# Lagrangian Particle Tracking: Model Development

M. Mahdavimanesh*[1], A.R. Noghrehabadi[2], M. Behbahaninejad [2], G. Ahmadi[3], M. Dehghanian[4]

[1]Department of Mechanical Engineering, Neyriz Branch, Islamic Azad University, Neyriz, Iran
[2] Department of Mechanical Engineering, Shahid Chamran University of Ahvaz, Ahvaz, Iran
[3] Department of Mechanical and Aeronautical Engineering, University of Clarkson, New York, US
[4] Department of Mechanical Engineering, Marvdasht Branch, Islamic Azad University, Marvdasht, Iran
mmmahdavimanesh@gmail.com, a.r.noghrehabadi@scu.ac.ir, bnmorteza@scu.ac.ir, gahmadi@clarkson.edu, dehghanian@miau.ac.ir

**Abstract:** The study of micro and nano particle-laden multiphase flow has received much attention due to its occurrence in a wide range of industrial and natural phenomena. Many of these flows are multi-dimensional systems involving strong mass, momentum and energy transfer between carrying fluid and particulate phase. The purpose of the present paper is to survey brief description of Eulerian-Lagrangian modeling of two-phase flow.

[M. Mahdavimanesh, A.R. Noghrehabadi, M. Behbahaninejad , G. Ahmadi, M. Dehghanian. **Lagrangian Particle Tracking: Model Development**. *Life Sci J* 2013;10(8s):34-41] (ISSN:1097-8135). http://www.lifesciencesite.com. 5

**Keywords:** Eulerian-Lagrangian framework; Turbulent two phase flow; Particle tracking; Two-way coupling

## 1. Introduction

A two-phase flow is defined as combination of continuous phase, e.g. gas or liquid, and disperse phase, e.g. particles or droplets. They are found in many industrial applications such as cyclone separators, jet mills, deposition in duct or pipe flows (most probably undesired deposition), dust precipitation or coating processes (desired deposition). They also occur in nature, e.g., desert sand storms, pollen in air and dispersion of pollutants. In order to model such a phenomenon, two theoretical approaches are considered, namely the Eulerian-Eulerian (two-fluids) and the Eulerian-Lagrangian approaches. The Eulerian description assumes continuum medium for discrete phase and solve conservative laws for both solid phase and gas phase [1-3]. With the rapid development of computational capabilities, the mixed Eulerian–Lagrangian approach attracted more and more attentions from many researchers. In this approach, the detailed particle motion behavior, which facilitates a better understanding of the physical phenomena, can be revealed by solving the Newtonian motion Equations in Lagrangian coordinate while solving continuity and momentum equations for continuum phase.

In this review we will focus on turbulent Lagrangian description of particle tracking to model two-phase flow and expose development process in modeling such phenomena.

## 2. Characteristics of Lagrangian description in turbulent flow

Empirical probes have shown that three impacts have to be allowed for, to predict particle dispersion precisely.

### *2.1 The inertia effect (IE)*

The experiments of Wells & Stock [3]and the direct numerical simulations of Squires & Eaton[4]indicate the possibility that the dispersion coefficient for heavy particles exceeds that for fluid particles. Reeks [5] and Pismen & Nir [6] have developed theories which predict that very heavy particles disperse more rapidly, in the long term, than fluid particles.

### *2.2 The crossing trajectory effect (CTE)*

This effect Udine [7],whereby particle dispersion is reduced in the presence of strong body forces due to particles rapidly passing through eddies.

### *2.3 The continuity effect (CE)*

The continuity effect Csanady [1] tells that the dispersion in the direction of the drift velocity exceeds the dispersion in the other two directions. The effect results from the fact that “longitudinal” and “transverse” length scales are different, which is in turn due to continuity equation of fluid motion [8].

## 3. Particle equation of motion

The surrounding fluid will interact with particles. The Lagrangian approach for the simulation of the disperse phase is based on

Newton's equation of motion. To date a number of such forces have been implemented in discrete particle simulations. The reader can refer to [9, 10] for the historical development of such forces. The equation of motion of particles is given by

$$m_p \frac{d\vec{u}^p}{dt} = \sum \vec{F} \quad (1)$$

Here, $F$ is sum of all forces that acting on particles which is represented in Table 1.

| Forces | Expression | References |
|---|---|---|
| Drag | $\vec{F}_{Drag} = \frac{\pi d_p^2}{8} \rho_f C_D \lvert \vec{u}-\vec{u}_p \rvert (\vec{u}-\vec{u}_p),\quad m_p = \frac{\pi}{6}\rho_p d_p^3,\quad C_D = \frac{24}{Re_p}(1.0+0.15\,Re^{0.687}),\quad Re_p = \frac{\rho_f \lvert \vec{u}-\vec{u}_p \rvert d_p}{\mu}$ | [9] |
| Gravity | $\vec{F}_{Gravity} = m_p \vec{g}$ | |
| Saffman | $\vec{F}_{saffman} = 1.61 d_p^2 (\mu\rho_f)^{1/2} \times \lvert \vec{\omega} \rvert^{-1/2} [(\vec{u}-\vec{u}_p)\times\vec{\omega}],\quad \vec{\omega} = \nabla\times\vec{u}$ | [11] |
| Magnus | $\vec{F}_{magnus} = \frac{\pi}{8} d_p^2 \rho_f \left[ (\frac{1}{2}\nabla\times\vec{u} - \nabla\times\vec{u}_p)\times(\vec{u}-\vec{u}_p) \right]$ | [12] |
| Virtual mass force | $\vec{F}_{VM} = C_{vm}\rho_f V_p \frac{d(\vec{u}-\vec{u}_p)}{dt}/2,\quad C_{Vm} = 2.1 - 0.132/(0.12 + A_c^2),\quad A_c = (\vec{u}-\vec{u}_p)^2/(d_p d(\vec{u}-\vec{u}_p)/dt)$ | [13] |
| Basset history | $\vec{F}_{Basset} = \frac{3}{2} d_p^2 \sqrt{\pi\rho_f \mu} \left[ \int_0^t \frac{d(\vec{u}-\vec{u}_p)}{dt}(t-t')^{-1/2}dt' + \frac{(\vec{u}-\vec{u}_p)_0}{\sqrt{t}} \right],\quad (\vec{u}-\vec{u}_p)_0 = \textit{initial velocity difference}$ | [14] |
| Brownian | $\vec{F}_{Brownian} = m_p \vec{G}\sqrt{\frac{\pi S_o}{\Delta t}},\quad So = \frac{216\upsilon k_{Boltzman}T}{\pi^2 \rho_f d_p^5 S^2 C_c},\quad C_c = 1 + \frac{2\lambda}{d_p}(1.257 + 0.4e^{-1.1d_p/2\lambda}),\quad \lambda = \upsilon(\frac{\pi M_G}{2RT_f})^1$ | [15] |
| Turbo-phoresis | $\vec{F}_{Turbo} = -\frac{1}{2}\rho_p \frac{\pi d_p^3}{6}\frac{\partial u_r'}{\partial r}$ | [16] |
| Thermo-phoresis | $\vec{F}_{Thermo} = -f\frac{C_T \upsilon}{T}\nabla^2 T,\quad f = \frac{3\pi\mu d_p}{C_c},\quad C_T = \frac{2.34(k_{Boltzman}/k_p + 2.18Kn)C_c}{(1+3.42Kn)(1+2k_{Boltzman}/k_p + 4.36Kn)}$ | [17, 18] |

*Table 1. Acting forces on particles*

The difference between the velocity of carrying fluid and of a particle moving in the carrying fluid causes the Drag force. The effect of gravity force on particle motion should be included, in the case where the free-fall velocity of particles and the velocity of carrying fluid are the same order of magnitude. The non-uniformity of the profile of averaged velocity of carrier fluid results in Saffman lifts force [11]. The Magnus force is due to the particle rotation. During particle motion in a fluid, particles of complex shape (a spherical) always rotate. The spherical particles will also rotate in a flow with a no uniform velocity profile. The added mass or virtual mass force is the inertia added to a particle because an accelerating or decelerating body must move some volume of surrounding fluid as it moves through it. The Basset history term is the drag caused by unsteady motion of the particle in a viscous medium. If the size of particle suspended in a fluid is very small, the motion of the particle affected by discrete nature of molecular motion of the fluid which is called the Brownian force. Turbo-phoresis force arises because of the non-uniformity of the profile of fluctuation velocity of carrier fluid. Thermo-phoresis force arises as a result of the non-uniformity of the temperature profile of carrier fluid.

## 4. Fluid-phase flow model

The conservation equations for the fluid flow are given by (in tensorial notation):

$$(\rho_f \phi)_{,t} + (\rho_f U_i \phi)_{,i} = (\Gamma \phi_{,i})_{,i} + S_\phi + S_{\phi p} \quad (2)$$

Where, $\rho_f$ is the gas density, $U_i$ are the Reynolds-averaged velocity components, and $\Gamma$ is an effective transport coefficient. The source terms $S_\phi$ and $S_{\phi p}$ are arisen from the transport equation and presence of particles, respectively. The $k$ (energy contained in velocity fluctuations)–ε (rate of transfer of kinetic fluctuation energy to heat by viscous friction) model is widely used for the simulation of turbulent fluid flows in practical applications. **Error! Reference source not found.** and Table 2. *Summary of terms in the general equation for the different variables that describe the fluid phase in Cartesian flow*demonstrate the different variables and source terms that describe the fluid phase; respectively. The direct influence of the dispersed phase on the continuous phase is usually taken into account by formulating appropriate source terms for all quantities under consideration. In the situation which mass loading of particles is low, the influence of particles on their carrier phase is negligible. Thus, the source terms that appear in the conservative equations as a result of particles are zero. In many situations, the mass loading particle to fluid ratio is too large to allow one to be satisfied with the one-way approach discussed in the previous subsection.

In the Euler-Lagrange approach the interaction between both phases requires an iterative solution procedure, which is usually called two-way coupling. The discrete form of Reynolds Stress Model equations and their relevant source terms were presented by Gouesbet and Berlemont[9] and Lain et al[10]. In addition to two-way coupling simulation, four way coupling simulation accounts for inter-particle collisions which was considered by many researchers [10, 19-23].

| | $\phi$ | $\Gamma$ | $S_\phi$ |
|---|---|---|---|
| Continuity | 1 | 0 | 0 |
| X-Momentum | $U$ | $\mu + \mu_t$ | $\frac{\partial}{\partial x}(\Gamma \frac{\partial U}{\partial x}) + \frac{\partial}{\partial y}(\Gamma \frac{\partial V}{\partial x}) + \frac{\partial}{\partial z}(\Gamma \frac{\partial W}{\partial x}) - \frac{\partial P}{\partial x} + \rho g_x$ |
| Y-Momentum | $V$ | $\mu + \mu_t$ | $\frac{\partial}{\partial x}(\Gamma \frac{\partial U}{\partial y}) + \frac{\partial}{\partial y}(\Gamma \frac{\partial V}{\partial y}) + \frac{\partial}{\partial z}(\Gamma \frac{\partial W}{\partial y}) - \frac{\partial P}{\partial y} + \rho g_y$ |
| Z-Momentum | $W$ | $\mu + \mu_t$ | $\frac{\partial}{\partial x}(\Gamma \frac{\partial U}{\partial z}) + \frac{\partial}{\partial y}(\Gamma \frac{\partial V}{\partial z}) + \frac{\partial}{\partial z}(\Gamma \frac{\partial W}{\partial z}) - \frac{\partial P}{\partial z} + \rho g_z$ |
| Turbulent Kinetic Energy | $k$ | $\mu + \frac{\mu_t}{\sigma_k}$ | $G_k - \rho\varepsilon$ |
| Viscous Dissipation Rate | $\varepsilon$ | $\mu + \frac{\mu_t}{\sigma_\varepsilon}$ | $\frac{\varepsilon}{k}(C_1 G_k - C_2 \rho\varepsilon)$ |
| Production of Turbulent Kinetic Energy | $G_k$ | $G_k = \mu_t \left\{ \begin{array}{l} 2\left[(\frac{\partial U}{\partial x})^2 + (\frac{\partial V}{\partial y})^2 + (\frac{\partial W}{\partial z})^2\right] + \\ (\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x})^2 + (\frac{\partial U}{\partial z} + \frac{\partial W}{\partial x})^2 + (\frac{\partial V}{\partial z} + \frac{\partial W}{\partial y})^2 \end{array} \right\}$ | |
| Eddy Viscosity | $\mu_t$ | $\mu_t = \rho c_\mu \frac{k^2}{\varepsilon}$ | |

*Table 2. Summary of terms in the general equation for the different variables that describe the fluid phase in Cartesian flow*

| | $\phi$ | $S_{\phi p}$ | **Reference** |
|---|---|---|---|
| Continuity | 1 | 0 | |
| X-Momentum | $U$ | $S_{Up} = -\sum_{k=1}^{N^p} [m_k N_K \Delta t_L \times \sum_{n=1}^{N^T} \{\frac{u_p)_k^{n+1} - u_p)_k^n}{\Delta t_L} - g_x (1 - \frac{\rho_f}{\rho_p})\}]$ | [9, 20-22, 24, 25] |
| Y-Momentum | $V$ | $S_{Vp} = -\sum_{k=1}^{N^p} [m_k N_k \Delta t_L \times \sum_{n=1}^{N^T} \{\frac{v_p)_k^{n+1} - v_p)_k^n}{\Delta t_L} - g_y (1 - \frac{\rho_f}{\rho_p})\}]$ | |
| Z-Momentum | $W$ | $S_{Wp} = -\sum_{k=1}^{N^p} [m_k N_k \Delta t_L \times \sum_{n=1}^{N^T} \{\frac{w_p)_k^{n+1} - w_p)_k^n}{\Delta t_L} - g_z (1 - \frac{\rho_f}{\rho_p})\}]$ | |
| Turbulent Kinetic Energy | $k$ | $S_{kp} = \overline{u_i^p S_{ui}} - U_i \bar{S}_{ui} = \overline{u^p S_{Up}} + \overline{v^p S_{Vp}} + \overline{w^p S_{Wp}} - (U \bar{S}_{Up} + V \bar{S}_{Vp} + W \bar{S}_{wp})$ | |
| Viscous Dissipation Rate | $\varepsilon$ | $S_{\varepsilon p} = C_3 \frac{\varepsilon}{k} S_k$ | |
| Constants | | $C_1 = 1.44;\ C_2 = 1.44;\ C_3 = 1.87;\ C_\mu = 0.09;\ \sigma_k = 1.0;\ \sigma_\varepsilon = 1.3$ | |

*Table 3.Fluid phase source terms in Cartesian flow*

## 5. Lagrangian modeling

A common feature of most Lagrangian methods used to date is the decomposition of the driving fluid velocity into the mean and the fluctuating parts. The mean fluid velocity field results from the continuous phase computation and is interpolated at particle locations. Then, a Lagrangian model gives the fluctuating component.

In order to develop stochastic models for the generation of fluctuations, one typically uses, together with particle parameters (such as $\tau_p$), a few quantities derived from the fluid variables, like the Eulerian length scale $L_E$ or the Lagrangian time scale $T_L$. The modeling of fluid fluctuation velocity is complicated, because of two effects that cause the fluid element and particle trajectories to differ: first, particle inertia that induces a relative instantaneous motion of particles with regard to their fluid neighborhood, and second, mean particle drift due to gravity.

### *5.1 Eddy Interaction Model*

Hutchinson et al [26]considered particles which have one-directional radial motion in a turbulent pipe flow. Their well-known model is called eddy life time model. The model uses a stochastic approach to predict the characteristic of discrete phase. The adequate characteristic of the eddy interaction model lies in its simplicity and the fact that the only statistics required by the model are representative length, time and velocity scales, whereas, in the autocorrelation models (will discuss in the next section), the forms of either the temporal (Lagrangian) or the spatial (Eulerian) velocity auto-correlation functions (or both of these) are required. In single eddy interaction model (SEIM) each of a number of individual particles is tracked through a series of interactions with fluid eddies whose length, "lifetime" and velocity can all be random variables. Their model modified by some researchers to account for various particles dispersion [27-29].

The effective eddy interaction time interval $t_i$ in which the velocities are kept constants are set to the minimum of the integral eddy life time $T_e$ and the time scale for the crossing of the eddies $t_c$ in order to account for the turbulence structure of the carrier phase as well as for the crossing trajectory effect[29]which is given by

$$t_c = -\tau_r \ln\left(1 - \frac{L_e}{\tau_r \left|\vec{u} - \vec{u}_p\right|}\right) \tag{3}$$

In this approach[30, 31]as shown in Figure, at the start of the interaction time between fluid and particle ($t = 0$), the particle will be assumed to be sitting at the center of the eddy with velocity of $U_{p0}$. During the interaction of eddy and non-fluid particle, instantaneous fluid velocity of eddy is remained constant in space and time within the eddy and is given by

$$u = U + u' \tag{4}$$

Where $u$ is the instantaneous eddy velocity, $U$ is the mean velocity and $u'$ is the fluctuating velocity which is given by

$$u' = \gamma_i \sqrt{\frac{2k}{3}} \tag{5}$$

Where $\gamma_i$ is the zero mean unit variance Gaussian random vector.

With proceed of time both the eddy and the non-fluid particle have moved in space. The eddy moves with its instantaneous fluid velocity while the non-fluid particle movement is governed by the Newtonian equation of motion (Eq. (1)). The non-fluid particle remains under the influence of that eddy until the interaction time exceeds the eddy lifetime ($T_e$), or distance between the center of eddy and non-fluid particle ($d(t_1)$) exceeds the eddy length $L_e$. So in this approach interaction time is given by

$$t_i = \min(T_e, t_c) \tag{6}$$

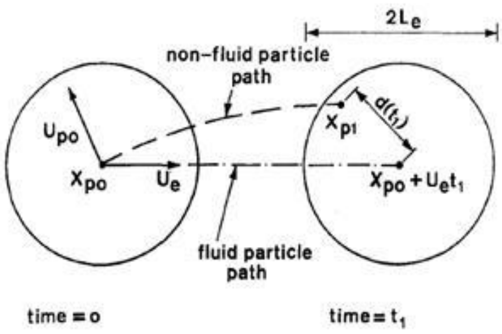


*Figure 1. Eddy interaction model (from[30, 31])*

Inertia and crossing trajectory and continuity effects have not been considered in original EIM. Some modifications have been taken to account by Graham[30, 31] to amend these deficiencies. The drawbacks of the method are twofold: a resulting velocity correlation coefficient is linear (rather than exponential) and the velocity record for a fluid particle is discontinuous (rather than continuous).

#### *5.1.1 Eddy scale determination*

As discussed earlier, to predict Particle motions in eddy interaction models three parameters are therefore determined: (i) eddy velocity, (ii) eddy lifetime and (iii) eddy length. Wang [32]have shown that the choice of eddy lifetime distribution in the eddy interaction model determines the form of the Lagrangian fluid velocity auto-correlation function.

Eddy scales used by Hutchinson and James et al[33]were obtained from empirical correlations using Laufer [34]pipe flow data. Gosman & Ioannides [27]assumed that the eddy length and lifetime are equal to the dissipation scales, given by

$$T_e = \sqrt{3/2}\, C_\mu^{3/4} \frac{k}{\varepsilon}, \quad L_e = C_\mu^{3/4} \frac{k^{3/2}}{\varepsilon} \tag{7}$$

Where $k$ and $\varepsilon$ is the turbulence kinetic energy and its rate of dissipation, respectively and $C_\mu = 0.09$.

Eddy scales in the near-wall turbulence studied by Kallio&Reeks[35]where determined from laws of similarity at the wall. Milojevic[29]proposed the following eddy length and time scales which is given by

$$T_e = C_T \frac{k}{\varepsilon}, \quad L_e = \sqrt{\frac{2}{3}} C_T \frac{k^{3/2}}{\varepsilon} \tag{8}$$

Typically a value of $C_T$ is in the range of 0.2 to 0.96.

Call & Kennedy[36] accounted for anisotropic turbulence by the use of a Reynolds stress turbulence model for the primary flow, thereby allowing for different eddy velocities in different coordinate directions. Graham & James[31]investigated the performance of different eddy interaction models with random length and time scales.

### *5.2 Auto-correlation method*

In analyzing turbulent two phase flow we almost deal with Lagrangian fluid velocity auto-correlation and Eulerian fluid velocity auto-correlation functions. These two functions represent correlations between fluctuations of velocity of carrier phase and have the following general form, respectively:

$$R_L(t,\tau) = \frac{\langle u_f(t) u_f(t+\tau)\rangle}{\langle u_f^2(t)\rangle} \tag{9}$$

$$R_E(x,t) = \frac{\langle u_f(x_0,t_0) u_f(x_0+x, t_0+t)\rangle}{\langle u_f^2(x_0,t_0)\rangle} \tag{10}$$

Where $u_f(t)$ and $u_f(t+\tau)$ are the fluid velocity at time $t$ and $t+\tau$, respectively and angled brackets indicate ensemble averaging over many such particles. Lagrangian velocity coloration function is one of the most important statistics of turbulent flows.

$R_L$ should satisfy some requirements as following requirements:

I. $\tau \to 0 \Rightarrow R_L \to 1, \quad \tau \to \infty \Rightarrow R_L \to 0$

II. $(dR_L/d\tau)_{\tau=0} = 0, \quad \left(d^2 R_L/d\tau^2\right)_{\tau=0} < 0$

Desjonqueres et al [37] proposed a Lagrangian method in which a correlation matrix is used in random process, which simulates the Lagrangian time correlations along the particle path. In the one dimensional formulation we have the following procedure in order to generate fluctuation velocities ([9, 38 and 39]):

Consider $U'$ as the fluctuating velocity in the different time steps as follow

$$U' = [u_x'(0), u_x'(\Delta t), ..., u_x'(i\,\Delta t), ..., u_x'(n\Delta t)] \tag{11}$$

Define uncorrelated zero mean and unit variance Gaussian vector $U'$ and $Y$ have the following relationship

$$U' = BY \tag{12}$$

There is a symmetric, positive-definite matrix $A$ which has the following relationship with $B$ as follow:

$$A = BB^T \tag{13}$$

Where, $B^T$ is the transpose matrix of $B$.

Element of matrix $A$ obtain simply by Frenkiel [40] family of the correlation as follow

$$a_{ij} = \exp\left(\frac{-|j-i|\Delta t}{(m^2+1)\tau_L}\right)\cos\left(\frac{m|j-i|\Delta t}{(m^2+1)\tau_L}\right) \tag{14}$$

where m is the loop parameter and m = 1 is a recommended value[9]. In the final step we use Cholesky factorization in order to $B$ obtain from $A$.

### *5.3 Lagrangian temporal construction Model*

Lu et al [41] proposed a Lagrangian model which represents satisfactory results in compared to experimental results. In their model, at time t, the particle and a corresponding fluid point start out from the same position $X_S$ (Figure 2). After one time step, they arrive at $X_f$ and $X_P$, respectively, and the distance between them is $\Delta s$. The relative coordinate system O-ΞΩΘ is chosen such that its original point, O, is located at the position $X_f$, and the Θ-axis passes through the position $X_P$ ([41-43]).

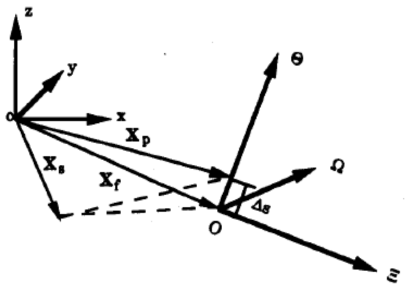


***Figure 2.*** *The locations of the particle and a fluid particle (from[42])*

With the aid of time series analysis, the normalized fluctuating component in the location of particle in the *i*-direction of relative coordinate system O-ΞΩΘ is obtained by (no summation convention is used):

$$W_i(X_P) = a_i b_i W_i(X_S) + \Psi_i \quad (i = 1,\ 2,\ 3) \tag{15}$$

Where

$$W_i = \frac{u_i'}{\sqrt{u_i'^2}} \quad (i = 1,\ 2,\ 3) \tag{16}$$

In Eq. (15), $\Psi_i$ are mean zero, $\sqrt{1-(a_i b_i)^2}$ variance Gaussian random numbers and $a_i$, $b_i$ are

Lagrangian auto-correlation and Eulerian spatial velocity correlations functions, respectively which they for example pick from the Frenkeil [40]family of auto-correlations as follow

$$a_i = \exp(\frac{-\Delta t}{\tau_i^L}),\ \ b_i = \exp(\frac{-\Delta s}{2\Lambda_i})\cos(\frac{\Delta s}{2\Lambda_i}),\ \ (i = 1, \quad (17)$$

Where are $\Lambda_i$ the length scales and $\tau_i^L$ are the Lagrangian time scales.

### *5.4 Kraichnan Fourier modes*

Kraichnan [44]suggested a simple method for generating a random field which resembles a pseudo-isotropic turbulence. In this approach fluctuation velocities are obtained as below

$$\vec{u}'(\overrightarrow{X_p},t) = \sqrt{\frac{2}{N}}\left\{\sum_{n=1}^{N}\vec{u}_1(\overrightarrow{k_n})\cdot\cos(\overrightarrow{k_n}.\ \overrightarrow{X_p}+\omega_n t)\right. \quad (18)$$

Where

$$\vec{u}_1(\overrightarrow{k_n}) = \vec{\zeta}_n \times \overrightarrow{k_n},\ \vec{u}_2(\overrightarrow{k_n}) = \vec{\xi}_n \times \overrightarrow{k_n} \quad (19)$$

And

$$\overrightarrow{k_n}.\ \vec{u}_1(\overrightarrow{k_n}) = \overrightarrow{k_n}.\ \overrightarrow{u_2}(\overrightarrow{k_n}) = 0 \quad (20)$$

Satisfy the incompressibility condition. The components of vectors $\overrightarrow{\xi_n}$ , $\overrightarrow{\zeta_n}$ and the values of frequency $\omega_n$ , were picked independently from a three or two dimensional Gaussian distribution with a standard deviation of unity. Each component of $\overrightarrow{k_n}$ , is a Gaussian random number with a standard deviation that depends on energy spectrum [44]. Here, N is the number of terms in the series that usually is considered as 100.

Some modifications were taken to account for anisotropy effects [21, 22 and 45].

### *5.5 Models based on the Langevin equation*

A following stochastic differential equation has been proposed to model the behavior of fluid velocities

$$\frac{du_i}{dt} = -\frac{u_i - U_i}{T_L} + (\frac{2\overline{u_i'^2}}{T_L})^{1/2}\psi_i(t) \quad (21)$$

Here, $\psi_i(t)$ is the Wiener process (white noise);it is a stochastic process of zero mean, $\langle\psi_i(t)\rangle = 0$ , a variance equal to the time interval $\langle(\psi_i(t))^2\rangle = dt$. The above is the Langevin equation first proposed to model the Brownian motion; in that context, it represents the equation of motion of a small particle in surrounding fluid [46, 47]. Some models presented by Pozorski & Minier [46] which account for main three effects in Lagrangian simulation, i.e. IE, CTE and CE.

## References


[1] Csanady, G.T., *"Turbulent Diffusion of Heavy Particles in the Atmosphere"*. 1963. p. 201-208.

[2] Tu, J.Y. and C.A. Fletcher, *"Numerical Computation of Turbulent Gas-Solid Particle Flow in 90 Bend "*. AIChE, 1995. 41(1): p. 2187-2196.

[3] Wells, M.R. and D.E. Stock, *"The Effects of Crossing Trajectories on the Dispersion of Particles in a Turbulent Flow"*. 1983, Cambridge Journals Online. p. 31-62.

[4] Squires, K.D. and J.K. Eaton, *"Measurements of Particle Dispersion Obtained from Direct Numerical Simulations of Isotropic Turbulence"*. 1991, Cambridge Journals Online. p. 1-35.

[5] Reeks, M.W., *"On the Dispersion of Small Particles Suspended in an Isotropic Turbulent Fluid"*. 1977, Cambridge Journals Online. p. 529-546.

[6] Pismen, L.M. and A. Nir, *"On the Motion of Suspended Particles in Stationary Homogeneous Turbulence"*. 1978, Cambridge Journals Online. p. 193-206.

[7] Yudine, M.I., "*Physical Considerations on Heavy-Particle Diffusion*", in *Advances in Geophysics*, H.E. Landsberg and J.V. Mieghem, Editors. 1959, Elsevier. p. 185-191.

[8] Hinze, J.O., *"Turbulence"*. 1975, New York: McGraw Hill.

[9] Gouesbet, G. and A. Berlemont, *"Eulerian and Lagrangian Approaches for Predicting the Behaviour of Discrete Particles in Turbulent Flows"*. Progress in Energy and Combustion Science, 1999. 25: p. 133–159.

[10] Lain, S., M. Sommerfeld, and J. Kussin, *"Experimental Studies and Modelling of Four-Way Coupling in Particle-Laden Horizontal Channel Flow"*. Journal of Heat and Fluid Flow, 2002. 23(1): p. 647–656.

[11] Saffman, P.G., *"Lift on a Small Sphere in a Slow Shear Flow"*. Journal of Fluid Mechanics, 1965. 22(1): p. 385–400.

[12] Rubinow, S.I. and J.B. Keller, *"The Transverse Force on a Spinning Sphere Moving in a Viscous Fluid"*. Journal of Fluid Mechanics, 1961. 11(1): p. 447–459.

[13] Odar, F. and W.S. Hamilton, *"Forces on a Sphere Accelerating in a Viscous Fluid"*. Journal of Fluid Mechanics, 1964. 18(1): p. 302–314.

[14] Reeks, M.W. and S. Mckee, *"The Dispersive Effects of Basset History*

*Forces on Particle Motion in a Turbulent Flow"*. Physics of Fluids, 1984. 27(1): p. 1573–1582.
[15] Ounis, H., G. Ahmadi, and J.B. McLaughlin, *"Dispersion and Deposition of Brownian Particles from Point Sources in a Simulated Turbulent Channel Flow"*. Journal of Colloid and Interface Science, 1991. 147(1): p. 233-250.
[16] Mednikov, E.P., *"Turbulentnyi Perenos I Osazhdenie Aerozolei (Turbulent Transport and Sedimentation of Aerosols)"*, in *Nauka*. 1981: Moscow.
[17] Soo, S.L., H.K. Ihrig, and A.F. El Kouh, *"Experimental Determination of Statistical Properties of Two-Phase Turbulent Motion"*. journal of ASME, 1960. 82(3): p. 609.
[18] Talbot, L., et al., *"Thermophoresis of Particles in a Heated Boundary Layer"*. Journal of Fluid Mechanics, 1980. 101(1): p. 737–758.
[19] Crowe, C.T., M. Sommerfeld, and Y. Tsuji, *"Multiphase Flows with Droplets and Particles"*. Vol. 1. 1997: CRC Press. 471.
[20] Lain, S., et al., *"Modelling Hydrodynamics and Turbulence in a Bubble Column Using the Euler–Lagrange Procedure"*. Journal of Multiphase Flow, 2002. 28: p. 1381–1407.
[21] Mansoori, Z., et al., *"Inter-Particle Heat Transfer in a Riser of Gas–Solid Turbulent Flows"*. Powder Technology, 2005. 159: p. 35 – 45.
[22] Nasr, H. and G. Ahmadi, *"The Effect of Two-Way Coupling and Inter-Particle Collisions on Turbulence Modulation in a Vertical Channel Flow"*. Journal of Heat and Fluid Flow, 2007. 28: p. 1507–1517.
[23] Saffar-Avval, M., et al., *"Gas–Solid Turbulent Flow and Heat Transfer with Collision Effect in a Vertical Pipe"*. Journal of Thermal Sciences, 2007. 46: p. 67–75.
[24] Lain, S., D. Bröder, and M. Sommerfeld, *"Experimental and Numerical Studies of the Hydrodynamics in a Bubble Column"*. Chemical Engineering Science, 1999. 54(21): p. 4913-4920.
[25] Lain, S. and M. Sommerfeld, *"Turbulence Modulation in Dispersed Two-Phase Flow Laden with Solids from a Lagrangian Perspective"*. Journal of Heat and Fluid Flow, 2003. 24: p. 616–625.
[26] Hutchinson, P., G.F. Hewitt, and A.E. Dukler, *"Deposition of Liquid or Solid Dispersions from Turbulent Gas Streams: A Stochastic Model"*. Chemical Engineering Journal, 1970. 26(1): p. 419-439.
[27] Gosman, A., D. and E. Ioannides, *"Aspects of Computer Simulation of Liquid-Fueled Combustors"*. Journal of Energy, 1983. 7(6): p. 482-490.
[28] Graham, D.I., *"On the Inertia Effect in Eddy Interaction Models"*. International Journal of Multiphase Flow, 1996. 22(1): p. 177-184.
[29] Milojevic, D., *"Lagrangian Stochastic-Deterministic (Lsd) Prediction of Particle Dispersion in Turbulence"*. Part. Part. Syst. Charact, 1990. 7: p. 181-190.
[30] Graham, D.I. and P.W. James, *"Turbulent Dispersion of Particles Using Eddy Interaction Models"*. International Journal of Multiphase Flow, 1996. 22(1): p. 157-175.
[31] Graham, D.I. and P.W. James, *" Dispersion of Solid Particles in Turbulent Flow through Pipe Expansion"*. Journal of Applied Mathematics in business & Industry, 1996. 7: p. 149-179.
[32] Wang, L.-P. and D.E. Stock, *"Stochastic Trajectory Models for Turbulent Diffusion: Monte Carlo Process Versus Markov Chains"*. Atmospheric Environment. Part A. General Topics, 1992. 26(9): p. 1599-1607.
[33] James, P.W., G.F. Hewitt, and P.B. Whalley, *"Droplet Motion in Two-Phase Flow"*, in *Proceedings of American Nuclear Society Topical Meeting on Reactor Thermohydraulics*. 1980: Saratoga Springs, NY. p. 1484-1503.
[34] Laufer, J., *"The Structure of Turbulence in a Fully Developed Pipe Flow"*. 1954, Naca Technical Report.
[35] Kallio, G.A. and M.W. Reeks, *"A Numerical Simulation of Particle Deposition in Turbulent Boundary Layers"*. International Journal of Multiphase Flow, 1989. 15(3): p. 433-446.
[36] Call, C.J. and I.M. Kennedy, *"Measurements and Simulations of Particle Dispersion in a Turbulent Flow"*. International Journal of Multiphase Flow, 1992. 18(6): p. 891-903.
[37] Desjonqueres, P., A. Berlemont, and G. Gouesbet, *"A Lagrangian Approach for the Prediction of Particle Dispersion in Turbulent Flows"*. Journal of Aerosol Science, 1988. 19(1): p. 99-103.
[38] Berlemont, A., P. Desjonqueres, and G. Gouesbet, *"Particle Lagrangian Simulation in Turbulent Flows"*. Journal of Muhiphase Flow, 1990. 16(1): p. 19-34.
[39] Berlemont, A., M.S. Grancher, and G. Gouesbet, *"Heat and Mass Transfer*

*Coupling between Vaporizing Droplets and Turbulence Using a Lagrangian Approach"*. International Journal of Heat and Mass Transfer, 1995. 38(16): p. 3023-3034.
[40] Frenkiel, F.N., *"Etude Statistique De La Turbulence-Fontions Spectrales Et Coefficients De Correlation"*. Rapport Technique, 1948. 34.
[41] Lu, Q.Q., J.R. Fontaine, and G. Aubertin, *"Particle Motion in Two-Dimensional Confined Turbulent Flows"*. Aerosol Science and Technology, 1992. 17(3): p. 169-185.
[42] Lu, Q.Q., J.R. Fontaine, and G. Aubertin, *"A Lagrangian Model for Solid Particles in Turbulent Flows"*. International Journal of Multiphase Flow, 1993. 19(2): p. 347-367.
[43] Lu, Q.Q., J.R. Fontaine, and G. Aubertin, *"Numerical Study of the Solid Particle Motion in Grid-Generated Turbulent Flows"*. International Journal of Heat and Mass Transfer, 1993. 36(1): p. 79-87.
[44] Kraichnan, R.H., *"Diffusion by a Random Velocity Field"*. Physics of Fluids, 1969. 13(1): p. 22-31.
[45] Li, A. and G. Ahmadi, *"Dispersion and Deposition of Spherical Particles from Point Sources in a Turbulent Channel Flow"*. Aerosol Science and Technology, 1992. 16(4): p. 209-226.
[46] Pozorski, J. and J.P. Minier, *"On the Lagrangian Turbulent Dispersion Models Based on the Langevin Equation"*. Journal of Multiphase Flow, 1998. 24: p. 913-945.
[47] Tian, L. and G. Ahmadi, *"Particle Deposition in Turbulent Duct Flows-Comparisons of Different Model Predictions"*. Journal of Aerosol Science, 2007. 38(4): p. 377-397.

4/2/2013