.b).

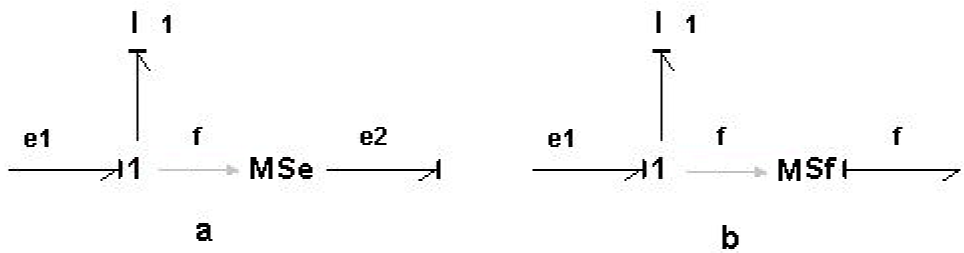

Figure 1: The bond graph model of integrator of effort

2.2 Derivative of effort

Obtaining the derivative of effort is the same as the previous one, except one difference: instead an inertia element a capacity element should be used. Its schema is shown in Figure 2. Its equation is as below:

$$e_1 = \frac{1}{C}\int f\,dt \Rightarrow \frac{de_1}{dt} = \frac{f}{C} \overset{C=1}{\Longrightarrow} \frac{de_1}{dt} = f = e_2 \qquad (2)$$

It can be seen again that the flow of the junction (*f*) keeps the value of the derivative of the effort (e_1) in itself. Again in two ways, a source of effort or flow is used that its output is equal to *f*.

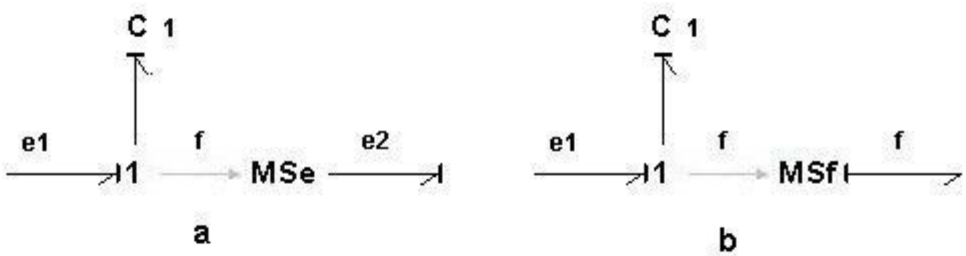

Figure 2: The bond graph model of derivative of effort

2.3 Integral of flow

For obtaining the integral of flow again a capacity elements should be used. But in the flow option instead of a one junction a zero junction should be used. Its schema could be seen in Figure 3. Because the junction has only one input and one output the effort and flow in both bonds are equal (e,f_1). Its equation becomes as below:

$$e = \frac{1}{C}\int f_1\,dt \Rightarrow \int f_1\,dt = Ce \overset{C=1}{\Longrightarrow} \int f_1\,dt = e = f_2 \qquad (3)$$

It can be seen in this equation that this time the effort of the junction (e) keeps the value of the integral of the flow (f_1) in itself. Thus, like the previous ones a source of effort or flow is used, that its output is equal to this effort.

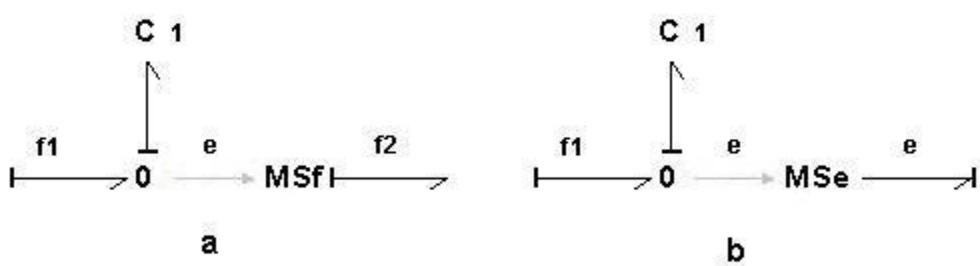

Figure 3: The bond graph model of integral of flow

2.4 Derivative of flow

The last option is the derivative of flow. This option is similar to the previous one, except that instead of a capacity element an inertia element is used. Its schema is presented in Figure 4. It can be seen that again there is a zero junction and its effort value is used to define the output of the source. The ruled equation over it is as below:

$$e = I\frac{df_1}{dt} \Rightarrow \frac{df_1}{dt} = \frac{e}{I} \overset{I=1}{\Longrightarrow} \frac{df_1}{dt} = e = f_2 \qquad (4)$$

It can be seen that the value of derivative of the flow (f_1) is equal to the effort of the junction (e). Thus a source of effort or flow is used that its output is equal to this effort.

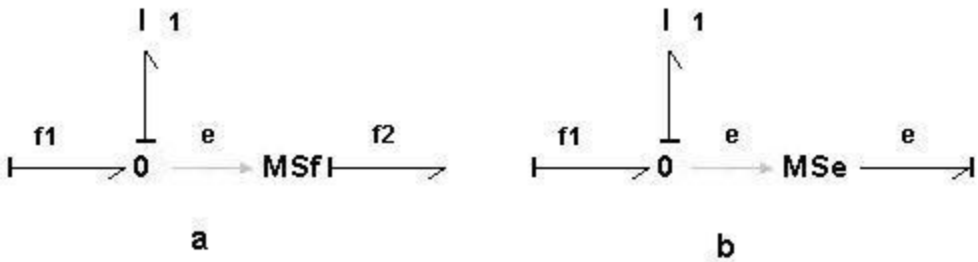

Figure 4: The bond graph model of derivative of flow

3. Modeling PID controller

In previous section the way of modeling integrator and derivative of the effort and flow variables in such way that their value can be reused in the model, was presented. Now by using this information a bond graph model for a PID controller can be designed. For modeling it the input error to the PID is considered in the form of effort or flow. Then the integral and derivative parts can be modeled by the ways described in previous section. The proportional part also can be modeled using a transformer. Then three outputs of these three parts should be added by a junction, and then the output of PID can be used by an output bond from the junction. But it must be considered that in this model if the parameters of the capacity and inertia elements be set to 1, it means that integral and derivative gains of PID are 1. Thus, the parameters of the capacity and inertia elements should be set in such way that satisfies the desired values of the gains of PID.

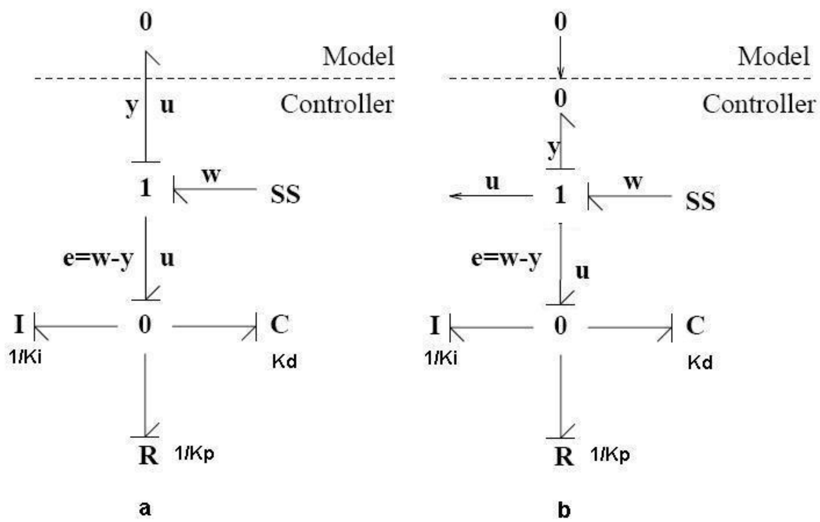

Figure 5: The bond graph model of PID for controlling effort

Although the PID controller can be modeled by this way but Peter J. Gawthrop has presented an easier way for modeling PID that it can be found in [Gawthrop, 1994]. In this plan again the input error to the PID must be considered in the form of effort or flow. But the difference is that the proportional part is modeled by a reluctance element and also the form of model is more integrated.

In Figure 5 this model is presented when the input (y) is effort. SS presents the desired value of system that it's in the form of effort too. The flow of one junction (u) becomes the output of the PID. The value of C, I and R parameters can be obtained from Equations 1-4, and they are seen in the figure. In Figure 5.a input and output, both are taken from one point, such that input is the effort of the bond and output is the flow of the bond. But in Figure 5.b they are taken from separate points, that in this case the value of flow of one junction must be transmitted by a data line. In Figure 6 the same model is shown when the input is in the form of flow, and in this case output is the effort of the zero junction (u).

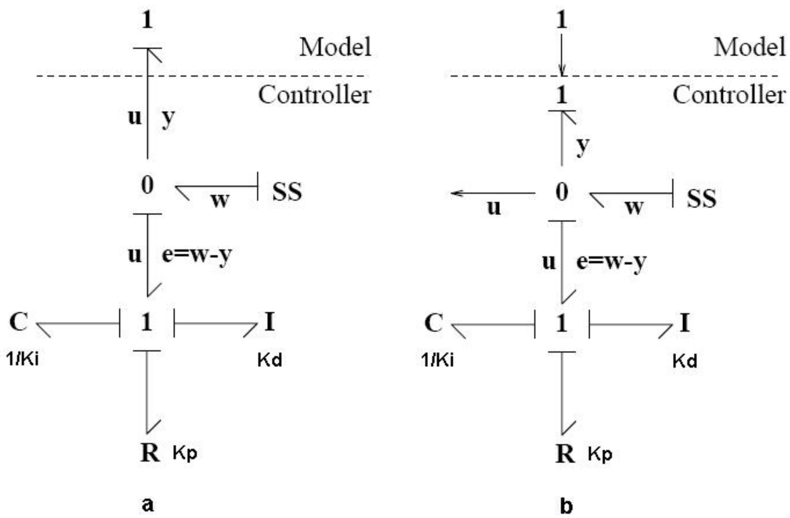

Figure 6: The bond graph model of PID for controlling flow

Now that the ways of modeling integrator and derivative and PID controller were presented, they can be used in modeling various systems. In the following sections for example the bond graph model of a closed-loop controlled system is designed entirely.

4. Modeling closed-loop controlling system of electrical motor

In this section as an example an electrical motor is considered, that is connected to a source of voltage, and is spinning a wheel (J_w). The bond graph model of this open loop system is shown in Figure 7. It can be seen that an electrical reluctance and self are connected to the motor.

For a closed-loop system two options of controlling velocity and position of the wheel are considered. In following sections first these two options are modeled. Then the adaptive control using bond graph, for the second option is shown.

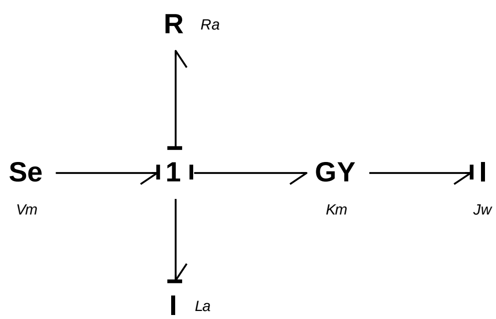

Figure 7: The bond graph model of an electrical motor

4.1 Controlling the velocity of the motor

In this section a closed-loop control system for controlling the angular velocity of an electrical motor is selected for investigation. The bond graph model of this system is shown in Figure 8. It is seen that, a source of flow is used that its output is equal to the negative of the flow of the wheel, and another source of flow presents the desired value of the velocity. These two values are added by a zero junction, so the flow of the output bond is the error value of the velocity.

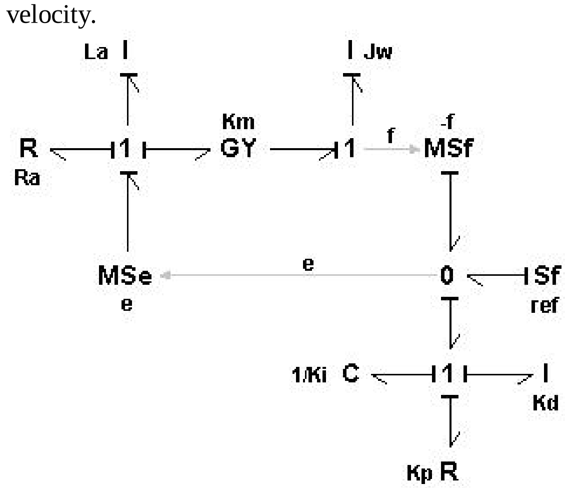

Figure 8: The bond graph model of the closed loop control system of the velocity of the electrical motor

Now the state equations of the system can be extracted from the model. the state variables are related to the self, the inertia of the wheel and the capacity element that represent integral gain of PID:

$$\dot{P}_{L_a} = -\left(\frac{K_d K_m}{J_w L_a} + \frac{R_a}{L_a}\right) P_{L_a} - \frac{K_p + K_m}{J_w} P_{J_w} + K_i Q_{K_i} + K_p.ref \quad (5)$$

$$\dot{P}_{J_w} = \frac{K_m}{L_a} P_{L_a} \quad (6)$$

$$\dot{Q}_{K_i} = ref - \frac{1}{J_w} P_{J_w} \quad (7)$$

The velocity of the wheel in every moment by these equations can be found ($\frac{P_{J_w}}{J_w}$). The equations in the matrix from will be:

$$\begin{bmatrix}\dot{P}_{L_a}\\ \dot{P}_{J_w}\\ \dot{Q}_{K_i}\end{bmatrix}=\begin{bmatrix}-\left(\frac{K_dK_m}{J_wL_a}+\frac{R_a}{L_a}\right) & -\left(\frac{K_p}{J_w}+\frac{1}{K_mJ_w}\right) & K_i\\ \frac{K_m}{L_a} & 0 & 0\\ 0 & -\frac{1}{J_w} & 0\end{bmatrix}\begin{bmatrix}P_{L_a}\\ P_{J_w}\\ Q_{K_i}\end{bmatrix}+\begin{bmatrix}K_p\\ 0\\ 1\end{bmatrix}ref \quad (8)$$

It can be seen that the PID gains are appeared in these equations.

4.2 Controlling the position of the motor

Now in this section we have the same system, but this time controlling the position of the wheel is desired. For obtaining the position of it, the integral of the velocity must be obtained. Thus, there is a need to take integral from the flow of the wheel that it can be done by the presented way in section 2.3 and using a capacity element, that one of the state variables of the system is related to this element. The bond graph model of this system is presented in Figure 9. In this model, e presents the integral of the velocity or the position. It is seen that this time the desired value for control system, is presented in the form of effort, so the bond graph model for controlling effort (Figure 5), for modeling the PID, should be used, that the output flow of it presents the output of the controller (f_2). Thus, a source of effort is used, that its output is equal to this flow.

The system's state equations become as below:

$$\dot{P}_{L_a}=-\frac{R_a}{L_a}P_{L_a}-\frac{K_d+K_m}{J_w}P_{J_w}-K_pQ_{int}+K_iP_{K_i}+K_p.ref \quad (9)$$

$$\dot{P}_{J_w}=\frac{K_m}{L_a}P_{L_a} \quad (10)$$

$$\dot{Q}_{int}=\frac{1}{J_w}P_{J_w} \quad (11)$$

$$\dot{P}_{K_i}=ref-Q_{int} \quad (12)$$

In these equations because the parameter of C_{int} is 1, so $Q_{int}=e$ and it presents the position.

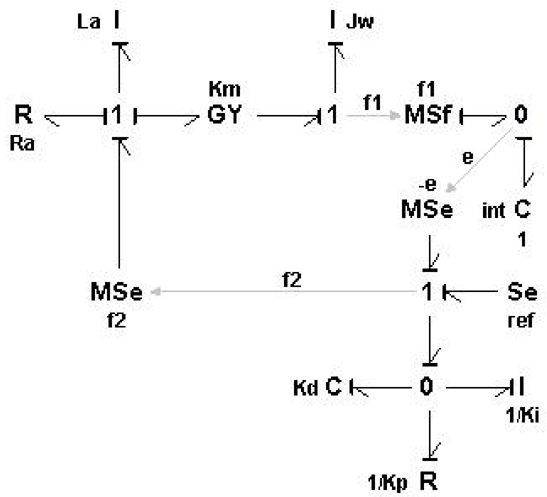

Figure 9: The bond graph model of the closed loop control system of the position of the electrical motor

The matrix form of the equations becomes:

$$\begin{bmatrix}\dot{P}_{L_a}\\ \dot{P}_{J_w}\\ \dot{Q}_{int}\\ \dot{P}_{K_i}\end{bmatrix}=\begin{bmatrix}-\frac{R_a}{L_a} & -\frac{K_d+K_m}{J_w} & -K_p & K_i\\ \frac{K_m}{L_a} & 0 & 0 & 0\\ 0 & \frac{1}{J_w} & 0 & 0\\ 0 & 0 & -1 & 0\end{bmatrix}\begin{bmatrix}P_{L_a}\\ P_{J_w}\\ Q_{int}\\ P_{K_i}\end{bmatrix}+\begin{bmatrix}K_p\\ 0\\ 0\\ 1\end{bmatrix}ref \quad (13)$$

4.3 Adaptive control of the electrical motor

In previous sections, the bond graph model for the electrical motor controlling system in two cases was designed, and the state equations were extracted. Now in this section it will be shown that, how these equations can be used in adaptive control of the system. It was seen that the gains of the PID, were present in the state equations. This specification can be used to designing a suitable PID controller for the system that satisfies the desired requirements. In the equation that contains the PID gains (Equations 5 and 9), in each moment if the system's state parameters and their derivatives be specified, it will be an equation with three unknown variables (PID gains). Thus, if this equation in three moments be established, the PID gains could be detected. Thus, if the curves of the changes of the state variables in a specified time interval be specified, the PID gains in each moment can be specified, such that satisfy these curves.

This operations for controlling the position option, is done here. Among the system's state variables, the controlled variable is Q_{int} that presents the position of the wheel. Thus, first its curve of the changes should be specified. Other system variables by defining initial values and knowing the value of Q_{int} could be detected in each moment. It is needed to define an equation for the position's curve, that satisfies the desired value for some of the system's parameters, such as settling time (t_s) and overshoot (M_p). This purpose can be performed using some control knowledge. If it is desired that the position follows a curve that has a specified overshoot, and finally becomes constant on the desired value in a specified time, this curve according to the time, can be defined by this equation [Trimmer, 1969]:

$$c(t) = ref(1 - \frac{e^{-\varepsilon \omega_n t}}{\sqrt{1-\varepsilon^2}} \sin(\omega_n \sqrt{1-\varepsilon^2} t + \cos^{-1} \varepsilon)) \quad (14)$$

In this equation ε is the *damping ratio*, and ω_n is the *undamped natural frequency*. The parameters settling time and overshoot, also can be defined according these two parameters as below:

$$t_s = 4T = \frac{4}{\varepsilon \omega_n} \quad \textit{(With 2\% tolerance)} \quad (15)$$

$$M_p = ref(1 + exp(-\frac{\varepsilon \pi}{\sqrt{1-\varepsilon^2}})) \quad (16)$$

Now by these equations and defining the settling time and overshoot, ε and ω_n can be obtained. Then using Equation 14, an equation for the position, according the time can be defined, that has the desired requirements. In this work it's desired to go from the position 0 to $\pi/3$ in 5 seconds and having an overshoot equal to 1.3. Using Equations 15 and 16, these values will be obtained for ε and ω_n:

$$\varepsilon \approx 0.4122 \quad , \quad \omega_n \approx 1.94$$

These values are used for the system's parameters:

$$L_a = 5\, mH$$
$$R_a = 0.1\, \Omega$$
$$K_m = 0.5$$
$$J_w = 0.07\, kgm^2$$

The initial values, for the state variables are set to zero. The result for the PID gains along the time is presented in Figure 10. It can be seen that the values, almost are constant along the time.

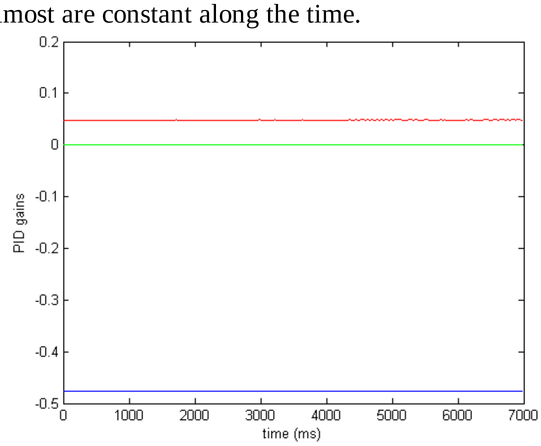

Figure 10: The gains of PID: K_p red, K_i green, K_d blue.

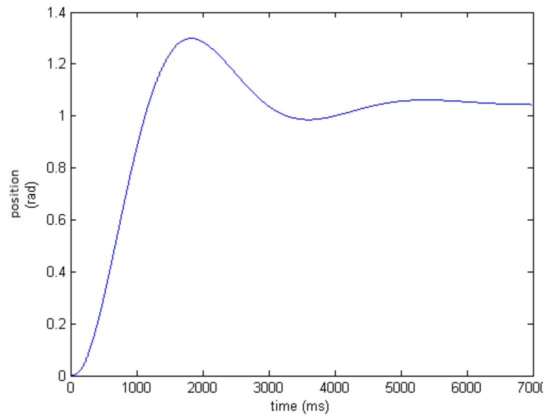

Figure 11: The position of the wheel along the time

Now the system can be tested by the obtained PID gains. The result is presented in Figure 11. It can be seen that the resulted gains, satisfy the desired requirements. Because the changes of the gains are very little, the average values of them can be used in whole time that almost has the same result.

The control method that was used here is an offline method because the position trajectory was predetermined. However, it can be used online too, because if the state variables in each moment and its previous moment, be known, and the desired values for them for the first future moment be specified too, by establishing equation 9 in these tree moments, the PID gains can be found for the first future moment.

5. Modeling Hydraulic system

In the previous sections a simple system (electrical motor) was modeled. Now in this section for another example, a more complicated system is investigated. The system that is considered is a bidirectional hydraulic system, that is controlled by a 5/3 valve. The schema of this system is shown in Figure 12. The valve that is used, works by magnetic force and a spring. It has 3 states consist of stop, left and right. It is shown in these 3 states in Figure 13. It has a spool that moves to the left and right by the force of the magnetic coil and the spring. It can be seen in the figure that in stop state, the input and two output ports, all are closed. When the spool moves to the left and right the input port becomes open to one of the pipes to the cylinder, and the other pipe becomes connected to the output. But the movement of the spool is rational. It means that a port in every moment is not essentially fully closed or open. The spool may be in such state that a percentage of the area of the port be open. Thus, by controlling the position of the spool, and therefore the open area of the port, the position of the piston can be controlled (for further details of hydraulic systems refer to [Shames, 2002]).

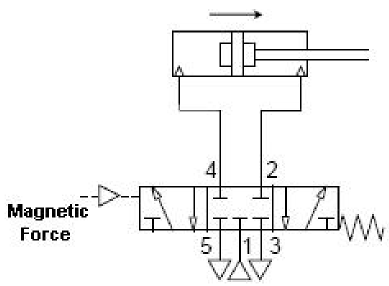

Figure 12: The hydraulic system

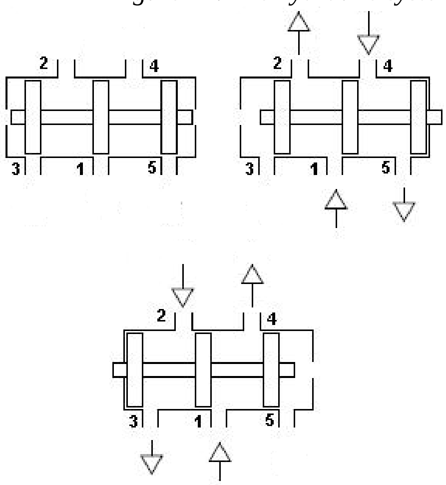

Figure 13: The used valve in three different states

5.1 Modeling closed-loop controlled system

Now that the performance of the system was explained, we try to model its closed-loop controlled system, by bond graph. The controlling of the system is performed by controlling the electrical current of the magnetic coil. The magnetic force that is applied to the valve is obtained by a coefficient multiplied to the electrical current. A PID controller is used to determine the value of the electrical current. In hydraulic systems the effort variable is the pressure (P) and the flow variable is the Debbie (Q). The bond graph model of the entire closed-loop system is presented in Figure 14.

The middle part of the figure presents the model of the valve. The flow of the one junction presents the velocity of the spool, that by obtaining its integral by the mentioned method the position of the spool can be found. But because here there is a C element for the spring, there is no need to use an additional C element (Q_{sp} presents the position of the spool). It can be seen that there is two sources of pressure (Se_1, Se_2). In every moment one of these sources is the input source, and the other is the output source. So for defining the switching of them in the model, two transformers are used that have conditional coefficients (n_1, n_2). Their values are defined as below:

$$n_1 = \begin{cases} 5 & if \quad Q_{sp} > 0 \\ 1 & if \quad Q_{sp} < 0 \end{cases} \qquad n_2 = \begin{cases} -1 & if \quad Q_{sp} > 0 \\ -5 & if \quad Q_{sp} < 0 \end{cases}$$

The value of n_2 is negative, because the second source moves the piston in the negative direction. When the absolute value of each of them is more, it means that this source is the input.

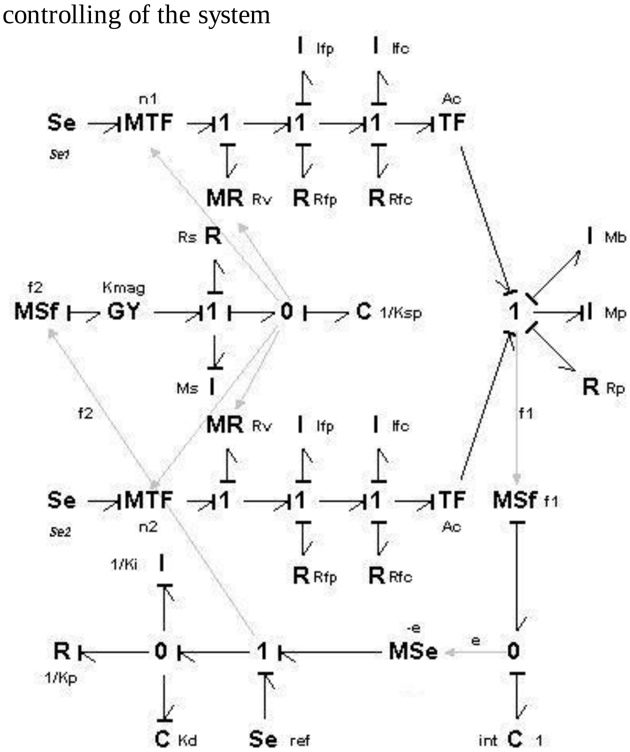

Figure 14: The bond graph model of closed-loop controlled hydraulic system

The pressure that is produced by the source is subjected to the reluctance of the valve (R_v), reluctance of the fluid in pipe and cylinder (R_{fp}, R_{fc}), and the inertia of the fluid in pipe and cylinder (I_{fp}, I_{fc}).These values are obtained by the following formulas:

$$R_v = \frac{1}{2} K\rho \frac{Q^2}{A^2} \quad (17)$$

$$R_f = \frac{f_f LQ}{DA^2} \quad \textit{Reluctance of fluid} \quad (18)$$

$$I_f = \frac{\rho L}{A} \quad \textit{Inertia of fluid} \quad (19)$$

In these equations A is the cross area, L is the length of the path and f_f is the friction coefficient of the fluid.

After these branches, the reminder of the effort (pressure) is converted to the force by a transformer that its coefficient is the cross area of the cylinder (A_c), because $F = P/A$. In continue, there is a one junction that two forces from the pressure sources are entered to it. It can be seen that, the output bonds from this junction are the reluctance of the friction of the piston (R_p), the mass of the piston (M_p) and the mass of the burden that is conveyed by the piston (M_b). The flow of this junction is the velocity of the piston, so its integral presents the position of the piston, that it is the variable that should be controlled. Taking integral from it is performed by the explained way and the effort of C_{int} presents the integral value (e). The desired or reference value of the system, is presented in the effort form that is subtracted from e and its output is delivered to the PID. As explained before, the flow of the junction before the PID presents the output of the PID (f_2). This value is transmitted to the source of flow that produces the electrical current of the coil.

Now the state equations of the system can be established. In this system there are 5 integral variables that are $Q_{sp}, P_s, Q_{int}, P_{K_i}$ and P_{pis}, that respectively are related to spring, spool, integrator of position, integral of PID and piston. The state equations will be obtained as below:

$$\dot{Q}_{sp} = \frac{P_s}{M_s} \quad (20)$$

$$\dot{P}_s = K_{mag}\left(K_i P_{K_i} + K_p(ref - Q_{int}) - \frac{K_d}{M_p} P_{pis}\right) - \frac{R_s}{M_s} P_s - K_{sp} Q_{sp} \quad (21)$$

$$\dot{Q}_{int} = \frac{P_{pis}}{M_p} \quad (22)$$

$$\dot{P}_{K_i} = ref - Q_{int} \quad (23)$$

$$\dot{P}_{pis} = \begin{cases} \frac{A_c}{\delta}(n_1 Se_1 - n_2 Se_2) - \frac{R_p}{M_p \delta} P_{pis} - \frac{(\frac{2\alpha}{Q_{sp}} + 2\beta + \gamma)A_c^2}{M_p \delta} P_{pis}^2 \ , \ P_{pis} > 0 \\ \frac{A_c}{\delta}(n_1 Se_1 - n_2 Se_2) - \frac{R_p}{M_p \delta} P_{pis} + \frac{(\frac{2\alpha}{Q_{sp}} + 2\beta + \gamma)A_c^2}{M_p \delta} P_{pis}^2 \ , \ P_{pis} < 0 \end{cases} \quad (24)$$

Where

$$\alpha = \frac{K\rho A_c}{2M_p D_p} \quad (25)$$

$$\beta = \frac{f_f L_p A_c}{D_p A_p^2 M_p} \quad (26)$$

$$\gamma = \frac{f_f L_c}{D_c A_c M_p} \quad (27)$$

$$\delta = 1 + \frac{\frac{2\rho L_p A_c^2}{A_p} + \rho A_c L_c + M_b}{M_p} \quad (28)$$

In these equations M_s is the mass of the spool, R_s is the reluctance of the friction of the spool, D_p is the diagonal of the pipe, L_p is the length of the pipe, A_p is the cross area of the pipe and D_c is the diagonal of the cylinder. In this work for simplicity the cross area of the input and output ports of the valve are considered in the square shape, so the open area of the valve is equal to $Q_{sp} D_p$. It is seen that in this system the state equations are nonlinear, and unlike the previous examples the matrix form of the equations can't be established.

5.2 Adaptive control of the hydraulic system

Now like the electrical motor example in this section adaptive control for the hydraulic system is performed to design a PID controller. Q_{int} presents the position of the piston and is the variable that should be controlled. Thus, like the previous example first a curve for its changes must be defined. This time we want define a curve that hasn't any overshoot. For this purpose ε must be equal to1 or more. But if it is exactly one in Equations 14 and 16 divide to zero will be occurred, and if it is more than one under the radical will be negative. Thus, for avoiding these problems it is considered near one (0.999). The desired settling time is 4 seconds and $\omega_n = 1.5$. The length of the piston is 1.5 meters and desired position is 1.15. If it is simulated using Equation 14, it can be seen that the response has the desired requirements (Figure 15).

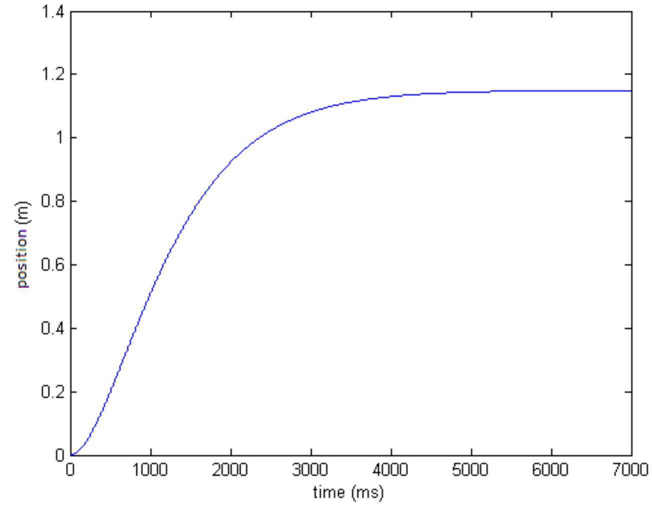

Figure 15: The position of the piston

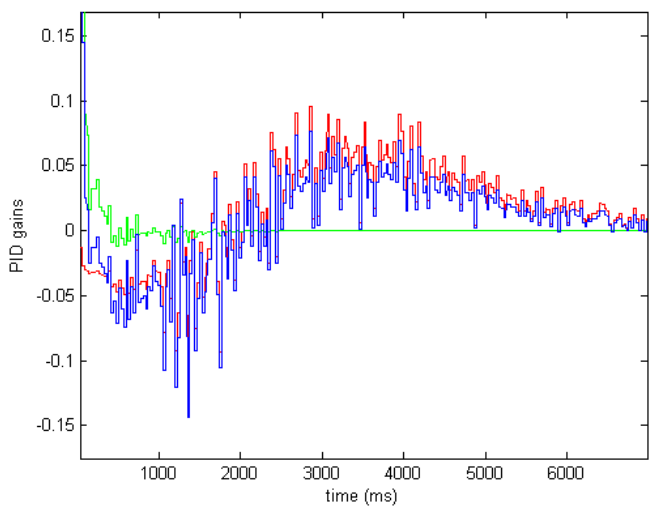

Figure 16: The gains of PID: K_p red, K_i green, K_d blue.

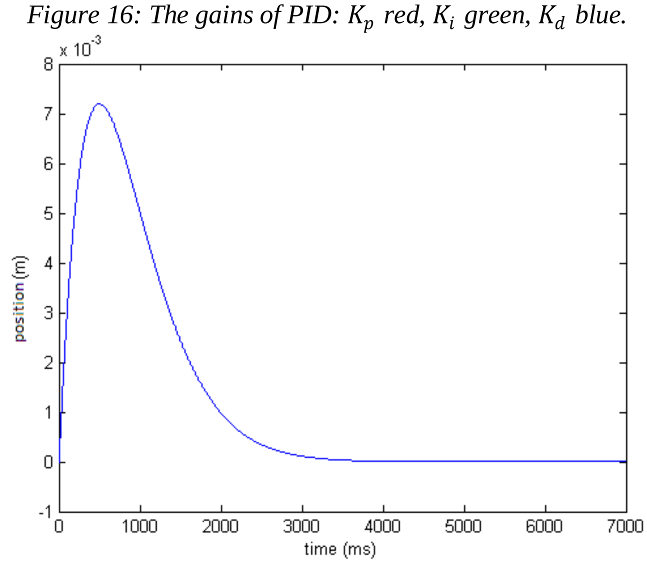

Figure 17: The movements of the spool along the time

The initial values for the system's state variables are considered 0. In each moment by knowing Q_{int} and its derivative, using Equations 22 and 23, P_{pis} and P_{K_i} can be obtained. Then using Equation 24 Q_{sp} will be obtained. Then by Equation 20 P_s is resulted (and then its derivative). Thus, in Equation 21 three unknown parameters will be remained that are the PID gains. Therefore by establishing this equation in three moments the gains can be found. These gains along the time are shown in Figure 16. It can be seen that despite the example of the electrical motor, this time the gains aren't constant along the time. In Figure 17 the movements of the spool is presented. As it was mentioned before, the cross area of the ports is considered in the square shape, and the length of its edge is 2 cm. It can be seen that the spool don't entirely open the input port in this operation.

In this example the advantages of the presented method is observed. A nonlinear hybrid system consist of solid mechanic, magnetic and hydraulic fields was modeled and required PID gains for gain schedule control of it, such that the desired output be produced, obtained. It was seen that using this method there is no need to linearize the system.

6. Conclusion

In this paper it was seen that first a new way for modeling the integrator and derivative of the effort and flow variables in bond graph modeling was presented. It has some useful effects. In many dynamical systems there is a need to the value of the integral and derivative of some system's variables, so obtaining the effort and flow variables as two key variables of various systems, is a useful thing. It was seen that by the presented way, these values can be obtained using bond graph elements, and it can be reused in the model. These specifications make it possible to model more various systems by bond graph, and increase the domain of bond graph usage. It was mentioned that one of the obvious instances of it, is PID controller, but it was modeled by bond graph, before. It was seen that by combining this method with our invented method a closed loop controlled system for various plants can be modeled entirely by bond graph, and it is one of its important usages. In continue for example two different plants that are controlled by PID were modeled by bond graph, and the system equations were extracted. It was seen that the PID gains are participated in these equations, and this specification allows us to use bond graph for designing PID for various systems. Thus, by using it bond graph can be used in adaptive control. It means that if the desired behavior of the system is defined and the closed loop system is modeled by bond graph, the PID gains along the time that satisfy the desired requirements can be obtained. Using this method adaptive control for nonlinear and hybrid systems can be performed without linearizing them. This task in this work was performed for two example systems.

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